Journal of Applied Fluid Mechanics, Vol. 9, No. 3, pp. 1247-1253, 2016.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.68.228.22936

Stability of Two Axially Superposed Fluids of Different Viscosities between Two Rotating Cylinders

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(Received April 27, 2014; accepted May 20, 2015)

ABSTRACT

The linear stability of two axially superposed immiscible fluids between two rotating coaxial cylinders is studied. The fluids are assumed to have equal density but different viscosities. The effect of viscosity ratio of the two fluids on the condition for onset of instability is studied. The critical Taylor number in the less viscous fluid for onset of instability is obtained as a function of the viscosity ratio. The two limiting values of this curve correspond to critical Taylor numbers of the one fluid configuration with height of fluid column either equal to that of the less viscous fluid or equal to the sum of those for both liquids. It is found that the variation of the critical Taylor number with viscosity ratio is small when the heights of the fluid columns are large compared to the gap between the cylinders but is significant when the heights are comparable with the gap. The marginal state is found to be stationary.

Keywords: Linear stability; Taylor Couette flow; Two fluid flow.

NOMENCLATURE

C_1	constant
C_2	constant
D	differential operator
D_*	differential operator
d	gap between two cylinders
L	length of each fluid column
m	viscosity ratio
p	pressure
p'	perturbed pressure
R_1	inner cylinder radius
R_2	outer cylinder radius
s	growth rate
Ta	Taylor number
Ta_c	critical Taylor number
t	time
u	radial velocity
u'	perturbed velocity in radial direction
V	velocity of unperturbed state
$\mathbf{v}$	velocity vector
v	tangential velocity
v'	perturbed velocity in tangential direction
w	axial velocity
w'	perturbed velocity in axial direction
ρ	density of the fluid
μ	coefficient of viscosity of the fluid.
Ω	angular speed of the inner cylinder
η	radius ratio
ν	kinematic viscosity
χ'	perturbed state of the interface
ψ'	stream function
subscripts	
1	fluid 1
2	fluid 2

1. INTRODUCTION

It is known that the flow between two rotating coaxial cylinders can be unstable to infinitesimal axisymmetric disturbances. For an inviscid fluid, instability occurs if the circulation decreases outward anywhere between the two cylinders (Chandrasekhar 1981; Rayleigh 1916). For a viscous fluid, instability occurs when a dimensionless quantity, now called the Taylor number, exceeds a critical value (Chandrasekhar 1981; Taylor 1923). The Taylor-Couette flow of two immiscible fluids between two coaxial cylinders has been studied (Joseph *et al*. 1984; Joseph and Renardy 1993; Peng and Zhu 2010) and different flow patterns were found to develop. It was assumed that, in the unperturbed state, the two liquids were separated by a cylindrical interface.

The inner cylinder was set in rotation while the outer cylinder was kept stationary. Taylor vortices were seen to form, and in different parameter regimes a variety of interesting flow patterns were found to develop.

In another study, the deformation of the interface between two immiscible fluids in a cylindrical container, with a rotating lid placed above the interface was investigated experimentally by Fujimoto and Takeda (2009), and with bottom wall rotating was numerically studied by Brady and Herrmann (2012). Fujimoto and Takeda (2009) observed that center of the interface rises as rotational speed is increased when upper fluid had higher viscosity. Whereas, Brady and Herrmann (2012) found that viscosity ratio plays a major role in defining the dynamics of the two fluids.

The dynamics of two immiscible fluids between two rotating coaxial cylinders has been studied by Bonn *et al*. (2004). Where, in the unperturbed state, the interface between the two liquids was planar and perpendicular to the axis of the cylinders. In their experiment the two liquids were separated by a horizontal interface while the axis of the cylinders was vertical. It was observed that, when centrifugal instability sets in, the interface would climb the inner cylinder when the lower fluid had lower viscosity.

No theoretical study of the stability of this configuration seems to have been reported. As a first step a linear stability analysis of this configuration has been carried out. This study also has practical importance, since such flow configurations are known to occur in emulsification. The manuscript is arranged as follows. Derivation of the equations governing linear stability and the method used for numerical solution is described in Sec. 2, Sec.3 illustrate the results obtained using this formulation and Sec.4 summarizes the findings.

