Peristaltic Transport with Convective Conditions of Heat and Mass Transfer

F. M. Abbasi¹†, T. Hayat², ³ and A. Alsaedi ³

¹ Department of Mathematics, COMSATS Institute of Information Technology, Islamabad 44000, Pakistan
² Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan
³ Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

†Corresponding Author Email: abbasisarkar@gmail.com
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ABSTRACT

The objective of present communication is to discuss the effect of mass convective condition on the peristaltic transport of viscous fluid in an asymmetric channel. Analysis has been carried out in the presence of Soret and Dufour effects. Comparative study of temperature and concentration fields in the presence and absence of convective conditions through heat and mass transfer is carefully examined. Numerical values of heat and mass transfer rates are computed and analyzed.

Keywords: Convective mass condition; Soret and dufour effects; Comparative analysis.

NOMENCLATURE

- \( d_1 + d_2 \): width of the channel
- \( H_1 \): upper wall
- \( a_2 \): amplitude of the wave at \( d_2 \)
- \( \lambda \): wavelength of the peristaltic waves
- \( \nu \): \( \bar{X} \)-component of the velocity
- \( t \): time
- \( \rho \): density of the fluid
- \( \nu \): kinematic viscosity
- \( T \): dimensional temperature
- \( \Phi \): dimensional heat generation/absorption
- \( C_{\text{z}} \): concentration susceptibility
- \( \bar{c} \): speed of the peristaltic wave
- \( H_3 \): lower wall
- \( b_2 \): amplitude of the wave at \( d_1 \)
- \( \alpha \): phase difference of the waves
- \( \bar{\nu} \): \( \bar{Y} \)-component of the velocity
- Subscripts: \( X, Y, i \)
- \( \bar{p} \): derivative w.r.t the mentioned component
- \( \bar{c}_p \): specific heat and constant pressure
- \( \kappa \): thermal conductivity
- \( D \): mass diffusivity
- \( T_{\text{m}} \): mean fluid temperature
- \( C \): dimensional concentration
- Lower case letters with overbar: quantities in the moving frame of reference \( (\bar{x}, \bar{y}) \)
- Lower case letters without overbar: dimensionless quantities in the moving frame of reference \( (x, y) \)
- \( \delta \): Wave number
- \( \bar{h}_{\text{L}} \): dimensionless peristaltic walls
- \( \alpha \): amplitude ratio for upper wall
- \( \bar{e} \): Reynolds number
- \( T_{\text{w}} \): temperature at upper and lower walls respectively
- \( \bar{\psi} \): dimensionless concentration

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respectively

\[ \beta \]  
\( \beta \) dimensionless heat generation/absorption parameter

\[ Pr \]  
Prandtl number

\[ D_f \]  
Dufour number

\[ S_r \]  
Soret number

\[ Q \]  
dimensional flow rate in the fixed frame

\[ \eta \]  
dimensionless flow rate in the fixed frame

\[ \overline{Q} \]  
time averaged mean flow rate

\[ k_{m,u2} \]  
wall mass transfer coefficients for upper and lower walls respectively

\[ Mt_{u2} \]  
mass transfer Biot numbers for the upper and lower walls respectively

\[ Br \]  
Brinkman number

\[ E \]  
Eckert number

\[ Sc \]  
Schmidt number

\[ \psi \]  
stream function

\[ q \]  
dimensional flow rate in the moving frame

\[ p \]  
dimensionless flow rate in the moving frame

\[ l_{u2} \]  
wall heat transfer coefficients for upper and lower walls respectively

\[ Bl_{u2} \]  
heat transfer Biot numbers for the upper and lower walls respectively

1. INTRODUCTION

There is growing interest of the recent investigators in the interaction of heat transfer with peristaltic motion. Such interest in fact stems because of relevance of such topic in physiology and industry. In particular the peristalsis through heat transfer is important in hemodialysis and oxygenation, cancer tumor treatment, tissue engineering, nuclear reactors, power generators and biomedical engineering. The simultaneous effects of heat and mass transfer are further important because oxygen and nutrients diffuse out of the blood vessels to the neighboring tissues. Although information on peristalsis with heat/mass transfer is quite sizeable but some recent contributions in this direction may be seen through the studies Mekheimer et al. (2008, 2010), Nadeem et al. (2009), Hayat et al. (2011, 2014, 2014a), Abbasi et al. 2014a, 2015b, Tripathi (2012) and Ali et al. (2010). In these preceding studies, the heat/mass transfer effects are not analyzed through convective wall conditions. Very few attempts have been recently made for the peristaltic flows with convective heat transfer conditions. The relevant works in this direction are by Abbasi et al. (2014b, 2015b, 2015c).

