Journal of Applied Fluid Mechanics, Vol. 9, No. 3, pp. 1321-1328, 2016.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.68.228.24778

Viscoelastic Flow and Heat Transfer over a Non-Linearly Stretching Sheet: OHAM Solution

M. Mustafa

School of Natural Sciences (SNS), National University of Sciences and Technology (NUST), Islamabad 44000, Pakistan

Email: meraj_mm@hotmail.com

(Received March 9, 2015; accepted June 24, 2015)

ABSTRACT

In this paper the viscoelastic flow and heat transfer over a non-linearly stretching sheet with the power law velocity of the form $u_w = cx^n$ is investigated for the first time. A prescribed power-law surface temperature distribution of the form $T_w = T_\infty + Ax^n$ is considered. Mathematical model is constructed through the constitutive equations of second grade fluid. The arising non-linear boundary value problem has been treated analytically by a powerful optimal homotopy analysis method (OHAM). The solutions are found in excellent agreement with the obtained numerical solutions in the case of Newtonian fluid. The results show that velocity and skin friction coefficient have direct relationship with the power-law index n. Further the thermal boundary layer becomes thinner when larger values of n are taken into account.

Keywords: Second grade fluid; Non-linearly stretching sheet; Heat transfer, Optimal homotopy analysis method (OHAM); Non-linear problem.

NOMENCLATURE

a, n	positive constants	T_w	wall temperature
C_f	skin friction coefficient	T_∞	ambient fluid temperature
f	dimensionless stream function	u_w	velocity of the stretching sheet along the $x-$ direction
$\hbar$	auxiliary (convergence control) parameter in OHAM	u, v	velocity components along the $x-, y-$ directions
k	thermal conductivity	'	differentiation with respect to η
K	local second grade fluid parameter	α_1	material fluid parameter
Nu_x	local Nusselt number	α	thermal diffusivity
Pr	Prandtl number	η	similarity variable
q	embedding parameter in OHAM	μ	dynamic viscosity
q_w	wall heat flux	ν_f	kinematic viscosity
Re_x	local Reynolds number	θ	dimensionless temperature
T	fluid temperature	ρ	fluid density
		τ_{xy}	wall shear stress

1 INTRODUCTION

Non-Newtonian fluid mechanics has emerged as one of the most important subjects of modern applied mechanics. Materials encountered in industry and medicine such as multigrade oils, composite materials, blood, polymers, liquid detergents, fruit juices, printing inks and industrial suspensions exhibit the shear-rate dependent viscosity and thus fall outside the classical model of

Newtonian fluids. The frequently discussed power-law fluid model has tendency to describe shear-thinning as well as shear-thickening effects. The former is a common characteristic of many non-Newtonian fluids including blood, polymers and composite materials. On the other hand, some fluids possess normal stress differences which can be described through second grade fluid model. In contrast to the power-law model, the second grade fluid model has been scarcely discussed in the literature due to its complex constitutive relationship between stress and shear rate. Vajravelu and Roper (1999) numerically investigated the flow and heat transfer in second grade fluid in the presence of viscous dissipation and heat generation/absorption. Later, Ariel (2001) discussed the axisymmetric flow of second grade fluid past a radially stretching surface using explicit finite difference scheme. Stagnation-flow of second grade fluid with variable wall heat flux was described by Massoudi (2003). Effects of heat transfer on the hydro-magnetic flow of second grade fluid past a stretching sheet were addressed by Liu (2004). Cortell (2006) extended this problem for permeable stretching sheet. Hayat and Sajid (2007) presented homotopy based analytic solution for second grade fluid flow caused by linearly stretching sheet. Abbas *et al*. (2008) described the flow of second grade fluid due to oscillatory stretching sheet. They presented both numerical and analytical solutions of the arising non-linear problem. Abel and Mahesha (2008) discussed the flow of second grade fluid with variable thermal conductivity, radiation and non uniform heat source/sink. Hayat *et al*. (2009) investigated the boundary layer flow over a flat plate with uniform free stream, the so-called Blasius flow, by considering second grade fluid. Simultaneous effects of heat and mass transfer on the flow of second grade fluid with Dufour and Soret effects were examined by Hayat *et al*. (2010). Recently a variety of two-dimensional flow problems concerning second grade fluid have been discussed in the literature (see Abel *et al*. (2010), Olajuwon (2011), Hayat *et al*. (2012), Raftari *et al*. (2013), Mustafa *et al*. (2014), Mastroberardino (2014), Weidman (2014) and Turkyilmazoglu (2014)).