2. FORMULATION

Two coaxial cylinders of radii R_1 and R_2 as shown in Fig. 1 have been considered. The space between the two cylinders contains two immiscible fluids. In the unperturbed state assuming that fluid 1 occupies the region $0 < z < L$, $R_1 < r < R_2$ while fluid 2 occupies the region $-L < z < 0$, $R_1 < r < R_2$, where a cylindrical coordinate system (r, θ, z) with the z-axis along the axis of the cylinders has been used and the plane z = 0 to coincide with the unperturbed interface between the two fluids. The two fluids are assumed to have the same density but different coefficients of viscosity. The outer cylinder is assumed to be stationary while the inner cylinder rotates with angular speed Ω.

Present aim is to determine at what value of Ω instability sets in for infinitesimal disturbances. In the original experiment of Taylor (Taylor 1923) for one fluid as well as in the recent experiments of Bonn *et al*. (2004) for two superposed fluids, instability is observed to first set in as an axisymmetric mode. Accordingly, present analysis is restricted to axisymmetric disturbances. Assuming that the fluids are incompressible, the governing equations in both fluids are

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v}\right) = -\nabla p + \mu\nabla^2\boldsymbol{v}, \tag{1}$$

$$\nabla\cdot\boldsymbol{v} = 0, \tag{2}$$

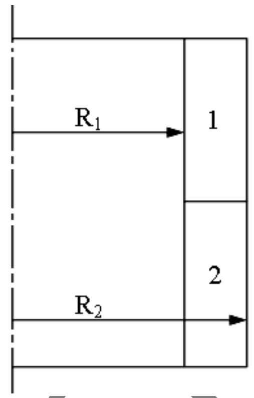


Fig. 1. Schematic figure of two fluid configuration.

where $\boldsymbol{v}$, p, ρ, and μ are the velocity, pressure, density and coefficient of viscosity of the fluid. When the outer cylinder is stationary and the inner cylinder rotates with angular speed Ω, a stationary solution of equations (1) and (2) is given by (Chandrasekhar, 1981)

$$\boldsymbol{v} = V(r)\,\mathbf{e}_\theta = \left(C_1 r + \frac{C_2}{r}\right)\mathbf{e}_\theta, \quad p = P(r), \tag{3}$$

Where

$$C_1 = -\frac{\Omega R_1^2}{R_2^2 - R_1^2} = -\frac{\Omega\eta^2}{1-\eta^2},\; C_2 = \frac{\Omega R_1^2 R_2^2}{R_2^2 - R_1^2} = \frac{\Omega R_1^2}{1-\eta^2},$$

$$\eta = \frac{R_1}{R_2}, \tag{4}$$

and the pressure profile satisfies

$$\frac{dP}{dr} = \frac{\rho V^2}{r}. \tag{5}$$

Since this solution does not depend on μ it is valid in both fluids. To make the problem tractable the modification of the flow profile due to the end walls at $z = \pm L$. Strictly this is valid provided we have end walls which allow differential rotation and have the same rotation profile as given by equation (3). Now considering a small axisymmetric perturbation from this steady flow described by

$$\boldsymbol{v} = u'(r,z,t)\mathbf{e}_r + [V(r) + v'(r,z,t)]\,\mathbf{e}_\theta + w'(r,z,t)\mathbf{e}_z,$$

$$p = P(r) + p'(r,z,t). \tag{6}$$

Substituting in equations (1) and (2) and linearizing in the perturbations yields

$$\frac{\partial u'}{\partial t}-\frac{2V}{r}v'=-\frac{\partial}{\partial r}\left(\frac{p'}{\rho}\right)+\nu\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}+\frac{\partial^2}{\partial z^2}\right)u', \quad (7)$$

$$\frac{\partial v'}{\partial t}+\left(\frac{dV}{dr}+\frac{V}{r}\right)u'=\nu\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}+\frac{\partial^2}{\partial z^2}\right)v', \quad (8)$$

$$\frac{\partial w'}{\partial t}=-\frac{\partial}{\partial z}\left(\frac{p'}{\rho}\right)+\nu\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right)w', \quad (9)$$

$$\frac{\partial u'}{\partial r}+\frac{u'}{r}+\frac{\partial w'}{\partial z}=0. \quad (10)$$