The main interest here is to examine the peristaltic flow in an asymmetric channel with convective mass condition. Problem formulation is made in presence of Soret and Dufour effects. Long wavelength and low Reynolds number assumptions are employed. Attention is focused to the exact solutions of temperature and concentration fields in presence/absence of convective heat and mass transfer conditions. This paper is organized as follows. Section two consists of flow equations and boundary conditions. The solution expressions are presented in section three. Section four analyzes the impact of pertinent parameters. Main observations are included in section five.

2. MATHEMATICAL ANALYSIS

Consider the peristaltic waves traveling along the walls of an asymmetric channel of width \( d_1 + d_2 \). An incompressible viscous fluid fills the space inside the channel. We select coordinate system in such a manner that \( X \)-axis lies along the length of the channel and the \( Y \)-axis is taken normal to \( X \)-axis. The peristaltic waves travel on the channel walls in the \( X \) direction with speed \( c \). The wall shapes are given as follows:

\[
H_1(X, \tilde{t}) = d_1 + a_1 \cos \left( \frac{2\pi}{\lambda} (X - \tilde{c} \tilde{t}) \right)
\]

upper wall,

\[
H_2(X, \tilde{t}) = -d_2 - b_1 \cos \left( \frac{2\pi}{\lambda} (X - \tilde{c} \tilde{t}) + \alpha \right)
\]

lower wall.

In above relations \( a_1 \) and \( b_1 \) are the amplitudes of the waves for upper and lower walls respectively, \( \lambda \) is the wavelength and \( \alpha \) is the phase difference of the waves. These waves are responsible for the disturbance in the channel. The equation of conservation of mass for two-dimensional incompressible flow is

\[
\overline{U}_X + \overline{V}_Y = 0. \quad (2)
\]

Here \( \overline{U} \) and \( \overline{V} \) are the longitudinal and transverse components of velocity. The scalar equations through momentum equation are

\[
\overline{U}_t + \overline{U} \overline{U}_X + \overline{V} \overline{U}_Y = -\frac{1}{\rho} \overline{P}_X + \psi \left( \overline{U}_X \overline{U} + \overline{U} \overline{U}_Y \right) \quad (3)
\]

\[
\overline{V}_t + \overline{U} \overline{V}_X + \overline{V} \overline{V}_Y = -\frac{1}{\rho} \overline{P}_T + \psi \left( \overline{V}_X \overline{V} + \overline{V} \overline{V}_Y \right) \quad (4)
\]
where \( \rho, \ \vec{P}, \ \mathbf{v} \) and \( \tilde{t} \) indicate the density, pressure, kinematic viscosity and time respectively. Subscripts denote the partial derivatives. Laws of energy and concentration yield

\[
C_p \left( \frac{\vec{T} \cdot \vec{U}}{T_0} + \frac{T_0}{TT} \right) = \frac{K}{\rho} \left[ \frac{T \cdot \vec{X}}{T_0} + \frac{T_0}{TT} \right] + \frac{\Phi}{\rho} + \mathbf{v} \left[ \frac{\left( \vec{U}^2 + \vec{F}^2 \right)}{T} + \left( \vec{U} + \vec{F} \right) \right] + \frac{DK_F}{\rho C_p} \left[ \frac{T_0}{TT} + \frac{T_0}{TT} \right]
\]

\[
C_t + \vec{U} \cdot \vec{C} + \vec{F} \cdot \vec{C} = D \left[ \frac{T \cdot \vec{X}}{T_0} + \frac{T_0}{TT} \right] + \frac{DK_F}{T_m} \left[ \frac{T_0}{TT} + \frac{T_0}{TT} \right]
\]