To the best of author's knowledge, the flow of viscoelastic fluid due to non-linearly stretching surface is not yet reported. This work is therefore undertaken to fill such a void. Heat transfer analysis is also carried out by assuming a power-law surface temperature distribution. Mathematical formulation involves the constitutive relationships of second grade fluid. The arising non-linear boundary value problem is tackled through a powerful analytic approach namely optimal homotopy analysis method (OHAM) (see Marinca and Herisanu (2008), Niu and Wang (2010), Liao (2010), Abbasbandy *et al*. (2011), Sheikholeslami *et al*. (2011), Mustafa *et al*. (2013a), Mustafa *et al*. (2013b), Sheikholeslami *et al*. (2014), Rashidi *et al*. (2014) and Hassan and Rashidi (2014)). Graphs are presented to examine the behaviors of parameters entering in the problem. Expressions of skin friction coefficient and local Nusselt number are evaluated and discussed in detail.

2 PROBLEM FORMULATION

Consider the two-dimensional flow of an incompressible second grade fluid and heat transfer over a plane surface coincident with the plane $y=0$. The sheet is stretched in its own plane with the velocity $u_w = cx^n$, where $c, n > 0$ are constants. Temperature across the sheet varies non-linearly in the form $T_w = T_\infty + Ax^n$, where A is a constant and T_∞ denotes the ambient fluid temperature. The boundary layer equations governing the flow of second grade and heat transfer in the absence of viscous dissipation and heat generation/absorption can be expressed as below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left(u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right), \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are the velocity components along the $x-$ and $y-$ directions respectively, ν is the kinematic viscosity, α_1 is the material fluid parameter, k is the thermal conductivity, ρC_p is the effective heat capacity of the fluid and T is the local fluid temperature. The boundary conditions are as under:

$$\begin{aligned} &u = u_w(x) = cx^n, \quad T = T_w(x) = T_\infty + Ax^n \text{ at } y = 0, \\ &u \to 0, \quad T \to T_\infty \text{ as } y \to \infty. \end{aligned} \tag{4}$$

Introducing the following similarity transformations

$$\begin{aligned} &\eta = \sqrt{\frac{c(n+1)}{2\nu}} x^{\frac{n-1}{2}} y, u = cx^n f'(\eta), \\ &v = -\sqrt{\frac{a\nu(n+1)}{2}} x^{\frac{n-1}{2}}\left(f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right), \\ &\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \end{aligned} \tag{5}$$

Eq. (1) is identically satisfied and Eqs. (2)-(4) become

$$f''' + ff'' - \frac{2n}{n+1}f'^2 + K\left[(3n-1)ff''' - \left(\frac{n+1}{2}\right)ff^{iv} - \left(\frac{3n-1}{2}\right)f''^2\right] = 0, \tag{6}$$

$$\frac{1}{\Pr}\theta'' + f\theta' - \frac{2n}{n+1}f'\theta = 0, \tag{7}$$

$$f(0)=0,\ f'(0)=1,\ \theta(0)=1,\ f'(\infty)\to 0,\ \theta(\infty)\to 0, \tag{8}$$

Where $K=\alpha_1 c x^{n-1}/\mu$ is the local second grade fluid parameter and $\Pr=\mu C_p/k$ is the Prandtl number.

The skin friction coefficient C_f and local Nusselt number Nu_x are defined as below:

$$C_f=\frac{\tau_w}{\rho u_w^{\,2}},\qquad Nu_x=\frac{x q_w}{T_w-T_\infty}, \tag{9}$$

where τ_w is the wall shear stress and q_w is the wall heat flux defined as:

$$\tau_w=\left[\mu\frac{\partial u}{\partial y}+\alpha_1\left(u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}-2\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}\right)\right]_{y=0},\quad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{10}$$

Using Eqs. (5) and (10) in Eq. (9), one obtains

$$\mathrm{Re}_x^{1/2}C_f=\sqrt{\frac{n+1}{2}}\left[1+K\left(\frac{7n-1}{2}\right)\right]f''(0),\quad \mathrm{Re}_x^{-1/2}Nu_x=-\sqrt{\frac{n+1}{2}}\theta'(0), \tag{11}$$

where $\mathrm{Re}_x=u_w x/\nu$ is the local Reynolds number.

2.1 Particular Cases

(i) When $K=0$, viscous flow and heat transfer due to non-isothermal non-linearly stretching sheet is obtained, as discussed by Cortell (2006). In this problem, when $\Pr=1$, the solution of f' is also a solution of θ.

(ii) When $n=0$, Eqs. (8)-(10) reduce to the well known Sakiadis flow problem in second grade fluid.

(iii) When $n=1$, flow of second grade fluid and heat transfer due to non-isothermal linearly stretching sheet is achieved.

