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On Some Fractional Systems of Difference Equations

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ABSTRACT. This paper deals with the solutions of the systems of difference equations

$$x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} \pm y_{n-3}y_n \pm y_n x_{n-2}}, y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} \pm x_{n-1}}, n \in \mathbb{N}_0,$$

where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, and initial values $x_{-2}, x_{-1}, x_0, y_{-3}, y_{-2}, y_{-1}, y_0$ are non-zero real numbers.

Keywords: System of difference equations, Form of the solutions, Periodicity.

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1. INTRODUCTION

Difference equations enter as approximations of continuous problems and as models describing life situations in many directions. Recently there has been a great interest in studying difference equations, see, for instance [1]-[12], [14], [17]-[19] and references cited therein, as well as in studying systems of difference equations (see, e.g. [13], [15], [16]).

In this paper we will investigate the solutions of the systems of difference equations

$$x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} \pm y_{n-3}y_n \pm y_n x_{n-2}}, y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} \pm x_{n-1}}, n \in \mathbb{N}_0,$$

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where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, and initial values $x_{-2}, x_{-1}, x_0, y_{-3}, y_{-2}, y_{-1}, y_0$ are non-zero real numbers.

Definition 1.1. A sequence $\{x_n\}_{n \geq -k}$ is said to be eventually periodic with period $p \in \mathbb{N}$ if there is an $n_0 \geq -k$ such that $x_{n+p} = x_n$, for all $n \geq n_0$. If $n_0 = -k$, we say that the sequence $\{x_n\}_{n \geq -k}$ is periodic with period p .

Definition 1.2. Let $\{F_n\}_{n \geq 0} = \{0, 1, 1, 2, 3, 5, 8, \dots\}$ be the Fibonacci sequence defined by

$$F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n, n \in \mathbb{N}_0.$$

Definition 1.3. Let $\{U_n\}_{n \geq 0} = \{0, 2, 4, 14, 40, \dots\}$ be the sequence defined by

$$U_0 = 0, U_{n+1} + U_n = 2 \cdot 3^n, n \in \mathbb{N}_0.$$

$$2. \text{ THE SYSTEM: } x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} + y_{n-3}y_n + y_n x_{n-2}}, y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} - x_{n-1}}$$

In this section, we study the solutions of the system of two difference equations

$$x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} + y_{n-3}y_n + y_n x_{n-2}}, y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} - x_{n-1}}, n \in \mathbb{N}_0, \quad (2.1)$$

with non-zero real initials conditions such that $y_{-3}x_{-2} + y_{-3}y_0 + y_0x_{-2} \neq 0$, $\frac{x_{-1}}{y_{-2}}, \frac{x_0}{y_{-1}}$ and $\frac{y_{-3}x_{-2}}{y_{-3}x_{-2} + y_{-3}y_0 + y_0x_{-2}} \notin \{(-1)^{n+1}U_n, n \geq 1\}$.

The following theorem is devoted to the form of the solutions of the system (2.1).

Theorem 2.1. Let $\{x_n\}_{n \geq -2}, \{y_n\}_{n \geq -3}$ be a solution of (2.1). Then, for $n \geq 0$, we have,

$$\begin{aligned} x_{3n-1} &= x_{-1} \left(\frac{1}{3}\right)^n, x_{3n} = x_0 \left(\frac{1}{3}\right)^n, x_{3n+1} = \frac{y_{-3}y_0 x_{-2}}{y_{-3}x_{-2} + y_{-3}y_0 + y_0x_{-2}} \left(\frac{1}{3}\right)^n, \\ y_{3n-2} &= \frac{y_{-2}x_{-1}}{y_{-2}U_n + (-1)^n x_{-1}}, y_{3n-1} = \frac{y_{-1}x_0}{y_{-1}U_n + (-1)^n x_0}, \\ y_{3n} &= \frac{y_{-3}y_0 x_{-2}}{(y_{-3}x_{-2} + y_0y_{-3} + y_0x_{-2})U_n + (-1)^n y_{-3}x_{-2}}. \end{aligned}$$

Proof. Form (2.1), we have

$$x_1 = \frac{y_{-3}y_0 x_{-2}}{y_{-3}x_{-2} + y_{-3}y_0 + y_0x_{-2}}.$$

So, the result holds for $n = 0$. Suppose that $n \geq 1$ and that our assumption holds for $n - 1$ that is,

$$x_{3n-4} = x_{-1} \left(\frac{1}{3}\right)^{n-1}, \quad (2.2)$$

$$x_{3n-3} = x_0 \left(\frac{1}{3}\right)^{n-1}, \quad (2.3)$$

$$x_{3n-2} = \frac{y_{-3}y_0x_{-2}}{y_{-3}x_{-2} + y_{-3}y_0 + y_0x_{-2}} \left(\frac{1}{3}\right)^n, \quad (2.4)$$

$$y_{3n-5} = \frac{y_{-2}x_{-1}}{y_{-2}U_{n-1} + (-1)^{n-1}x_{-1}}, \quad (2.5)$$

$$y_{3n-4} = \frac{y_{-1}x_0}{y_{-1}U_{n-1} + (-1)^{n-1}x_0}, \quad (2.6)$$

$$y_{3n-3} = \frac{y_{-3}y_0x_{-2}}{(y_{-3}x_{-2} + y_0y_{-3} + y_0x_{-2})U_{n-1} + (-1)^{n-1}y_{-3}x_{-2}}. \quad (2.7)$$