(5)

(6)

Here the second term on right side of Eq. (5) is the heat generation/absorption term, third term is due to consideration of viscous dissipation and the last term is due to Dufour effect. Further \( C_p \) is the specific heat at constant pressure, \( T \) the temperature, \( K \) the thermal conductivity of the fluid, \( D \) the mass diffusivity, \( K_F \) the thermal diffusivity ratio, \( C_t \) the concentration susceptibility, \( C \) the concentration and \( T_m \) the fluid mean temperature. In order to transform our problem from the fixed frame (laboratory frame) to a frame of reference moving with the wave with speed \( \vec{c} \) (wave frame) we use the following transformations:

\[
\vec{x} = \vec{X} - c_1 \tilde{t}, \quad \vec{y} = \vec{Y}, \quad \vec{u} = \vec{U} - c_1, \quad \vec{v} = \vec{V}, \quad \rho(x, y) = \rho(X, Y, \tilde{t})
\]

(7)

in which \( \vec{u}, \vec{v} \) and \( \rho \) are the velocity components and pressure in wave frame \((\vec{X}, \vec{Y})\). Considering the dimensionless quantities

\[
x = \frac{x}{\lambda}, \quad y = \frac{y}{\lambda}, \quad u = \frac{u}{c_1}, \quad v = \frac{v}{c_1}, \quad \delta = \frac{\delta}{\lambda}, \quad H_1 = \frac{H_1}{d_1}, \quad H_2 = \frac{H_2}{d_1}, \quad a = \frac{a}{d_2}, \quad b = \frac{b}{d_2}, \quad p = \frac{p}{d_3}, \quad \mu = \frac{\mu}{\rho}, \quad \beta = \frac{\beta}{K T_0}, \quad \eta = \frac{\eta}{C_p},
\]

(8)

and applying the long wavelength and low Reynolds number approximations we have

\[
\psi_{yy}, \quad p_x = \psi_{yy}, \quad p_y = 0,
\]

(9)

(10)

\[
\frac{1}{Sc} \frac{\partial \phi_{yy}}{\partial y} + \frac{\partial^2 \phi_{yy}}{\partial y^2} + \frac{Pr D_f (\phi_{yy})}{\partial y} + \beta = 0,
\]

(11)

where continuity equation is identically satisfied, \( \mu \) denotes the dynamic viscosity, \( \psi \) the stream function, \( Re \) the Reynolds number, \( Br \) the Brinkman number, \( E \) the Eckert number, \( Pr \) the Prandtl number, \( \delta \) the wave number, \( \phi \) dimensionless concentration, \( \theta \) the dimensionless temperature, \( T_0, C_0 \) the temperature and concentration of the upper wall and \( T_1, C_1 \) the temperature and concentration of the lower wall respectively. Equation (10) also indicates that \( p \neq p(y) \).

Taking \( \vec{H}_i \) \((i = 1, 2)\) as functions of \( \vec{X} \) and \( \tilde{t} \), the dimensionless volume flow rate in laboratory frame is

\[
Q = \int_{H_1}^{H_2} \psi dY
\]

(13)

and in wave frame we have

\[
q = \int_{0}^{1} u(x, y) dy
\]

(14)

in which \( h_i \) \((i = 1, 2)\) are functions of \( X \) alone. From Eqs. (8) (13) and (14) we can write

\[
Q = q + c h_1 (x) - c h_2 (x).
\]

(15)

The time averaged flow over a period \( T_f \) is given by

\[
\bar{Q} = \frac{1}{T_f} \int_{0}^{T_f} Q dt,
\]

(16)

which implies that

\[
\bar{Q} = q + cd_1 - cd_2.
\]

(17)

Defining \( \eta \) and \( F \) as the dimensionless mean flows in laboratory and wave frames by

\[
\eta = \frac{Q}{cd_1} = F - q, \quad \eta = F + d,
\]

(18)

and using Eqs. (16) and (18) one has

\[
F = \int_{0}^{h_1} \frac{\partial \psi}{\partial y} dy.
\]

(19)

where

\[
F = \int_{0}^{1} \frac{\partial \phi_{yy}}{\partial y} dy.
\]

(20)