3 OPTIMAL HOMOTOPY ANALYSIS METHOD (OHAM)

In order to derive analytic solutions of Eqs. (6)-(8) by optimal homotopy analysis method (OHAM), we choose the following initial guesses f_0 and θ_0 of $f(\eta)$ and $\theta(\eta)$ as under:

$$f_0(\eta)=1-\exp(-\eta),\quad \theta_0(\eta)=\exp(-\eta), \tag{12}$$

and the auxiliary linear operators are selected as below:

$$\mathbf{L}_f(f)=\frac{d^3 f}{d\eta^3}-\frac{df}{d\eta},\qquad \mathbf{L}_\theta(\theta)=\frac{d^2\theta}{d\eta^2}-\theta. \tag{13}$$

If $q\in[0,1]$ is an embedding parameter and $\hbar$ the non-zero auxiliary parameter, then generalized homotopic equations corresponding to (8)-(10) can be expressed as follows:

$$(1-q)\mathbf{L}_f[\hat f(\eta,q)-f_0(\eta)]=q\hbar\mathbf{N}_f\left[\hat f(\eta;q)\right], \tag{14}$$

$$(1-q)\mathbf{L}_\theta[\Theta(\eta;q)-\theta_0(\eta)]=q\hbar\mathbf{N}_\theta\left[\hat f(\eta;q),\Theta(\eta;q)\right], \tag{15}$$

$$\hat f(\eta;q)\Big|_{\eta=0}=0,\ \frac{\partial\hat f(\eta;q)}{\partial\eta}\bigg|_{\eta=0}=1,\ \Theta(\eta;q)\Big|_{\eta=0}=1, \tag{16}$$

$$\frac{\partial\hat f(\eta;q)}{\partial\eta}\bigg|_{\eta\to\infty}=0,\ \Theta(\eta;q)\Big|_{\eta\to\infty}=0,$$

in which the non-linear operators $\mathbf{N}_f$ and $\mathbf{N}_\theta$ through Eqs. (8) and (9) are

$$\mathbf{N}_f\left[\hat f(\xi,\eta;q)\right]=\frac{\partial^3\hat f(\eta;q)}{\partial\eta^3}+\hat f(\eta;q)\frac{\partial^2\hat f(\eta;q)}{\partial\eta^2}-\left(\frac{2n}{n+1}\right)\left(\frac{\partial\hat f(\eta;q)}{\partial\eta}\right)^2+K\left[\begin{array}{l}(3n-1)\dfrac{\partial\hat f(\eta;q)}{\partial\eta}\dfrac{\partial^3\hat f(\eta;q)}{\partial\eta^3}-\\ \left(\dfrac{n+1}{2}\right)\hat f(\eta;q)\dfrac{\partial^4\hat f(\eta;q)}{\partial\eta^4}\\ -\left(\dfrac{3n-1}{2}\right)\left(\dfrac{\partial^2\hat f(\eta;q)}{\partial\eta^2}\right)^2\end{array}\right], \tag{17}$$

$$\mathbf{N}_\theta\left[\hat f(\eta;q),\Theta(\eta;q)\right]=\frac{1}{\Pr}\frac{\partial^2\Theta(\eta;q)}{\partial\eta^2}+\hat f(\eta,p)\frac{\partial\Theta(\eta;q)}{\partial\eta}-\left(\frac{2n}{n+1}\right)\frac{\partial\hat f(\eta;q)}{\partial\eta}\Theta(\eta;q), \tag{18}$$

By Taylor series expansion one obtains

$$\hat f(\eta;q)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta)q^m;\quad f_m(\eta)=\frac{1}{m!}\frac{\partial^m\hat f(\eta;q)}{\partial\eta^m}\bigg|_{q=0}, \tag{19}$$

$$\Theta(\eta;q)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta)q^m;\quad \theta_m(\eta)=\frac{1}{m!}\frac{\partial^m\Theta(\eta;q)}{\partial\eta^m}\bigg|_{q=0}. \tag{20}$$