From (2.1) and (2.2), we have

$$\begin{aligned} x_{3n-1} &= \frac{y_{3n-5}y_{3n-2}x_{3n-4}}{y_{3n-5}x_{3n-4} + y_{3n-5}y_{3n-2} + y_{3n-2}x_{3n-4}} \\ &= \frac{y_{3n-5} \left(\frac{y_{3n-5}x_{3n-4}}{2y_{3n-5} - x_{3n-4}}\right) x_{3n-4}}{y_{3n-5}x_{3n-4} + y_{3n-5} \left(\frac{y_{3n-5}x_{3n-4}}{2y_{3n-5} - x_{3n-4}}\right) + \left(\frac{y_{3n-5}x_{3n-4}}{2y_{3n-5} - x_{3n-4}}\right) x_{3n-4}} \\ &= \frac{1}{3}x_{3n-4} = x_{-1} \left(\frac{1}{3}\right)^n. \end{aligned}$$

From (2.1) and (2.3), we get

$$\begin{aligned} x_{3n} &= \frac{y_{3n-4}y_{3n-1}x_{3n-3}}{y_{3n-4}x_{3n-3} + y_{3n-4}y_{3n-1} + y_{3n-1}x_{3n-3}} \\ &= \frac{y_{3n-4} \left(\frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} - x_{3n-3}}\right) x_{3n-3}}{y_{3n-4}x_{3n-3} + y_{3n-4} \left(\frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} - x_{3n-3}}\right) + \left(\frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} - x_{3n-3}}\right) x_{3n-3}} \\ &= \frac{1}{3}x_{3n-3} = x_0 \left(\frac{1}{3}\right)^n. \end{aligned}$$

From (2.1) and (2.4), we get

$$\begin{aligned} x_{3n+1} &= \frac{y_{3n-3}y_3x_{3n-2}}{y_{3n-3}x_{3n-2} + y_{3n-3}y_3 + y_3x_{3n-2}} \\ &= \frac{y_{3n-3} \left(\frac{y_{3n-3}x_{3n-2}}{2y_{3n-3} - x_{3n-2}}\right) x_{3n-2}}{y_{3n-3}x_{3n-2} + y_{3n-3} \left(\frac{y_{3n-3}x_{3n-2}}{2y_{3n-3} - x_{3n-2}}\right) + \left(\frac{y_{3n-3}x_{3n-2}}{2y_{3n-3} - x_{3n-2}}\right) x_{3n-2}} \\ &= \frac{1}{3}x_{3n-2} = \frac{y_{-3}y_0x_{-2}}{y_{-3}x_{-2} + y_{-3}y_0 + y_0x_{-2}} \left(\frac{1}{3}\right)^n. \end{aligned}$$

From (2.1), (2.2) and (2.5), we have

$$\begin{aligned}
y_{3n-2} &= \frac{y_{3n-5}x_{3n-4}}{2y_{3n-5} - x_{3n-4}} = \frac{\left(\frac{y_{-2}x_{-1}}{y_{-2}U_{n-1}+(-1)^{n-1}x_{-1}}\right)\left(\frac{x_{-1}}{3^{n-1}}\right)}{2\left(\frac{y_{-2}x_{-1}}{y_{-2}U_{n-1}+(-1)^{n-1}x_{-1}}\right) - \left(\frac{x_{-1}}{3^{n-1}}\right)} \\
&= \frac{y_{-2}x_{-1}}{2 \cdot 3^{n-1}y_{-2} - y_{-2}U_{n-1} + (-1)^n x_{-1}} = \frac{y_{-2}x_{-1}}{y_{-2}(2 \cdot 3^{n-1} - U_{n-1}) + (-1)^n x_{-1}} \\
&= \frac{y_{-2}x_{-1}}{y_{-2}U_n + (-1)^n x_{-1}}.
\end{aligned}$$

From (2.1), (2.3) and (2.6), we get

$$\begin{aligned}
y_{3n-1} &= \frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} - x_{3n-3}} = \frac{\left(\frac{y_{-1}x_0}{y_{-1}U_{n-1}+(-1)^{n-1}x_0}\right)\left(\frac{x_0}{3^{n-1}}\right)}{2\left(\frac{y_{-1}x_0}{y_{-1}U_{n-1}+(-1)^{n-1}x_0}\right) - \left(\frac{x_0}{3^{n-1}}\right)} \\
&= \frac{y_{-1}x_0}{2 \cdot 3^{n-1}y_{-1} - y_{-1}U_{n-1} + (-1)^n x_0} = \frac{y_{-1}x_0}{y_{-1}(2 \cdot 3^{n-1} - U_{n-1}) + (-1)^n x_0} \\
&= \frac{y_{-1}x_0}{y_{-1}U_n + (-1)^n x_0}.
\end{aligned}$$

From (2.1), (2.4) and (2.7), we have

$$\begin{aligned}
y_{3n} &= \frac{y_{3n-3}x_{3n-2}}{2y_{3n-3} - x_{3n-2}} \\
&= \frac{\left(\frac{y_{-3}y_0x_{-2}}{(y_{-3}x_{-2}+y_0y_{-3}+y_0x_{-2})U_{n-1}+(-1)^{n-1}y_{-3}x_{-2}}\right)\left(\frac{y_{-3}y_0x_{-2}}{3^{n-1}(y_{-3}x_{-2}+y_{-3}y_0+y_0x_{-2})}\right)}{2\left(\frac{y_{-3}y_0x_{-2}}{(y_{-3}x_{-2}+y_0y_{-3}+y_0x_{-2})U_{n-1}+(-1)^{n-1}y_{-3}x_{-2}}\right) - \left(\frac{y_{-3}y_0x_{-2}}{3^{n-1}(y_{-3}x_{-2}+y_{-3}y_0+y_0x_{-2})}\right)} \\
&= \frac{y_{-3}y_0x_{-2}}{(2 \cdot 3^{n-1} - U_{n-1})(y_0x_{-2} + y_0y_{-3} + x_{-2}y_{-3}) + (-1)^n x_{-2}y_{-3}} \\
&= \frac{y_{-3}y_0x_{-2}}{U_n(y_0x_{-2} + y_0y_{-3} + x_{-2}y_{-3}) + (-1)^n x_{-2}y_{-3}}.
\end{aligned}$$