Assuming that in the perturbed state the interface is given by $z=\chi'(r,t)$. The linearized kinematic boundary condition is

$$\frac{\partial \chi'}{\partial t}=w' \text{ at } \quad z=0. \quad (11)$$

Continuity of velocity at the interface requires

$$u_1'=u_2',\ v_1'=v_2',\ w_1'=w_2' \quad \text{at} \quad z=0, \quad (12)$$

where the subscripts 1 and 2 represent values in fluid 1 and fluid 2, respectively. Continuity of tangential components of stress requires

$$\mu_1\left(\frac{\partial u_1'}{\partial z}+\frac{\partial w_1'}{\partial r}\right)=\mu_2\left(\frac{\partial u_2'}{\partial z}+\frac{\partial w_2'}{\partial r}\right) \quad \text{at } z=0, \quad (13)$$

$$\mu_1\frac{\partial v_1'}{\partial z}=\mu_2\frac{\partial v_2'}{\partial z} \quad \text{at} \quad z=0. \quad (14)$$

Continuity of normal stress at the interface requires

$$-p_1'+2\mu_1\frac{\partial w_1'}{\partial z}=-p_2'+2\mu_2\frac{\partial w_2'}{\partial z} \quad \text{at} \quad z=0 \quad (15)$$

where the effect of surface tension at the interface between the two liquids has been neglected. Differentiating with respect to *r* and using equation (7) to eliminate pressure we obtain

$$-\mu_1\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}+\frac{\partial^2}{\partial z^2}\right)u_1'+2\mu_1\frac{\partial^2 w_1'}{\partial r\partial z}$$
$$=-\mu_2\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}+\frac{\partial^2}{\partial z^2}\right)u_2'+2\mu_2\frac{\partial^2 w_2'}{\partial r\partial z}$$

at $\quad z=0. \quad (16)$

The no slip boundary condition on the walls are

$$u'=v'=w'=0 \quad \text{at } r=R_1 \text{ and } R_2 \text{ and } z=\pm L \quad (17)$$

Obtaining u' and w' from a stream function $\psi'(r,z,t)$ by

$$u'=\frac{\partial \psi'}{\partial z},\ w'=-\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)\psi'. \quad (18)$$

Then equation (10) is identically satisfied. Eliminating p' between equations (7) and (9) and using equation (18) yields

$$\frac{\partial}{\partial t}\left(DD_*+\frac{\partial^2}{\partial z^2}\right)\psi'-\frac{2V}{r}\frac{\partial v'}{\partial z}=\nu\left(DD_*+\frac{\partial^2}{\partial z^2}\right)^2\psi', \quad (19)$$

where the operator D and D* are defined as

$$D=\frac{\partial}{\partial r},\quad D_*=\frac{\partial}{\partial r}+\frac{1}{r}. \quad (20)$$

Using equations (18) and (20), equation (8) can be written as

$$\frac{\partial v'}{\partial t}+\left(\frac{dV}{dr}+\frac{V}{r}\right)\frac{\partial \psi'}{\partial z}=\nu\left(DD_*+\frac{\partial^2}{\partial z^2}\right)v'. \quad (21)$$