The convective boundary condition for the temperature is defined as follows:

\[
- \frac{K}{\rho \lambda} \frac{\partial T}{\partial y} = \eta (T - T_0).
\]
in which \( K \) is the thermal conductivity, \( l \) is the wall heat transfer coefficient and \( T_w \) is the temperature of the wall. This condition includes in form of heat transfer coefficient \( l \) the material properties of the wall into the problem of heat transfer. The asymmetry of channel demands to choose different heat transfer coefficients for the upper and lower walls, i.e. \( l_1 \) for the upper and \( l_2 \) for the lower wall. We can also check the behavior of temperature when \( l_1 = l_2 \). Analogues to the heat transfer at the boundary we use the condition for the mass transfer

\[-D \frac{\partial C}{\partial y} = k_m (C - C_w),\]

Here \( k_m \) is the mass transfer coefficient. Such coefficient is used to describe the ratio between actual mass flux of a species into or out of the flowing fluid and the driving force that causes such flux and \( C_w \) the concentration at the wall.

The dimensionless boundary conditions can be expressed as follows:

\[
\begin{align*}
\psi &= \frac{F}{2}, \quad \psi_y = -1, \quad y = h_1, \\
\psi &= \frac{F}{2}, \quad \psi_y = -1, \quad y = h_2,
\end{align*}
\]

where

\[
\begin{align*}
h_1(x) &= 1 + a \cos(2\pi x), \\
h_2(x) &= -d - b \cos(2\pi x + \alpha), \\
B_i &= \frac{\int_0^1 d \xi}{K}, \quad B_{12} = \frac{\int_0^1 d \xi}{K}, \quad M_{1} = \frac{k_{m1} d_1}{D} \text{ and } M_{2} = \frac{k_{m2} d_2}{D}.
\end{align*}
\]

In above equations \( l_1 \), \( l_2 \), \( k_{m1} \), and \( k_{m2} \) are the dimensionless transfer coefficients, \( B_i \), \( B_{12} \) are heat transfer Biot-numbers and \( M_{i1,2} \) are the mass transfer Biot-numbers.

\[ \text{3. SOLUTION EXPRESSIONS} \]

Now we consider the following two cases.

**Case I: With convective condition at the boundary**

The obtained closed form solutions for the temperature and concentration are given by

\[
\begin{align*}
\theta &= \frac{1}{[2 - 1 + A((h_1 - h_2)^2 B_{12}^2 + B_{12}^2 B_{24} - B_{12} B_{24})]^{x}} \\
\beta &= \frac{1}{[2 - 1 + A((h_1 - h_2)^2 (M_{11} + M_{22} - (h_1 - h_2) M_{12})]^{x}} \\
&= -A_{10} - A_{12} + A_{13} + A_{15} \beta + A_{16} + h_1^2 A_{16}.
\end{align*}
\]

**Case II: Without convective condition at the boundary**

In this case the physical quantities are denoted by asterisk. The resulting problems here are

\[
\begin{align*}
\psi^*_{y_{yy}} &= 0, \\
\theta^*_{yy} + Br \psi^*_{yy} &= 0, \\
\frac{1}{Sc} \theta^*_{yy} + Sr \theta^*_{yy} &= 0,
\end{align*}
\]

where

\[
\begin{align*}
\psi^* &= \frac{F}{2}, \quad \psi^*_y = -1, \quad \theta^* = 0, \quad \psi^* = 0, \quad at \ y = h_1, \\
\psi^* &= -\frac{F}{2}, \quad \psi^*_y = -1, \quad \theta^* = 1, \quad \psi^* = 1, \quad at \ y = h_2.
\end{align*}
\]

The solutions can be presented into the following forms:

\[
\begin{align*}
\theta^* &= \frac{h_1 - y}{2[h_1 - h_2]^2 (1 + A)} \\
&\left( B_1 - y B_1 + y^2 B_1 - y^3 B_1 + (h_1 - h_2)^2 (h_1 - y) \beta \right)
\end{align*}
\]
The values of $A_i$ and $B_i$ appearing in the solution expressions can be obtained by the usual computations.