The proof is complete. \square

Corollary 2.2. *Every solution of system (2.1) is such that*

$$\lim_{n \rightarrow +\infty} x_n = \lim_{n \rightarrow +\infty} y_n = 0.$$

$$3. \text{ THE SYSTEM: } x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} + y_{n-3}y_n + y_n x_{n-2}}, \quad y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} + x_{n-1}}$$

In this section, we get the form of the solutions of the system

$$x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} + y_{n-3}y_n + y_n x_{n-2}}, \quad y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} + x_{n-1}}, \quad n \in \mathbb{N}_0, \quad (3.1)$$

with non-zero real initials conditions such that $\frac{y_{-1}}{x_0}, \frac{y_{-2}}{x_{-1}} \notin \left\{ -\frac{F_{3n}}{F_{3n+1}}, n \geq 1 \right\}$

and $\frac{y_{-3}y_0+y_0x_{-2}}{y_{-3}x_{-2}} \notin \left\{ -\frac{F_{3n+2}}{F_{3n+1}}, n \geq 0 \right\}$.

Theorem 3.1. Let $\{x_n\}_{n \geq -2}$, $\{y_n\}_{n \geq -3}$ be a solution of (3.1). Then for $n = 0, 1, \dots$,

$$\begin{aligned} x_{3n} &= \frac{x_0 y_{-1}}{y_{-1} F_{3n+1} + x_0 F_{3n}}, \quad x_{3n+1} = \frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} F_{3n+2} + y_0 (y_{-3} + x_{-2}) F_{3n+1}}, \\ x_{3n+2} &= \frac{x_{-1} y_{-2}}{y_{-2} F_{3n+4} + x_{-1} F_{3n+3}}, \quad y_{3n} = \frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} F_{3n+1} + y_0 (y_{-3} + x_{-2}) F_{3n}}, \\ y_{3n+1} &= \frac{y_{-2} x_{-1}}{y_{-2} F_{3n+3} + x_{-1} F_{3n+2}}, \quad y_{3n+2} = \frac{y_{-1} x_0}{y_{-1} F_{3n+3} + x_0 F_{3n+2}}. \end{aligned}$$

Proof. From (3.1), we have

$$x_1 = \frac{y_{-3} y_0 x_{-2}}{y_{-3} x_{-2} + y_{-3} y_0 + y_0 x_{-2}} = \frac{y_{-3} y_0 x_{-2}}{y_{-3} x_{-2} F_2 + y_0 (y_{-3} + x_{-2}) F_1}, \quad (3.2)$$

$$y_1 = \frac{y_{-2} x_{-1}}{2y_{-2} + x_{-1}} = \frac{y_{-2} x_{-1}}{y_{-2} F_3 + x_{-1} F_2}, \quad (3.3)$$

$$y_2 = \frac{y_{-1} x_0}{2y_{-1} + x_0} = \frac{y_{-1} x_0}{y_{-1} F_3 + x_0 F_2}. \quad (3.4)$$

From (3.1)-(3.4), we get

$$\begin{aligned} x_2 &= \frac{y_{-2} y_1 x_{-1}}{y_{-2} x_{-1} + y_{-2} y_1 + y_1 x_{-1}} = \frac{y_{-2} \left(\frac{y_{-2} x_{-1}}{2y_{-2} + x_{-1}} \right) x_{-1}}{y_{-2} x_{-1} + y_{-2} \left(\frac{y_{-2} x_{-1}}{2y_{-2} + x_{-1}} \right) + \left(\frac{y_{-2} x_{-1}}{2y_{-2} + x_{-1}} \right) x_{-1}} \\ &= \frac{x_{-1} y_{-2}}{3y_{-2} + 2x_{-1}} = \frac{x_{-1} y_{-2}}{y_{-2} F_4 + x_{-1} F_3}, \end{aligned}$$

$$\begin{aligned} x_3 &= \frac{y_{-1} y_2 x_0}{y_{-1} x_0 + y_{-1} y_2 + y_2 x_0} = \frac{y_{-1} \left(\frac{y_{-1} x_0}{2y_{-1} + x_0} \right) x_0}{y_{-1} x_0 + y_{-1} \left(\frac{y_{-1} x_0}{2y_{-1} + x_0} \right) + \left(\frac{y_{-1} x_0}{2y_{-1} + x_0} \right) x_0} \\ &= \frac{x_0 y_{-1}}{3y_{-1} + 2x_0} = \frac{x_0 y_{-1}}{y_{-1} F_4 + x_0 F_3}, \end{aligned}$$

$$\begin{aligned} y_3 &= \frac{y_0 x_1}{2y_0 + x_1} = \frac{y_0 \left(\frac{y_{-3} y_0 x_{-2}}{y_{-3} x_{-2} + y_{-3} y_0 + y_0 x_{-2}} \right)}{2y_0 + \frac{y_{-3} y_0 x_{-2}}{y_{-3} x_{-2} + y_{-3} y_0 + y_0 x_{-2}}} = \frac{y_{-3} y_0 x_{-2}}{3y_{-3} x_{-2} + 2y_0 (y_{-3} + x_{-2})} \\ &= \frac{y_{-3} y_0 x_{-2}}{y_{-3} x_{-2} F_4 + y_0 (y_{-3} + x_{-2}) F_3}. \end{aligned}$$