For the linear perturbations assuming normal modes with time dependence of the form e^{st}. Further transforming to non-dimensional variables using

$$r\to rd,\ z\to zd,\ s\to s\frac{\nu_2}{d^2},\ \psi'\to\psi(2\Omega d^2),$$
$$v'\to v\frac{\nu_2}{d},\ \chi'\to\chi\left(\frac{2\Omega d^3}{\nu_2}\right), \quad (22)$$

where $d=R_2-R_1$ is the radial distance between the two cylinders. Writing equations (19) and (21) separately in the two fluids in non-dimensional form

$$\left[\left(DD_*+\frac{\partial^2}{\partial z^2}\right)-ms\right]\left(DD_*+\frac{\partial^2}{\partial z^2}\right)\psi_1=-m\frac{C_2}{\Omega d^2}\left(\frac{1}{r^2}+\frac{C_1 d^2}{C_2}\right)\frac{\partial v_1}{\partial z}, \quad (23)$$

$$\left[\left(DD_*+\frac{\partial^2}{\partial z^2}\right)-s\right]\left(DD_*+\frac{\partial^2}{\partial z^2}\right)\psi_2=-\frac{C_2}{\Omega d^2}\left(\frac{1}{r^2}+\frac{C_1 d^2}{C_2}\right)\frac{\partial v_2}{\partial z}, \quad (24)$$

$$\left[\left(DD_*+\frac{\partial^2}{\partial z^2}\right)-ms\right]v_1=-mTa\frac{\partial \psi_1}{\partial z}, \quad (25)$$

$$\left[\left(DD_*+\frac{\partial^2}{\partial z^2}\right)-s\right]v_2=-Ta\frac{\partial \psi_2}{\partial z}, \quad (26)$$

Where

$$Ta=-\frac{4C_1\Omega d^4}{\nu_2^2},\qquad m=\frac{\mu_2}{\mu_1}. \quad (27)$$

Here *Ta* is the Taylor number in fluid 2 which is assumed to be less viscous. Therefore, the Taylor number in fluid 1 is smaller than in fluid 2. The kinematic boundary condition equation (11) in nondimensional form is

$$s\chi=-D_*\psi \quad \text{at} \quad z=0 \quad (28)$$

Substituting from equation (18) in equations (12)-(14) and (16), and writing the boundary conditions at the interface in nondimensional form yields

$$\frac{\partial \psi_1}{\partial z} = \frac{\partial \psi_2}{\partial z} \quad \text{at} \quad z = 0 \tag{29}$$

$$v_1 = v_2 \quad \text{at} \quad z = 0 \tag{30}$$

$$D_* \psi_1 = D_* \psi_2 \quad \text{at} \quad z = 0 \tag{31}$$

$$\left(DD_* - \frac{\partial^2}{\partial z^2} \right) \psi_1 = m \left(DD_* - \frac{\partial^2}{\partial z^2} \right) \psi_2 \quad \text{at } z = 0 \tag{32}$$

$$\frac{\partial v_1}{\partial z} = m \frac{\partial v_2}{\partial z} \quad \text{at} \quad z = 0 \tag{33}$$

$$-\frac{1}{m}\left(3DD_* + \frac{\partial^2}{\partial z^2} \right) \frac{\partial \psi_1}{\partial z} = -\left(3DD_* + \frac{\partial^2}{\partial z^2} \right) \frac{\partial \psi_2}{\partial z}$$

$$\text{at} \quad z = 0 \tag{34}$$

Equation (17) in nondimensional form is

$$\frac{\partial \psi_1}{\partial z} = v_1 = D_* \psi_1 = 0 \quad \text{at} \quad r = \frac{R_1}{d} \quad \text{and} \quad \frac{R_2}{d} \quad \text{for} \quad 0 < z < \frac{L}{d}$$

$$\frac{\partial \psi_2}{\partial z} = v_2 = D_* \psi_2 = 0 \quad \text{at} \quad r = \frac{R_1}{d} \quad \text{and} \quad \frac{R_2}{d} \quad \text{for} \quad -\frac{L}{d} < z < 0$$

$$\frac{\partial \psi_1}{\partial z} = v_1 = D_* \psi_1 = 0 \quad \text{at} \quad z = \frac{L}{d} \quad \text{for} \quad \frac{R_1}{d} < z < \frac{R_2}{d}$$

$$\frac{\partial \psi_2}{\partial z} = v_2 = D_* \psi_2 = 0 \quad \text{at} \quad z = -\frac{L}{d} \quad \text{for} \quad \frac{R_1}{d} < z < \frac{R_2}{d} \tag{35}$$