The final solutions can be obtained by substituting $q=1$ in the above equations. The functions f_m and θ_m can be determined from the deformation of Eqs. (8)-(10). Explicitly *m*th-order deformation equations corresponding to Eqs. (8)-(10) are as under:

$$\mathbf{L}_f\left[f_m(\eta)-\chi_m f_{m-1}(\eta)\right]=\hbar\mathbf{R}_m^f(\eta), \qquad (21)$$

$$\mathbf{L}_\theta\left[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)\right]=\hbar\mathbf{R}_m^\theta(\eta), \qquad (22)$$

$$f_m(0)=0,\ \left.\frac{df_m(\eta)}{d\eta}\right|_{\eta=0}=0,\ \left.\frac{d\theta_m(\eta)}{d\eta}\right|_{\eta=0}=-\gamma(1-\theta_m(0)), \qquad (23)$$

$$\left.\frac{df_m(\eta)}{d\eta}\right|_{\eta\to+\infty}=0,\ \theta_m(\infty)=0,$$

$$\mathbf{R}_m^f(\eta)=\frac{d^3 f_{m-1}}{d\eta^3}+\sum_{k=0}^{m-1}\left[f_{m-1-k}\frac{d^2 f_k}{d\eta^2}-\left(\frac{2n}{n+1}\right)\frac{df_{m-1-k}}{d\eta}\frac{df_k}{d\eta}\right] \qquad (24)$$
$$+K\sum_{k=0}^{m-1}\left[\begin{array}{l}(3n-1)\dfrac{df_{m-1-k}}{d\eta}\dfrac{d^3 f_k}{d\eta^3}-\\ \left(\dfrac{n+1}{2}\right)f_{m-1-k}\dfrac{d^4 f_k}{d\eta^4}-\\ \left(\dfrac{3n-1}{2}\right)\dfrac{d^2 f_{m-1-k}}{d\eta^2}\dfrac{d^2 f_k}{d\eta^2}\end{array}\right],$$

$$\mathbf{R}_m^\theta(\eta)=\frac{1}{\Pr}\frac{d^2\theta_{m-1}}{\partial\eta^2}+\sum_{k=0}^{m-1}\left(f_{m-1-k}\frac{d\theta_k}{\partial\eta}-\left(\frac{2n}{n+1}\right)\frac{df_{m-1-k}}{\partial\eta}\theta_k\right), \qquad (25)$$

$$\chi_m=\begin{cases}0, & m\le 1,\\ 1, & m>1.\end{cases} \qquad (26)$$

The optimal values of the convergence control parameter $\hbar$ can be determined by minimizing the squared residuals of the governing Eqs. (6) and (7), $\zeta^f{}_M, \zeta^\theta{}_M$ in the domain $\eta\in[0,\infty)$

$$\zeta^f{}_M=\int_0^\infty\left[\mathbf{N}_f\left(\sum_{j=0}^{M}f_j(\eta)\right)\right]^2 d\eta, \qquad (27)$$

$$\zeta^\theta{}_M=\int_0^\infty\left[\mathbf{N}_\theta\left(\sum_{j=0}^{M}f_j(\eta),\sum_{j=0}^{M}\theta_j(\eta)\right)\right]^2 d\eta. \qquad (28)$$

Similar kind of error has also been considered in previous studies (see Liao (2010)). The smaller ζ_M's, the more accurate the *M*-th order approximation of the solution. First of all we plot the so called $\hbar$ -curves and the squared residuals given in (27) and (28) in Figs. 1-3. A sample of optimal values of $\hbar$ for functions f and θ for different values of parameter n has been given in Table 1.

4 RESULTS AND DISCUSSION

This section focuses on the physical behaviors of the involved paramaters on the velocity and temperature distributions. Figs. 4 and 5 compare the 15th-order OHAM solutions for different values of n and $\Pr$ with the corresponding numerical solutions when $K=0$. Here the numerical solutions have been derived by MATLAB built in routine *bvp4c*. It can be seen that both the solutions coincide for both small and large values of n and $\Pr$.

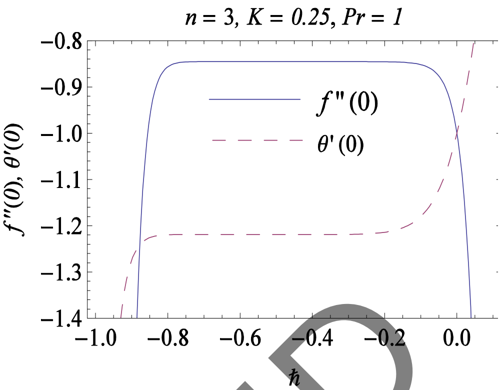

Fig. 1. $\hbar-$ **curves of** $f''(0)$ **and** $\theta'(0)$ **at 20th – order of approximations.**

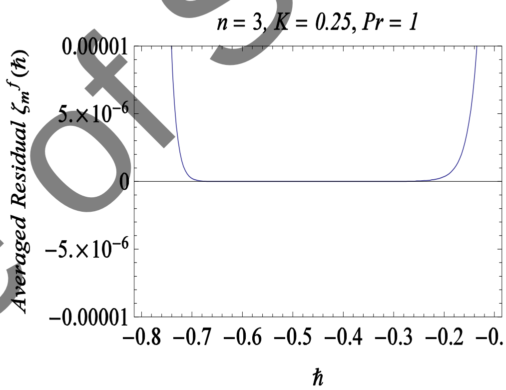

Fig. 2. Averaged square residual for the function f.