Similarly we get

$$\begin{aligned} x_4 &= \frac{y_{-3} y_0 x_{-2}}{5x_{-2} y_{-3} + 3y_0 x_{-2} + 3y_0 y_{-3}} = \frac{y_{-3} y_0 x_{-2}}{y_{-3} x_{-2} F_5 + y_0 (y_{-3} + x_{-2}) F_4}, \\ y_4 &= \frac{x_{-1} y_{-2}}{8y_{-2} + 5x_{-1}} = \frac{y_{-2} x_{-1}}{y_{-2} F_6 + x_{-1} F_5}, \\ x_5 &= \frac{x_{-1} y_{-2}}{13y_{-2} + 8x_{-1}} = \frac{x_{-1} y_{-2}}{y_{-2} F_7 + x_{-1} F_6}, \\ y_5 &= \frac{x_0 y_{-1}}{8y_{-1} + 5x_0} = \frac{y_{-1} x_0}{y_{-1} F_6 + x_0 F_5}. \end{aligned}$$

So, the result holds for $n = 0, 1$. Suppose that $n \geq 2$ and that our assumption holds for $n - 2, n - 1$ that is,

$$x_{3n-6} = \frac{x_0 y_{-1}}{y_{-1} F_{3n-5} + x_0 F_{3n-6}}, \quad (3.5)$$

$$x_{3n-5} = \frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} F_{3n-4} + y_0 (y_{-3} + x_{-2}) F_{3n-5}}, \quad (3.6)$$

$$x_{3n-4} = \frac{x_{-1} y_{-2}}{y_{-2} F_{3n-2} + x_{-1} F_{3n-3}}, \quad (3.7)$$

$$y_{3n-6} = \frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} F_{3n-5} + y_0 (y_{-3} + x_{-2}) F_{3n-6}}, \quad (3.8)$$

$$y_{3n-5} = \frac{y_{-2} x_{-1}}{y_{-2} F_{3n-3} + x_{-1} F_{3n-4}}, \quad (3.9)$$

$$y_{3n-4} = \frac{y_{-1} x_0}{y_{-1} F_{3n-3} + x_0 F_{3n-4}}, \quad (3.10)$$

$$x_{3n-3} = \frac{x_0 y_{-1}}{y_{-1} F_{3n-2} + x_0 F_{3n-3}}, \quad (3.11)$$

$$x_{3n-2} = \frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} F_{3n-1} + y_0 (y_{-3} + x_{-2}) F_{3n-2}}, \quad (3.12)$$

$$x_{3n-1} = \frac{x_{-1} y_{-2}}{y_{-2} F_{3n+1} + x_{-1} F_{3n}}, \quad (3.13)$$

$$y_{3n-3} = \frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} F_{3n-2} + y_0 (y_{-3} + x_{-2}) F_{3n-3}}, \quad (3.14)$$

$$y_{3n-2} = \frac{y_{-2} x_{-1}}{y_{-2} F_{3n} + x_{-1} F_{3n-1}}, \quad (3.15)$$

$$y_{3n-1} = \frac{y_{-1} x_0}{y_{-1} F_{3n} + x_0 F_{3n-1}}. \quad (3.16)$$

From (3.1), (3.10), (3.11) and (3.16), we get

$$\begin{aligned} x_{3n} &= \frac{y_{3n-4} y_{3n-1} x_{3n-3}}{y_{3n-4} x_{3n-3} + y_{3n-4} y_{3n-1} + y_{3n-1} x_{3n-3}} \\ &= \frac{x_0 y_{-1}}{F_{3n} y_{-1} + x_0 F_{3n-1} + x_0 F_{3n-3} + x_0 F_{3n-4} + F_{3n-2} y_{-1} + F_{3n-3} y_{-1}} \\ &= \frac{x_0 y_{-1}}{y_{-1} (F_{3n} + (F_{3n-2} + F_{3n-3})) + x_0 (F_{3n-1} + (F_{3n-3} + F_{3n-4}))} \\ &= \frac{x_0 y_{-1}}{y_{-1} (F_{3n} + F_{3n-1}) + x_0 (F_{3n-1} + F_{3n-2})} \\ &= \frac{x_0 y_{-1}}{y_{-1} F_{3n+1} + x_0 F_{3n}}. \end{aligned}$$

From (3.1), (3.12) and (3.14), we have

$$\begin{aligned}
 y_{3n} &= \frac{y_{3n-3}x_{3n-2}}{2y_{3n-3} + x_{3n-2}} \\
 &= \frac{y_{-3}y_0x_{-2}}{y_0x_{-2}(2F_{3n-2} + F_{3n-3}) + y_0y_{-3}(2F_{3n-2} + F_{3n-3}) + x_{-2}y_{-3}(2F_{3n-1} + F_{3n-2})} \\
 &= \frac{y_{-3}y_0x_{-2}}{y_0x_{-2}(F_{3n-2} + F_{3n-1}) + y_0y_{-3}(F_{3n-2} + F_{3n-1}) + x_{-2}y_{-3}(F_{3n-1} + F_{3n})} \\
 &= \frac{y_{-3}y_0x_{-2}}{y_0x_{-2}F_{3n} + y_0y_{-3}F_{3n} + x_{-2}y_{-3}F_{3n+1}}.
 \end{aligned}$$