The stream function ψ is defined correct to an additive constant. Therefore, one can choose ψ to be zero at one point on a boundary wall. Then using equation (35) it can easily be reasoned out that ψ will be zero on all the walls bounding the flow region. Then equation (35) can be written as

$$\psi_1 = v_1 = D\psi_1 = 0 \quad \text{at}\, r = \frac{R_1}{d} \quad \text{and}\, \frac{R_2}{d} \quad \text{for}\, 0 < z < \frac{L}{d}$$

$$\psi_2 = v_2 = D\psi_2 = 0 \quad \text{at}\, r = \frac{R_1}{d} \quad \text{and} \quad \frac{R_2}{d} \quad \text{for} \quad -\frac{L}{d} < z < 0$$

$$\frac{\partial \psi_1}{\partial z} = v_1 = \psi_1 = 0 \quad \text{at} \quad z = \frac{L}{d} \quad \text{for} \quad \frac{R_1}{d} < z < \frac{R_2}{d}$$

$$\frac{\partial \psi_2}{\partial z} = v_2 = \psi_2 = 0 \quad \text{at} \quad z = -\frac{L}{d}\, \text{for}\, \frac{R_1}{d} < z < \frac{R_2}{d} \tag{36}$$

Further from equations (31) and (36) it follows that

$$\psi_1 = \psi_2 \quad \text{at} \quad z = 0 \tag{37}$$

which now replaces equation (31). Equations (23)-(26), together with matching conditions (28)-(30), (32)-(34) and (37), and boundary conditions (36) constitute the eigenvalue problem governing linear stability. This can be solved for the eigenvalue *s* if all other parameters are specified. For simplicity a narrow gap approximation $d << R_1, R_2$ has been used. Then one can use the approximations $D_* \approx D$ and $(C_2/\Omega d^2)(1/r^2 + C_1 d^2/C_2) \approx (1-\xi)$, where ξ is a nondimensional radial coordinate defined by $r = R_1/d + \xi$. A finite difference approximation has been used in order to solve the eigenvalue problem and follow a procedure very similar to that used in an earlier study (Bhattacharyya and Gupta 2004).

3. RESULTS

As a first check the analysis was carried out to compute the critical Taylor number when there is only one fluid occupying the entire length. The critical Taylor number vs length of the cylinders is shown in Fig. 2. The results show that as the length of the cylinders becomes large the critical Taylor number approaches the value calculated for cylinders of infinite length. This is in agreement with experimental observations of Koschmieder (1993).

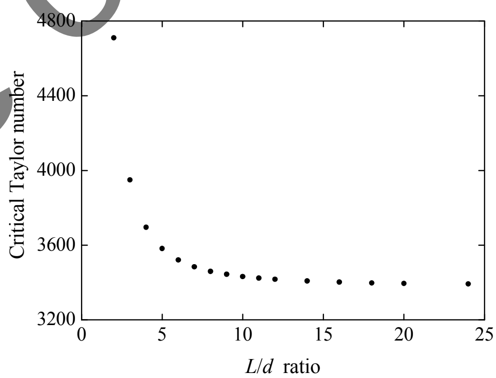


Fig. 2. Variation of critical Taylor number with *L/d* ratio, for single fluid.

For *L/d* = 16, the pattern of streamlines is shown in Fig. 3. This clearly shows the formation of Taylor vortices and the size of the vortices in the axial direction agrees with what theoretical calculations predict at the onset of instability.

Now considering the situation where there are two axially superposed fluids. First considering fluid column heights to be large and compare with the results for one fluid column of infinite height. The critical Taylor number in the lower fluid for onset of instability Ta_c vs the ratio of viscosities *m*, for *L/d* = 8 is shown in Fig. 4. When *m* = 1 there is no jump in material property and as expected the critical Taylor number is same as for one fluid in the entire flow domain. When *m* < 1 the lower fluid is less viscous and, therefore, the Taylor number in the lower fluid is higher than in the upper fluid. Bonn *et al*. (2004) assumed that instability occurs when the Taylor number in the less viscous fluid exceeds the critical Taylor number obtained from computation for one fluid.