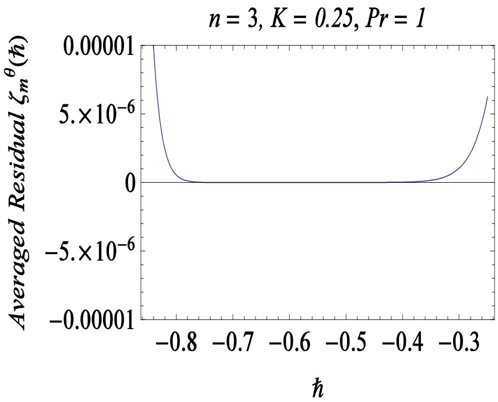

Fig. 3. Averaged residual for the function θ **.**

Fig. 6 presents the variation in velocity distribution with an increase in power-law index n . The bigger values of n imply larger velocity of the stretching sheet. Due to this reason the $x-$ component of velocity in the neighborhood of the sheet increases when n is increased. It may be noted that profiles descend to zero at larger values of η when n is increased indicating an augmentation in the boundary layer thickness. Fig. 7 is prepared to examine the impact of local second grade fluid parameter K on the hydrodynamic boundary layer. The velocity approaches zero value at large distance from the sheet when K is increased. This indicates that boundary layer thickness is an increasing function of K.

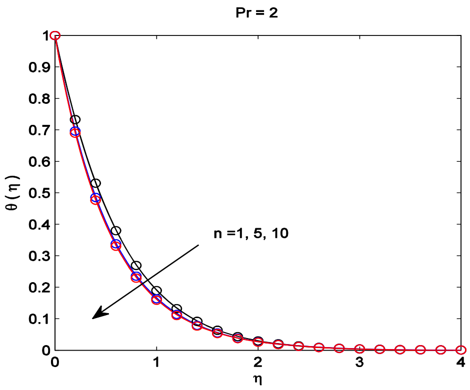

Fig. 4. Temperature profiles for different values of n in Newtonian fluid case ($K = 0$). Lines: 15th –order OHAM solution; Circles: Numerical solution.

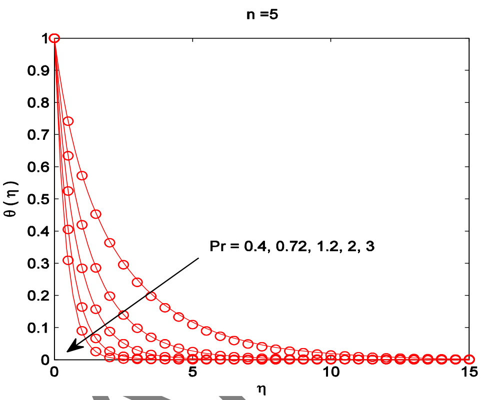

Fig. 5. Temperature profiles for different values of Pr in Newtonian fluid case ($K = 0$). Lines: 15th –order OHAM solution; Circles: Numerical solution.

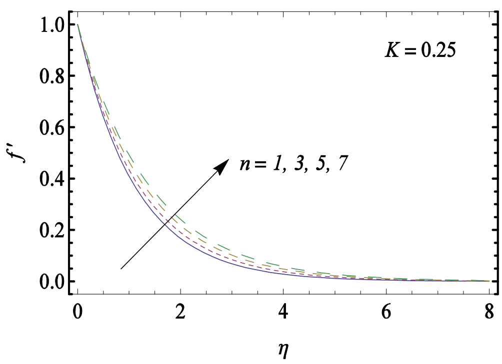

Fig. 6. Variation of f' with n.

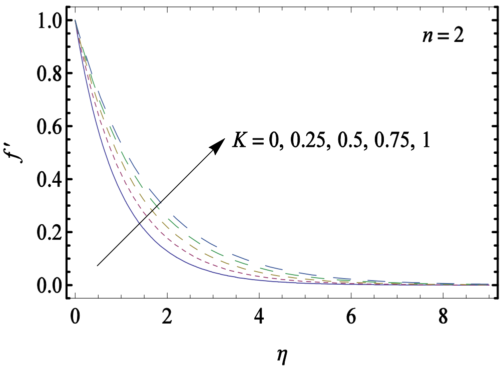

Fig. 7. Variation of f' with K.