From (3.1), (3.12) and (3.14), we get

$$\begin{aligned}
 x_{3n+1} &= \frac{y_{3n-3}y_{3n}x_{3n-2}}{y_{3n-3}x_{3n-2} + y_{3n-3}y_{3n} + y_{3n}x_{3n-2}} \\
 &= \frac{y_{3n-3} \left(\frac{y_{3n-3}x_{3n-2}}{2y_{3n-3} + x_{3n-2}} \right) x_{3n-2}}{y_{3n-3}x_{3n-2} + y_{3n-3} \left(\frac{y_{3n-3}x_{3n-2}}{2y_{3n-3} + x_{3n-2}} \right) + \left(\frac{y_{3n-3}x_{3n-2}}{2y_{3n-3} + x_{3n-2}} \right) x_{3n-2}} \\
 &= \frac{y_{3n-3}x_{3n-2}}{3y_{3n-3} + 2x_{3n-2}} \\
 &= \frac{\left(\frac{y_0y_{-3}x_{-2}}{y_{-3}x_{-2}F_{3n-2} + y_0(y_{-3} + x_{-2})F_{3n-3}} \right) \left(\frac{y_0y_{-3}x_{-2}}{y_{-3}x_{-2}F_{3n-1} + y_0(y_{-3} + x_{-2})F_{3n-2}} \right)}{3 \left(\frac{y_0y_{-3}x_{-2}}{y_{-3}x_{-2}F_{3n-2} + y_0(y_{-3} + x_{-2})F_{3n-3}} \right) + 2 \left(\frac{y_0y_{-3}x_{-2}}{y_{-3}x_{-2}F_{3n-1} + y_0(y_{-3} + x_{-2})F_{3n-2}} \right)} \\
 &= \frac{y_{-3}y_0x_{-2}}{y_{-3}x_{-2}(3F_{3n-1} + 2F_{3n-2}) + y_0(y_{-3} + x_{-2})(3F_{3n-2} + 2F_{3n-3})} \\
 &= \frac{y_{-3}y_0x_{-2}}{y_{-3}x_{-2}(F_{3n-1} + 2F_{3n}) + y_0(y_{-3} + x_{-2})(F_{3n-2} + 2F_{3n-1})} \\
 &= \frac{y_{-3}y_0x_{-2}}{y_{-3}x_{-2}(F_{3n+1} + F_{3n}) + y_0(y_{-3} + x_{-2})(F_{3n} + F_{3n-1})} \\
 &= \frac{y_{-3}y_0x_{-2}}{y_{-3}x_{-2}F_{3n+2} + y_0(y_{-3} + x_{-2})F_{3n+1}}.
 \end{aligned}$$

From (3.1), (3.13) and (3.15), we have

$$\begin{aligned}
 y_{3n+1} &= \frac{y_{3n-2}x_{3n-1}}{2y_{3n-2} + x_{3n-1}} \\
 &= \frac{\left(\frac{y_{-2}x_{-1}}{y_{-2}F_{3n} + x_{-1}F_{3n-1}} \right) \left(\frac{x_{-1}y_{-2}}{y_{-2}F_{3n+1} + x_{-1}F_{3n}} \right)}{2 \left(\frac{y_{-2}x_{-1}}{y_{-2}F_{3n} + x_{-1}F_{3n-1}} \right) + \frac{x_{-1}y_{-2}}{y_{-2}F_{3n+1} + x_{-1}F_{3n}}} \\
 &= \frac{y_{-2}x_{-1}}{y_{-2}(F_{3n} + 2F_{3n+1}) + x_{-1}(2F_{3n} + F_{3n-1})} \\
 &= \frac{y_{-2}x_{-1}}{y_{-2}(F_{3n+2} + F_{3n+1}) + x_{-1}(F_{3n} + F_{3n+1})} \\
 &= \frac{y_{-2}x_{-1}}{y_{-2}F_{3n+3} + x_{-1}F_{3n+2}}
 \end{aligned}$$

From (3.1), (3.13) and (3.15), we get

$$\begin{aligned}
x_{3n+2} &= \frac{y_{3n-2}y_{3n+1}x_{3n-1}}{y_{3n-2}x_{3n-1} + y_{3n-2}y_{3n+1} + y_{3n+1}x_{3n-1}} \\
&= \frac{y_{3n-2} \left(\frac{y_{3n-2}x_{3n-1}}{2y_{3n-2} + x_{3n-1}} \right) x_{3n-1}}{y_{3n-2}x_{3n-1} + y_{3n-2} \left(\frac{y_{3n-2}x_{3n-1}}{2y_{3n-2} + x_{3n-1}} \right) + \left(\frac{y_{3n-2}x_{3n-1}}{2y_{3n-2} + x_{3n-1}} \right) x_{3n-1}} \\
&= \frac{y_{3n-2}x_{3n-1}}{3y_{3n-2} + 2x_{3n-1}} \\
&= \frac{\left(\frac{y-2x-1}{y-2F_{3n}+x-1F_{3n-1}} \right) \left(\frac{x-1y-2}{y-2F_{3n+1}+x-1F_{3n}} \right)}{3 \left(\frac{y-2x-1}{y-2F_{3n}+x-1F_{3n-1}} \right) + 2 \left(\frac{x-1y-2}{y-2F_{3n+1}+x-1F_{3n}} \right)} \\
&= \frac{y-2x-1}{y-2(2F_{3n} + 3F_{3n+1}) + x-1(3F_{3n} + 2F_{3n-1})} \\
&= \frac{y-2x-1}{y-2(2F_{3n+2} + F_{3n+1}) + x-1(F_{3n} + 2F_{3n+1})} \\
&= \frac{y-2x-1}{y-2(F_{3n+2} + F_{3n+3}) + x-1(F_{3n+2} + F_{3n+1})} \\
&= \frac{y-2x-1}{y-2F_{3n+4} + x-1F_{3n+3}}.
\end{aligned}$$