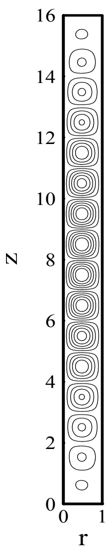


Fig. 3. Contour plots of stream function for single fluid, for *L/d* = 16.

Figure 4 shows that the critical Taylor number in the less viscous fluid for onset of instability increases slightly with decrease in the viscosity ratio. However, the difference is not more than 2%. Therefore, for this configuration the assumption made by Bonn *et al*. (2004) seems justified.

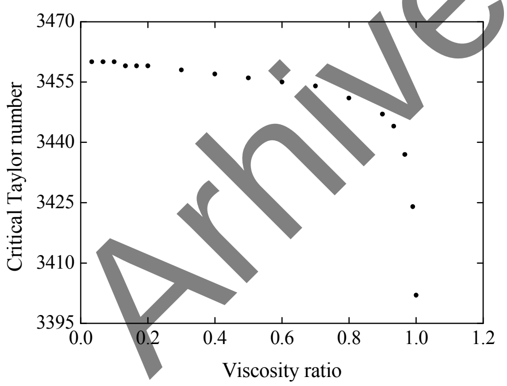


Fig. 4. Variation of critical Taylor number with viscosity ratio, for *L/d* = 8.

Next, examining how the eigenmode varies with change in the viscosity ratio. The radial component of velocity at $r = (R_1 + R_2)/2$ as a function of z is shown in Fig.5. For $m = 0.2$, Fig.5 (a) shows that at the onset of instability, the disturbance is practically confined to the less viscous fluid. Now considering values of m close to 1, as seen in Fig.5(b), for $m = 0.9$, the disturbance extends into the more viscous fluid but decays after roughly two Taylor cells. For $m = 0.99$, Fig.5(c) shows that the disturbance extends further into the more viscous fluid.

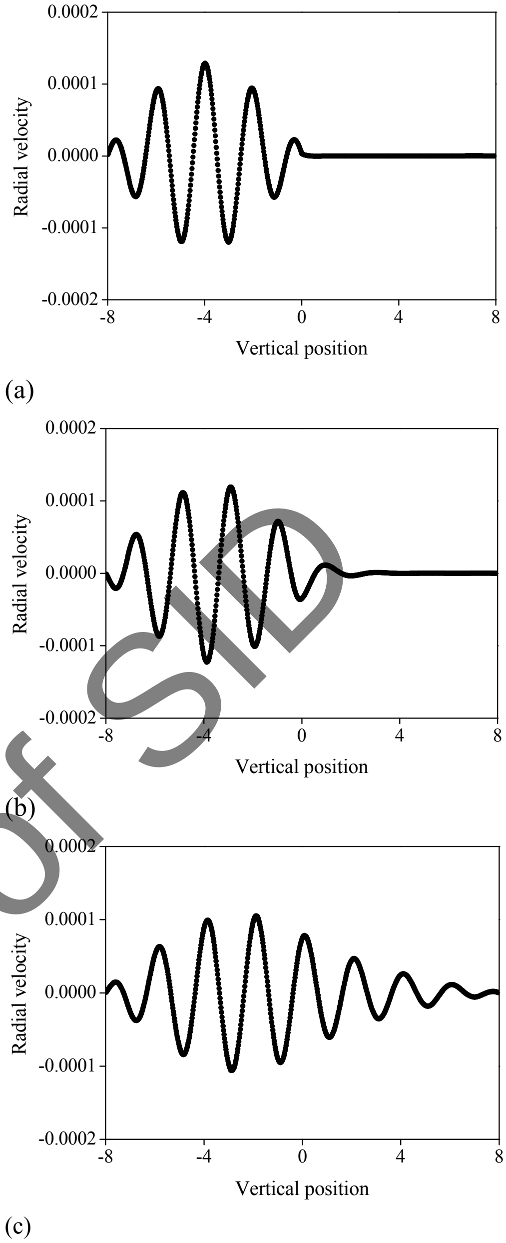
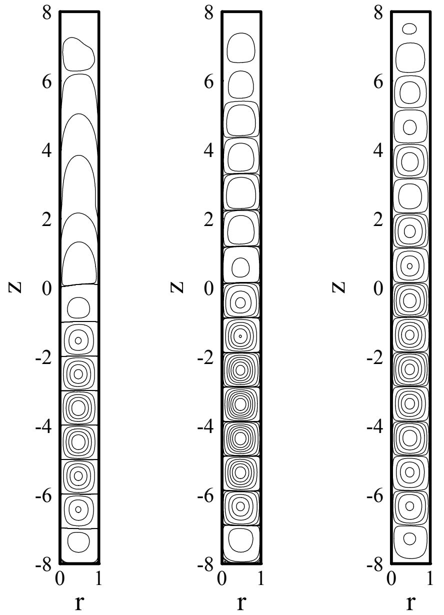