Fig. 8 plots the skin friction coefficient $\mathrm{Re}_x^{1/2} C_f$ against the viscoelastic fluid parameter K for different values of n. It is quite obvious that larger values of n indicates larger sheet velocity which requires stronger driving force at the wall. Consequently the wall shear stress increases with an increase in n. For a fixed value of n, wall shear stress reduces when K is increased. From the industrial point of view, this outcome is undesirable since the power required in displacing the fluid over the sheet increases when K is increased.

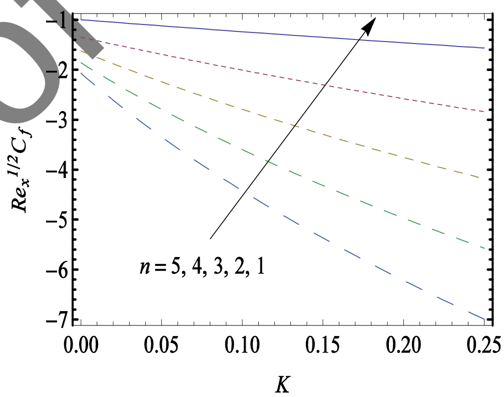

Fig. 8. Variation of $\mathrm{Re}_x^{1/2} C_f$ with n and K.

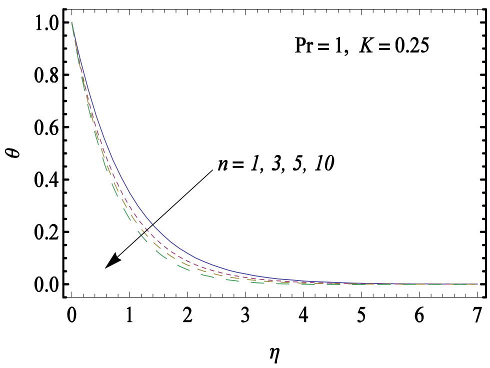

Fig. 9. Variation of θ with n.

The variation in temperature distribution with an increase in power-law index n can be observed from Fig. 9. Both temperature θ and thermal boundary layer thickness are decreasing functions of power-law index n. From the physical point of

Table 1 Optimal values of the auxiliary parameter $\hbar$ at 15th-order of approximations for different values of n when $K = 0.25$ and $\Pr = 1$

n	Optimal $\hbar$ for f	Minimum ζ^f_M	Optimal $\hbar$ for θ	Minimum ζ^θ_M
0	-1.025	1.52×10^{-11}	-1.184	7.96×10^{-12}
1	-0.738	2.26×10^{-27}	-1.198	6.11×10^{-16}
2	-0.738	1.020×10^{-17}	-0.879	1.37×10^{-13}
3	-0.585	8.24×10^{-15}	-0.678	1.11×10^{-11}
4	-0.480	3.94×10^{-13}	-0.557	1.61×10^{-10}
5	-0.408	5.06×10^{-12}	-0.478	1.59×10^{-9}

Table 2 Values of skin friction coefficient $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ when $\Pr = 1$

n	K	$\mathrm{Re}_x^{1/2} C_f$	$\mathrm{Re}_x^{-1/2} Nu_x$
0 (*Sakiadis flow problem*)	0	-0.44375	0.44375
	0.25	-0.38276	0.44634
	0.5	-0.32416	0.44847
	0.75	-0.26742	0.45024
1 (*Linearly stretching sheet problem*)	0	-1.00000	1.00000
	0.25	-1.56525	1.02789
	0.5	-2.04124	1.04841
	0.75	-2.45677	1.06430
2 (*Non-linearly stretching sheet problem*)	0	-1.34845	1.34845
	0.25	-2.84036	1.41148
	0.5	-3.97463	1.44857
	0.75	-4.91622	1.47385

view, bigger values of n enhance the intensity of cold fluid at the ambient towards the hot stretching surface due to increased fluid motion in the $x-$direction adjacent to the sheet. In Fig. 10, we present the behavior of local second grade fluid parameter K on the temperature θ. Temperature at a point above the sheet decreases with an increase in K. Fig. 11 illustrates the effects of Prandtl number $\Pr$ on the thermal boundary layer. A bigger Prandtl number fluid has relatively weaker thermal diffusivity and hence it yields shorter penetration depth of the temperature θ. Further the temperature profiles become steeper when $\Pr$ is increased indicating a growth in the magnitude of local Nusselt number.