From (3.1), (3.10), (3.11) and (3.16), we have

$$\begin{aligned}
y_{3n+2} &= \frac{y_{3n-1}x_{3n}}{2y_{3n-1} + x_{3n}} = \frac{y_{3n-1} \left(\frac{y_{3n-4}y_{3n-1}x_{3n-3}}{y_{3n-4}x_{3n-3} + y_{3n-4}y_{3n-1} + y_{3n-1}x_{3n-3}} \right)}{2y_{3n-1} + \frac{y_{3n-4}y_{3n-1}x_{3n-3}}{y_{3n-4}x_{3n-3} + y_{3n-4}y_{3n-1} + y_{3n-1}x_{3n-3}}} \\
&= \frac{y_{3n-4}y_{3n-1}x_{3n-3}}{2x_{3n-3}y_{3n-1} + 3x_{3n-3}y_{3n-4} + 2y_{3n-1}y_{3n-4}} \\
&= \frac{y-1x_0}{y-1(F_{3n} + 2F_{3n+1}) + x_0(F_{3n-1} + 2F_{3n})} \\
&= \frac{y-1x_0}{y-1(F_{3n+2} + F_{3n+1}) + x_0(F_{3n+1} + F_{3n})} \\
&= \frac{y-1x_0}{y-1F_{3n+3} + x_0F_{3n+2}}.
\end{aligned}$$

The proof is complete. \square

Corollary 3.2. Let $\{x_n\}_{n \geq -2}$, $\{y_n\}_{n \geq -3}$ be a solution of (3.1) with $y_{-2}x_{-1}$, $y_{-1}x_0$ and $y_{-3}x_{-2}(y_{-3}y_0 + y_0x_{-2})$ positive. Then

$$\lim_{n \rightarrow +\infty} x_n = \lim_{n \rightarrow +\infty} y_n = 0.$$

4. THE SYSTEM $x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} - y_{n-3}y_n - y_n x_{n-2}}$, $y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} + x_{n-1}}$

In this section we study the dynamic of the solutions of the system

$$x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} - y_{n-3}y_n - y_n x_{n-2}}, \quad y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} + x_{n-1}}, \quad n = 0, 1, \dots, \quad (4.1)$$

with non-zero real initials conditions with $y_{-3}x_{-2} - y_0(y_{-3} + x_{-2}) \neq 0$, $\frac{y_{-2}}{x_{-1}}$, $\frac{y_{-1}}{x_0}$ and $\frac{y_{-3}x_{-2} - y_0(y_{-3} + x_{-2})}{y_{-3}x_{-2}} \notin \left\{ -\frac{1}{2n}, n \geq 1 \right\}$.

Theorem 4.1. Let $\{x_n\}_{n \geq -2}$, $\{y_n\}_{n \geq -3}$ be a solution of system (4.1), then $\{x_n\}_{n \geq -2}$ is eventually periodic. That is,

$$x_{n+3} = x_n, n \geq -1.$$

Moreover, if

$$x_{-2} = \frac{2y_0y_{-3}}{y_{-3} - y_0}, y_{-3} - y_0 \neq 0,$$

then the sequence $\{x_n\}_{n \geq -2}$ will be periodic.

Proof. It follows from (4.1) that

$$\begin{aligned} x_{n+3} &= \frac{y_{n-1}y_{n+2}x_n}{y_{n-1}x_n - y_{n-1}y_{n+2} - y_{n+2}x_n} \\ &= \frac{y_{n-1} \left(\frac{y_{n-1}x_n}{2y_{n-1} + x_n} \right) x_n}{y_{n-1}x_n - y_{n-1} \left(\frac{y_{n-1}x_n}{2y_{n-1} + x_n} \right) - \left(\frac{y_{n-1}x_n}{2y_{n-1} + x_n} \right) x_n} = \frac{\frac{y_{n-1}^2 x_n^2}{x_n + 2y_{n-1}}}{\frac{y_{n-1}^2 x_n}{x_n + 2y_{n-1}}} \\ &= x_n. \end{aligned}$$

Now using

$$x_{-2} = \frac{2y_0y_{-3}}{y_{-3} - y_0},$$

we get

$$x_1 = x_{-2}. \quad \square$$

Theorem 4.2. Let $\{x_n\}_{n \geq -2}$, $\{y_n\}_{n \geq -3}$ be a solution of (4.1). Then for $n = 0, 1, 2, \dots$, we have

$$\begin{aligned} x_{3n} &= x_0, \\ x_{3n+1} &= \frac{y_0y_{-3}x_{-2}}{y_{-3}x_{-2} - y_0(y_{-3} + x_{-2})}, \\ x_{3n+2} &= x_{-1}, \\ y_{3n} &= \frac{y_0y_{-3}x_{-2}}{(2n+1)y_{-3}x_{-2} - 2ny_0(y_{-3} + x_{-2})}, \\ y_{3n+1} &= \frac{y_{-2}x_{-1}}{2(n+1)y_{-2} + x_{-1}}, \\ y_{3n+2} &= \frac{y_{-1}x_0}{2(n+1)y_{-1} + x_0}. \end{aligned}$$

Proof. From (4.1), we have

$$x_1 = \frac{y_0y_{-3}x_{-2}}{y_{-3}x_{-2} - y_0(y_{-3} + x_{-2})}, x_2 = x_{-1},$$

and

$$y_1 = \frac{y_{-2}x_{-1}}{2y_{-2} + x_{-1}}, y_2 = \frac{y_{-1}x_0}{2y_{-1} + x_0}.$$