Fig. 5. Variation of radial component of velocity with vertical position at $r = (R_1 + R_2) / 2$, for two fluid when *L/d* = 8 and (a) *m* = 0.2, (b) *m* = 0.9, (c) *m* = 0.99.

The patterns of streamlines for different values of m are shown in Fig.6. It was observed that when m is close to 1 the size of the cells in the two fluids are very similar. As the ratio of viscosities decreases the axial dimension of the cells in the more viscous fluid becomes progressively larger.

The position of the perturbed interface for $m = 0.2$ is shown in Fig.7. This shows there is a rise of the interface near the inner cylinder. However, since the displacement is an eigenvector its negative which shows depression near the inner cylinder would also be a valid solution. Therefore, present analysis does not explain the phenomenon of rod climbing completely.



(a) m = 0.2 (b) m = 0.9 (c) m = 0.99

Fig. 6. Contour plots of stream function for two fluid when L/d = 8 and (a) m = 0.2, (b) m = 0.9, (c) m = 0.99.

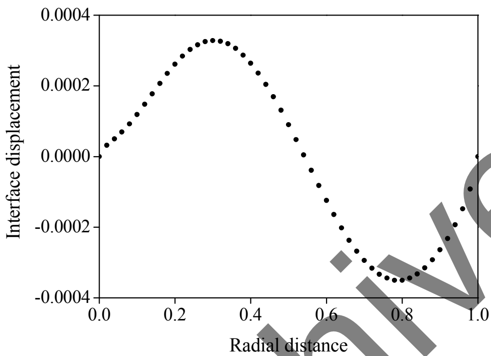


Fig. 7. Variation of interface displacement with radial distance when L/d = 8.

In the experimental setup of Bonn *et al*. (2004) the total height of the two liquid columns was 90 mm while the gap between the cylinders was 25 mm. Accordingly L/d = 45/25 = 1.8 has been taken for numerical calculation. The critical Taylor number in the lower fluid for onset of instability Ta_c vs the ratio of viscosities m, for this value of L/d is shown in Fig. 8.

The radial component of velocity vs z at $r = (R_1 + R_2)/2$ for the same value of L/d, and three values of m are shown in Fig.9.

Now the above results for the two fluid configurations have been compared with results of one fluid. For one fluid with L/d = 1.8 the critical Taylor number is 5150. If one consider one fluid with L/d = 3.6, so that it occupies the total height of the two fluids in the two fluid configuration, the critical Taylor number is 3787. In two fluid configuration, when m = 1 a fluid of same viscosity is occupying the entire length and, therefore, the critical Taylor number is the same as for one fluid with L/d = 3.6. When $m \rightarrow 0$, the upper fluid has very large viscosity compared to the lower fluid. Consequently the perturbation gets damped out in the upper fluid and is effectively confined to the lower fluid. Accordingly in this limit the critical Taylor number is same as for the one fluid configuration with L/d = 1.8.