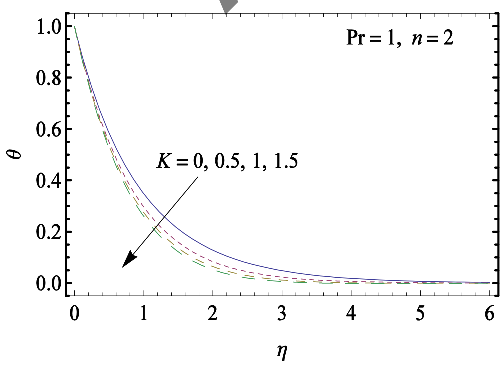

Fig. 10. Variation of θ with K.

The variation in local Nusselt number Nu_x with K and n can be observed from Fig. 12. The magnitude of local Nusselt number or equivalently the rate of heat transfer from the sheet is a decreasing function of K.

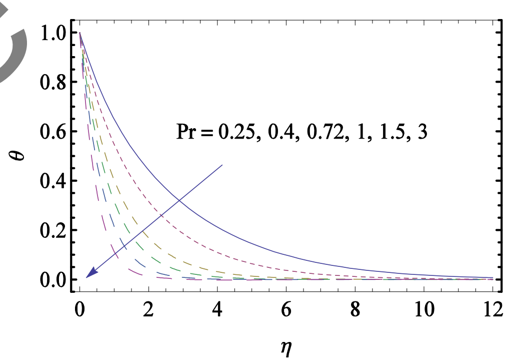

Fig. 11. Variation of θ with $\Pr$.

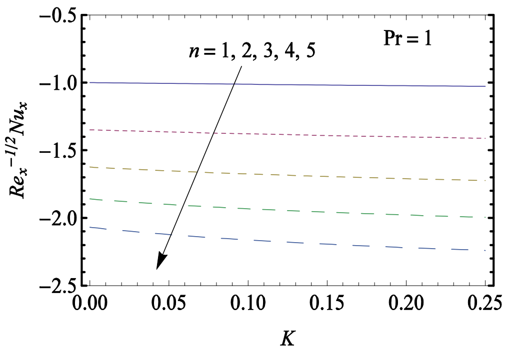

Fig. 12. Variation of $\mathrm{Re}_x^{-1/2} Nu_x$ with n and K.

Table 2 gives the numerical values of skin friction coefficient and local Nusselt number for different values of local second grade fluid parameter K. The cases of Sakiadis flow $(n=0)$, linearly stretching sheet $(n=1)$ and non-linearly stretching sheet with $n=2$ are considered. We notice that magnitude of skin friction coefficient increases when K is increased in the linearly and non-linearly stretching sheet problems. Interestingly, opposite trend is observed in the Sakiadis flow problem.

5 CONCLUDING REMARKS

For the first time, the flow and heat transfer of viscoelastic fluid due to non-linearly stretching sheet is considered. Analytic solutions are computed by powerful analytical approach namely optimal homotopy analysis method (OHAM). The main observations of this work are outlined below:

(i) Analytic solutions are found in excellent agreement with the numerical solutions in a limiting sense.

(ii) Velocity increases and temperature decreases when the power-law index n is incremented.

(iii) An increase in the local second grade fluid parameter K increases the velocity and decreases the temperature distribution.

(iv) Both skin friction coefficient and local Nusselt number have direct and linear relationship with the local second grade fluid parameter K.

(v) The well known Sakiadis flow problem for second grade fluid can be obtained as special case of present study by substituting $n=0$.

REFERENCES

Abbabsandy, S., E. Shivanianand and K. Vajravelu (2011). Mathematical properties of h-curve in the frame work of the homotopy analysis method. *Communications in Nonlinear Science and Numerical Simulation*. 16, 4268-4275.

Abbas, A., Y. Wang, T. Hayat and M. Oberlack (2008). Hydromagnetic flow in a viscoelastic fluid due to the oscillatory stretching surface. *International Journal of Non-Linear Mechanics* 43, 783-793.

Abel, M. S. and N. Mahesha (2008). Heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. *Applied Matehmatical Modelling* 32, 1965–1983.

Abel, M. S., N. Mahesha and S. B. Malipatil (2010). Heat transfer due to MHD slip flow of a second-grade liquid over a stretching sheet through a porous medium with non uniform heat source/sink. *Chemical Engineering Communications* 198, 191-213.

Ariel, P. D. (2001). Axisymmetric flow of a second grade fluid past a stretching sheet. *International Journal of Engineering Science* 39, 529-553.

Cortell, R. (2006). Flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to suction and to a transverse magnetic field. *International Journal of Heat and Mass Transfer* 49, 1851-1856.

Hassan, H. and M. M. Rashidi (2014). An analytic solution of micropolar flow in a porous channel with mass injection using homotopy analysis method. *International Journal of Numerical Methods for Heat and Fluid Flow* 24, 419-437.