So, the result holds for $n = 0$. Suppose $n > 0$ and that our assumption holds for $n - 1$ that is,

$$x_{3n-3} = x_0, \quad (4.2)$$

$$x_{3n-2} = \frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} - y_0 (y_{-3} + x_{-2})}, \quad (4.3)$$

$$x_{3n-1} = x_{-1}, \quad (4.4)$$

$$y_{3n-3} = \frac{y_0 y_{-3} x_{-2}}{(2n-1)y_{-3} x_{-2} - 2(n-1)y_0 (y_{-3} + x_{-2})}, \quad (4.5)$$

$$y_{3n-2} = \frac{y_{-2} x_{-1}}{2n y_{-2} + x_{-1}}, \quad (4.6)$$

$$y_{3n-1} = \frac{y_{-1} x_0}{2n y_{-1} + x_0}. \quad (4.7)$$

From (4.1) and (4.2), we have

$$\begin{aligned} x_{3n} &= \frac{y_{3n-4} y_{3n-1} x_{3n-3}}{y_{3n-4} x_{3n-3} - y_{3n-4} y_{3n-1} - y_{3n-1} x_{3n-3}} \\ &= \frac{y_{3n-4} \left(\frac{y_{3n-4} x_{3n-3}}{2y_{3n-4} + x_{3n-3}} \right) x_{3n-3}}{y_{3n-4} x_{3n-3} - y_{3n-4} \left(\frac{y_{3n-4} x_{3n-3}}{2y_{3n-4} + x_{3n-3}} \right) - \left(\frac{y_{3n-4} x_{3n-3}}{2y_{3n-4} + x_{3n-3}} \right) x_{3n-3}} \\ &= \frac{\frac{(y_{3n-4} x_{3n-3})^2}{2y_{3n-4} + x_{3n-3}}}{\frac{y_{3n-4}^2 x_{3n-3}}{2y_{3n-4} + x_{3n-3}}} \\ &= x_{3n-3} \\ &= x_0. \end{aligned}$$

From (4.1), (4.3) and (4.5), we get

$$\begin{aligned} y_{3n} &= \frac{y_{3n-3} x_{3n-2}}{2y_{3n-3} + x_{3n-2}} \\ &= \frac{\left(\frac{y_0 y_{-3} x_{-2}}{(2n-1)y_{-3} x_{-2} - 2(n-1)y_0 (y_{-3} + x_{-2})} \right) \left(\frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} - y_0 (y_{-3} + x_{-2})} \right)}{2 \left(\frac{y_0 y_{-3} x_{-2}}{(2n-1)y_{-3} x_{-2} - 2(n-1)y_0 (y_{-3} + x_{-2})} \right) + \frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} - y_0 (y_{-3} + x_{-2})}} \\ &= \frac{\frac{(y_{-3} y_0 x_{-2})^2}{[(2n-1)y_{-3} x_{-2} - 2(n-1)y_0 (y_{-3} + x_{-2})][y_{-3} x_{-2} - y_0 (y_{-3} + x_{-2})]}}{\frac{(x_{-2} y_{-3} y_0)[(2n+1)x_{-2} y_{-3} - 2n y_0 (x_{-2} + y_{-3})]}{[y_0 x_{-2} + y_0 y_{-3} - x_{-2} y_{-3}][(2n-1)y_{-3} x_{-2} - 2(n-1)y_0 (y_{-3} + x_{-2})]}} \\ &= \frac{y_{-3} y_0 x_{-2}}{(2n+1)x_{-2} y_{-3} - 2n y_0 (x_{-2} + y_{-3})}. \end{aligned}$$

From (4.1) and (4.3), we have

$$\begin{aligned}
 x_{3n+1} &= \frac{y_{3n-3}y_{3n}x_{3n-2}}{y_{3n-3}x_{3n-2} - y_{3n-3}y_{3n} - y_{3n}x_{3n-2}} \\
 &= \frac{y_{3n-3} \left(\frac{y_{3n-3}x_{3n-2}}{2y_{3n-3}+x_{3n-2}} \right) x_{3n-2}}{y_{3n-3}x_{3n-2} - y_{3n-3} \left(\frac{y_{3n-3}x_{3n-2}}{2y_{3n-3}+x_{3n-2}} \right) - \left(\frac{y_{3n-3}x_{3n-2}}{2y_{3n-3}+x_{3n-2}} \right) x_{3n-2}} \\
 &= \frac{\frac{(y_{3n-3}x_{3n-2})^2}{2y_{3n-3}+x_{3n-2}}}{\frac{y_{3n-3}^2 x_{3n-2}}{2y_{3n-3}+x_{3n-2}}} \\
 &= x_{3n-2} \\
 &= \frac{y_0 y_{-3} x_{-2}}{y_{-3} x_{-2} - y_0 (y_{-3} + x_{-2})}.
 \end{aligned}$$

From (4.1), (4.4) and (4.6), we have

$$\begin{aligned}
 y_{3n+1} &= \frac{y_{3n-2}x_{3n-1}}{2y_{3n-2} + x_{3n-1}} = \frac{\left(\frac{y_{-2}x_{-1}}{2ny_{-2}+x_{-1}} \right) x_{-1}}{2 \left(\frac{y_{-2}x_{-1}}{2ny_{-2}+x_{-1}} \right) + x_{-1}} = \frac{\frac{y_{-2}x_{-1}^2}{2ny_{-2}+x_{-1}}}{\frac{(2(n+1)y_{-2}+x_{-1})x_{-1}}{2ny_{-2}+x_{-1}}} \\
 &= \frac{y_{-2}x_{-1}}{2(n+1)y_{-2} + x_{-1}}.
 \end{aligned}$$

From (4.1) and (4.4), we get

$$\begin{aligned}
 x_{3n+2} &= \frac{y_{3n-2}y_{3n+1}x_{3n-1}}{y_{3n-2}x_{3n-1} - y_{3n-2}y_{3n+1} - y_{3n+1}x_{3n-1}} \\
 &= \frac{y_{3n-2} \left(\frac{y_{3n-2}x_{3n-1}}{2y_{3n-2}+x_{3n-1}} \right) x_{3n-1}}{y_{3n-2}x_{3n-1} - y_{3n-2} \left(\frac{y_{3n-2}x_{3n-1}}{2y_{3n-2}+x_{3n-1}} \right) - \left(\frac{y_{3n-2}x_{3n-1}}{2y_{3n-2}+x_{3n-1}} \right) x_{3n-1}} \\
 &= \frac{\frac{(y_{3n-2}x_{3n-1})^2}{2y_{3n-2}+x_{3n-1}}}{\frac{y_{3n-2}^2 x_{3n-1}}{2y_{3n-2}+x_{3n-1}}} \\
 &= x_{3n-1} \\
 &= x_{-1}.
 \end{aligned}$$