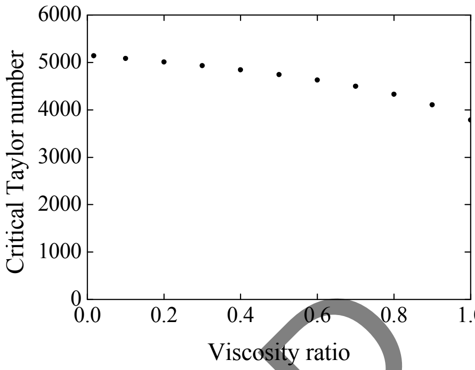
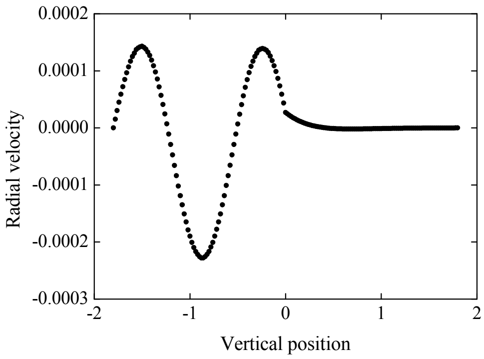


Fig. 8. Variation of critical Taylor number with viscosity ratio, for L/d = 1.8.

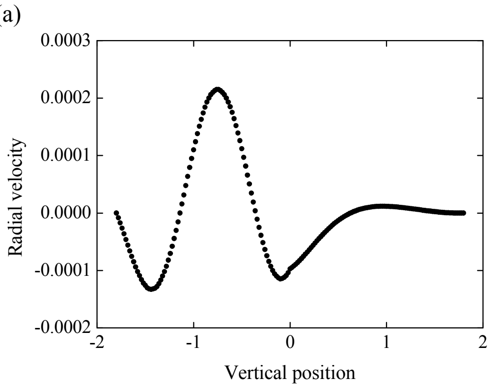
In all these numerical studies it was found that the unstable mode has $Im(s) = 0$. Thus the marginal state is stationary. For the Taylor-Couette problem, where there is only one fluid, the principle of exchange of stability is known to hold (Drazin and Reid, 1981; Yih, 1972a, and 1972b) unless the two cylinders rotate in opposite directions. When there are two axially superposed fluids and the outer cylinder is stationary while the inner cylinder rotates, for the considered parameter values, the marginal state is again found to be stationary when surface tension at the interface is neglected.

4. CONCLUSION

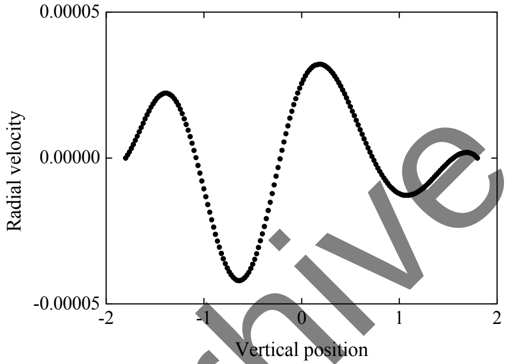
In this study the linear stability of two axially superposed fluids between two coaxial cylinders when the outer cylinder is stationary and the inner cylinder rotates about its axis with constant angular speed has been considered. It is observed that the more viscous fluid has a stabilizing effect on the onset of centrifugal instability. When m = 1, the critical Taylor number in the less viscous fluid for onset of instability is the same as for a one fluid configuration with column height equal to the total height of the two fluid columns in the two fluid configuration. When $m \rightarrow 0$, the critical Taylor number is the same as for a one fluid configuration with column height equal to the height of the less viscous fluid column in the two fluid configurations. These two values may not be very different when L/d is large but can be significantly different when L/d is small. For such a configuration and for values of m different from 0 and 1, a linear stability as carried out here is necessary to accurately predict the onset of instability. Present study also shows a displacement of the interface but this does not explain completely the rod climbing reported in Bonn *et al*. (2004). The linear stability reported here hope to provide a first step towards a theoretical understanding of the experimental results reported by Bonn *et al*. (2004).



(a)



(b)



(c)

Fig. 9. Variation of radial component of velocity with vertical position at $r = (R_1 + R_2) / 2$, for two fluid when $L/d = 1.8$ and (a) $m = 0.2$, (b) $m = 0.6$, (c) $m = 0.99$.

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