Hayat, T. and M. Sajid (2007). Analytic solution for axisymmetric flow and heat transfer of a second grade fluid past a stretching sheet. *International Journal of Heat and Mass Transfer* 50, 75-84.

Hayat, T., M. Mustafa and I. Pop (2010). Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid. *Communications in Nonlinear Science and Numerical Simulation* 15, 1183-1196.

Hayat, T., M. Mustafa and M. Sajid (2009). Influence of thermal radiation on Blasius flow of a second grade fluid. *Zeitschrift Fur Naturforschung* 64, 827-833.

Hayat, T., Z. Iqbal and M. Mustafa (2012). Flow of a second grade fluid over a stretching surface with Newtonian heating. *Journal of Mechanics* 28, 209-216.

Liao, S. (2010). An optimal homotopy-analysis approach for strongly nonlinear differential equations. *Communications in Nonlinear Science and Numerical Simulation* 15, 2003-2016.

Liu, I. C. (2004). Flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to a transverse magnetic field. *International Journal of Heat and Mass Transfer* 47, 4427-4437.

Marinca, V. and N. Herisanu (2008). Application of optimal homotopy asymptotic method for solving nonlinear equations arising in hear transfer. International *Communications in Heat and Mass Transfer* 35, 710-715.

Massoudi, M. (2003). Boundary layer flow of a second grade fluid with variable heat flux at the wall. *Applied Mathematics and Computation* 143, 201-212.

Mastroberardino, A. (2014). Comment on "Heat transfer in MHD viscoelastic boundary layer flow over a stretching sheet with thermal radiation and non-uniform heat source/sink". *Communications in Nonlinear Science and Numerical Simulation* 19, 1638-1643.

Mustafa, M., M. A. Farooq, T. Hayat and A. Alsaedi (2013). Numerical and series solutions for stagnation-point flow of nanofluid over an exponentially stretching sheet. *PLoS ONE* 8, doi:10.1371/journal.pone. 0061859.

Mustafa, M., M. Nawaz, T. Hayat and A. Alsaedi (2014). MHD boundary layer flow of second grade nanofluid over a stretching sheet with convective boundary condition. *Journal of Aerospace Engineering* 27.

Mustafa, M., T. Hayat and S. Obaidat (2013). Boundary layer flow of a nanofluid over an exponentially stretching sheet with convective boundary conditions. *International Journal of Numerical Methods for Heat and Fluid Flow* 23, 945-959.

Niu, Z. and C. Wang (2010). A one-step optimal homotopy analysis method for nonlinear differential equations. *Communications in Nonlinear Science and Numerical Simulation* 15, 2026-2036.

Olajuwon, B. I. (2011). Convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion. *International Communications in Heat and Mass Transfer* 38, 377-382.

Raftari, B., F. Parvaneh and K. Vajravelu (2013). Homotopy analysis of the magnetohydrodynamic flow and heat transfer of a second grade fluid in a porous channel. *Energy* 59, 625-632.

Rashidi, M. M., N. Freidoonimehr, A. Hosseini, O. A. Bég and T. K. Hung (2014). Homotopy simulation of nanofluid dynamics from a non-linearly stretching isothermal permeable sheet with transpiration. *Meccanica* 49, 469-482.

Sheikholeslami, M., H. R. Ashorynejad, D. Domairry and I. Hashim (2012). Investigation of the laminar viscous flow in a semi-porous channel in the presence of uniform magnetic field using optimal homotopy asymptotic method. *Sains Malaysiana* 41, 1281-1285.

Sheikholeslami, M., R. Ellahi, H. R. Ashorynejad, G. Domairry and T. Hayat (2014). Effects of heat transfer in flow of nanofluids over a permeable stretching wall in a porous medium. *Journal of Computational and Theoretical Nanoscience* 11, 486-496.

Turkyilmazoglu, M. (2014). Three dimensional MHD flow and heat transfer over a stretching/shrinking surface in a viscoelastic fluid with various physical effects. *International Journal of Heat and Mass Transfer* 78, 150-155.

Vajravelu, K. and T. Roper (1999). Flow and heat transfer in a second grade fluid over a stretching sheet. *International Journal of Non-Linear Mechanics* 34, 1031-1036.

Weidman, P. (2014). Comment on “Heat transfer in MHD viscoelastic boundary layer flow over a stretching sheet with thermal radiation and non-uniform heat source/sink”, M.M. Nandeppanavar, K. Vajravelu & M.S. Abel [Commun Nonlinear Sci Numer Simulat 16 (2011) 3578–3590]. *Communications in Nonlinear Science and Numerical Simulation* 19, 3412-3417.