4. RESULTS AND DISCUSSION

Our interest in this section is to analyze the behavior of influential parameters. Plots are presented and analyzed for $\theta$, $\theta^*$, $\phi$ and $\phi^*$. A comparative study in presence and absence of convective condition is made. Also the impact of Biot-numbers is examined. Two tables are given for the numerical values of transfer rates at the upper wall.

Fig. 1 (a & b) are plotted for the variations of $D_f$ and $\Pr$ on $\theta$. It is found that temperature is large when compared to $\theta^*$. Consideration of convective boundary condition (for fixed values of...
Fig. 5. Effect of $D_f$ and $Pr$ on the concentration profile $\phi$ and $\phi^*$ when $\eta=1.6$, $a=0.5$, $b=0.4$, 
$\beta=0.5$, $d=1.2$, $Sc=0.5$, $Sr=0.5$, $Mi_1=1$ and $Mi_2=2$.

Fig. 6. Effect of $Sr$ on the concentration profile $\phi$ and $\phi^*$ when $\eta=1.6$, $a=0.5$, $b=0.4$, $\beta=0.5$, 
$d=1.2$, $Sc=0.5$, $Pr=0.5$, $Mi_1=1$ and $Mi_2=2$.

Fig. 7. Effect of $Mi$ on the concentration profile $\phi$ when $\eta=1.6$, $a=0.5$, $b=0.4$, $\beta=0.5$, 
$d=1.2$, $Sc=0.5$ and $Sr=0.5$.

$B_{11}$ and $B_{12}$ do not affect the behavior of any parameter on the temperature. However the temperature increases in the Figs. 1-3. It is found that the temperature increases by increasing $D_f$, $Pr$, $Sr$, $Sc$ and $\beta$. Fig. 4 showed that the temperature profile is decreasing function of heat transfer Biot number.

Figs. 5-7 are plotted to analyze the behavior of concentration profile for different parameters. The dimensionless concentration profile is found to decrease with an increase in $D_f$, $Pr$ and $Sr$. Such decrease is large for the case of $Sr$ when compared with $D_f$ and $Pr$. The concentration field in presence of convective mass condition has been noted less than in its absence.

Numerical values for the heat transfer rate at the upper wall are given in Table 1. It is found that heat transfer rate at the upper wall is increased with an increase in Dufour, Prandtl and Soret numbers. Values of $\theta$ are relatively higher than the values of $(\phi')$ at the boundary. It means that the transfer rate is higher when one takes into account the convective heat transfer at the boundary. Further, the heat transfer rate is an increasing function of heat transfer Biot number.

Table 2 has been prepared for the concentration transfer rate at the upper wall. Such rate increases through an increase in $D_f$, $Pr$ and $Sr$. For Dufour number the value of $\phi'$ is greater than that of $(\phi')^*$, but when $Pr$ and $Sr$ are increased the value of $\phi'$ decreases by a small amount. It is also noticed that increasing the value of mass transfer
Biot-number decreases the transfer rate at the boundary.

Table 1 Effects of various parameters on heat transfer rate at the upper wall

<table>
<thead>
<tr>
<th>$D_f$</th>
<th>$Pr$</th>
<th>$Sr$</th>
<th>$Bi_1$</th>
<th>$Bi_2$</th>
<th>$\theta'(h_i) - (\theta')'(h_i)$</th>
</tr>
</thead>
<tbody>
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<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
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</tr>
<tr>
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Table 2 Effects of various parameters on mass transfer rate at the upper wall

<table>
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<tr>
<th>$D_f$</th>
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<th>$Sr$</th>
<th>$Mi_1$</th>
<th>$Mi_2$</th>
<th>$\phi(h_i) - (\phi')'(h_i)$</th>
</tr>
</thead>
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5. CONCLUSIONS

The peristaltic flow with convective conditions of heat and mass transfer is addressed. The main results can be summarized as follows:

- There is enhancement of temperature in presence of convective heat transfer condition.
- Effect of convective mass condition is to decrease a concentration field.
- Heat transfer rate at the boundary is higher in presence of convective condition.
- Heat transfer rate at the boundary increases with an increase in the heat transfer Biot number $Bi$.
- The mass transfer rate at the boundary decreases by increasing $Mi$.

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