Finally, from (4.1), (4.2) and (4.5), we have

$$\begin{aligned}
 y_{3n+2} &= \frac{y_{3n-1}x_{3n}}{2y_{3n-1} + x_{3n}} \\
 &= \frac{y_{3n-1} \left(\frac{y_{3n-4}y_{3n-1}x_{3n-3}}{y_{3n-4}x_{3n-3} - y_{3n-4}y_{3n-1} - y_{3n-1}x_{3n-3}} \right)}{2y_{3n-1} + \left(\frac{y_{3n-4}y_{3n-1}x_{3n-3}}{y_{3n-4}x_{3n-3} - y_{3n-4}y_{3n-1} - y_{3n-1}x_{3n-3}} \right)} \\
 &= \frac{y_{3n-1} \left(\frac{y_{3n-4} \left(\frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} + x_{3n-3}} \right) x_{3n-3}}{y_{3n-4}x_{3n-3} - y_{3n-4} \left(\frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} + x_{3n-3}} \right) - \left(\frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} + x_{3n-3}} \right) x_{3n-3}} \right)}{2y_{3n-1} + \left(\frac{y_{3n-4} \left(\frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} + x_{3n-3}} \right) x_{3n-3}}{y_{3n-4}x_{3n-3} - y_{3n-4} \left(\frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} + x_{3n-3}} \right) - \left(\frac{y_{3n-4}x_{3n-3}}{2y_{3n-4} + x_{3n-3}} \right) x_{3n-3}} \right)} \\
 &= \frac{y_{3n-1}x_{3n-3}}{2y_{3n-1} + x_{3n-3}} \\
 &= \frac{\left(\frac{y_{-1}x_0}{2ny_{-1} + x_0} \right) x_0}{2 \left(\frac{y_{-1}x_0}{2ny_{-1} + x_0} \right) + x_0} \\
 &= \frac{\frac{y_{-1}x_0^2}{2ny_{-1} + x_0}}{\frac{(2(n+1)y_{-1} + x_0)x_0}{2ny_{-1} + x_0}} \\
 &= \frac{y_{-1}x_0}{2(n+1)y_{-1} + x_0}.
 \end{aligned}$$

The proof is complete. \square

Corollary 4.3. *Every solution of system (4.1) is such that*

$$\lim_{n \rightarrow +\infty} y_n = 0.$$

5. THE SYSTEM $x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} - y_{n-3}y_n - y_n x_{n-2}}$, $y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} - x_{n-1}}$

Consider the system

$$x_{n+1} = \frac{y_{n-3}y_n x_{n-2}}{y_{n-3}x_{n-2} - y_{n-3}y_n - y_n x_{n-2}}, \quad y_{n+1} = \frac{y_{n-2}x_{n-1}}{2y_{n-2} - x_{n-1}}, \quad n \in \mathbb{N}_0 \quad (5.1)$$

with non-zero real initial conditions with $y_0 (y_{-3} + x_{-2}) \notin \left\{ \pm y_{-3}x_{-2}, \frac{y_{-3}x_{-2}}{2} \right\}$, $y_{-2} \notin \left\{ \frac{x_{-1}}{2}, 2x_{-1} \right\}$, $y_{-1} \notin \left\{ \frac{x_0}{2}, 2x_0 \right\}$.

The following theorem describes the form of the solutions of system (5.1).

Theorem 5.1. Let $\{x_n\}_{n \geq -2}$, $\{y_n\}_{n \geq -3}$ be a solution of (5.1). Then, for $n = 0, 1, \dots$,

$$\begin{aligned} x_{6n-1} &= x_{-1} \left(\frac{-1}{3}\right)^n, x_{6n} = x_0 \left(\frac{-1}{3}\right)^n, \\ x_{6n+1} &= \frac{y_{-3}y_0x_{-2}}{y_{-3}x_{-2} - y_0(y_{-3} + x_{-2})} \left(\frac{-1}{3}\right)^n, x_{6n+2} = \frac{y_{-2}x_{-1}}{y_{-2} - 2x_{-1}} \left(\frac{-1}{3}\right)^n, \\ x_{6n+3} &= \frac{y_{-1}x_0}{y_{-1} - 2x_0} \left(\frac{-1}{3}\right)^n, x_{6n+4} = -\frac{y_{-3}y_0x_{-2}}{y_{-3}x_{-2} + y_0(y_{-3} + x_{-2})} \left(\frac{-1}{3}\right)^n, \\ y_{6n-2} &= y_{-2} \left(\frac{-1}{3}\right)^n, y_{6n-1} = y_{-1} \left(\frac{-1}{3}\right)^n, \\ y_{6n} &= y_0 \left(\frac{-1}{3}\right)^n, y_{6n+1} = \frac{y_{-2}x_{-1}}{2y_{-2} - x_{-1}} \left(\frac{-1}{3}\right)^n, \\ y_{6n+2} &= \frac{y_{-1}x_0}{2y_{-1} - x_0} \left(\frac{-1}{3}\right)^n, y_{6n+3} = \frac{y_{-3}y_0x_{-2}}{y_{-3}x_{-2} - 2y_0(y_{-3} + x_{-2})} \left(\frac{-1}{3}\right)^n. \end{aligned}$$

Corollary 5.2. Every solution of system (5.1) is such that

$$\lim_{n \rightarrow +\infty} x_n = \lim_{n \rightarrow +\infty} y_n = 0.$$

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