Magneto-Hydrodynamic Flow in a Two-Dimensional Inclined Rectangular Enclosure Heated and Cooled on Adjacent Walls

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(Received September 8, 2014; accepted December 19, 2014)

ABSTRACT

Steady, laminar, natural-convection flow in the presence of a magnetic field in an inclined rectangular enclosure heated from one side and cooled from the adjacent side was considered. The governing equations were solved numerically for the stream function, vorticity and temperature using the finite-volume method for various Grashof and Hartman numbers and inclination angles and magnetic field directions. The results show that the orientation and the strength and direction of the magnetic field have significant effects on the flow and temperature fields. Counterclockwise inclination induces the formation of multiple eddies inside the enclosure significantly affecting the temperature field. Circulation inside the enclosure and therefore the convection become stronger as the Grashof number increases while the magnetic field suppresses the convective flow and the heat transfer rate.

Keywords: Natural convection; Magnetic field; Inclined rectangular enclosure finite-volume; Lorentz force.

NOMENCLATURE

\( A \) aspect ratio
\( B \) magnetic field \([T]\)
\( b \) term source
\( e_B \) unitary vector of the direction of \( B \)
\( F_x \) lorentz force in the \( x \)-direction
\( F_y \) lorentz force in the \( y \)-direction
\( G \) gravitational acceleration, \([m.s^{-2}]\)
\( Gr \) Grashof number \( = \frac{\alpha H^3}{\nu^3} \)
\( H \) height of the cavity, \([m]\)
\( Ha \) Hartmann number \( = \frac{B_0 H \sqrt{\alpha}}{\nu} \)
\( J_x \) electric current in the \( x \)-direction, \([A.m^{-2}]\)
\( J_y \) electric current in the \( y \)-direction, \([A.m^{-2}]\)
\( L \) length of the enclosure, \([m]\)
\( Nu \) Nusselt average number
\( P \) pressure, \([N.m^{-2}]\)
\( Pr \) Prandtl number \( = \frac{\nu}{\alpha} \)
\( Ra \) Rayleigh number \( = \frac{\alpha B_0^2 H^3}{\nu^3} \)
\( R_m \) Reynolds magnetic number
\( S_\phi \) term source
\( T \) temperature, \([K]\)
\( T_C \) cold temperature \([K]\)
\( T_H \) hot temperature \([K]\)
\( U \) dimensionless velocity in the \( x \)-direction
\( U \) velocity in the \( x \)-direction, \([m.s^{-1}]\)
\( V \) dimensionless velocity, \([m.s^{-1}]\)
\( \alpha \) angle of orientation magnetic filed
\( \gamma \) angle of inclination
\( \nu \) velocity in the \( y \)-direction, \([m.s^{-1}]\)
\( \phi \) function
\( \Gamma \) diffusivity term

1. INTRODUCTION

Natural convection in closed enclosures has been extensively studied numerically and experimentally. The study of thermal convection in inclined enclosures is motivated by a desire to find out what effect slope would have on certain thermally driven flows which are found in many engineering applications.

These applications include: building systems containing multi-layered walls, double windows, and air gaps in unventilated spaces; energy systems such as solar collectors, storage devices, furnaces, heat exchangers, and nuclear reactors; material processing equipment such as melting and crystal growth reactors. Thermally driven flows are also found in large scale geophysical, astrophysical, and environmental phenomena.
Most of the research work that has been carried out in this area was focused on enclosures that were differentially heated in one direction (vertically or horizontally) with adiabatic side walls in the other direction. Rather little work has been carried out considering more complex thermal boundary conditions that are normally found in most of the aforementioned practical applications. In these applications, the imposed temperature gradient is neither horizontal nor vertical. Ostrach (1972), in his review on natural convection in enclosures, noted that configurations with more complex boundary conditions can be viewed as an exception among the works on this topic.

When the fluid is electrically conducting and exposed to a magnetic field the Lorentz force is also active and interacts with the buoyancy force in governing the flow and temperature fields. Employment of an external magnetic field has increasing applications in material manufacturing industry as a control mechanism since the Lorentz force suppresses the convection currents by reducing the velocities. Study and thorough understanding of the momentum and heat transfer in such a process is important for the better control and quality of the manufactured products. The study of Oreper and Szekely (1983) shows that the magnetic field suppresses the natural-convection currents and the magnetic field strength is one of the most important factors for crystal formation. Ozoe and Maruo (1987) numerically investigated the natural convection of a low Prandtl number fluid in the presence of a magnetic field and obtained correlations for the Nusselt number in terms of Rayleigh, Prandtl and Hartmann numbers. Garandet and al. (1992) proposed an analytical solution to the governing equations of magnetohydrodynamics to be used to model the effect of a transverse magnetic field on natural convection in a two-dimensional cavity. Rudraiah and al. (1995) numerically investigated the effect of a transverse magnetic field on natural-convection flow inside a rectangular enclosure with isothermal vertical walls and adiabatic horizontal walls and found out that a circulating flow is formed with a relatively weak magnetic field and that the convection is suppressed and the rate of convective heat transfer is decreased when the magnetic field strength increases. Alchaar and al. (1995) numerically investigated the natural convection in a shallow cavity heated from below in the presence of an inclined magnetic field and showed that the convection modes inside the cavity strongly depend on both the strength and orientation of the magnetic field and that horizontally applied magnetic field is the most effective in suppressing the convection currents. Al-Najem and al. (1998) used the power law control volume approach to determine the flow and temperature fields under a transverse magnetic field in a tilted square enclosure with isothermal vertical walls and adiabatic horizontal walls at Prandtl number of 0.71 and showed that the suppression effect of the magnetic field on convection currents and heat transfer is more significant for low inclination angles and high Grashof numbers. Mehmet Cem Ece and Elif Büyükgöz (2006) proposed laminar natural convection flows in the presence of a magnetic field in an inclined rectangular enclosure heated from the left vertical wall and cooled from the top wall while the other walls are kept adiabatic. The boundary conditions considered have a practical importance in cooled ceiling applications. The object of the study is to obtain numerical solutions for the velocity and temperature fields inside the enclosure and to determine the effects of the magnetic field strength and direction, the aspect ratio and the inclination of the enclosure on the transport phenomena. H. Wang and M.S. Hamed (2005) numerically investigated the combined effect of various bidirectional temperature gradients and angles of inclination on flow mode-transition and on hysteresis phenomenon (multi-steady solutions) in rectangular enclosures. Such combined effect, to the authors’ knowledge, has not been investigated yet.

S.K. Ghosh and I. Pop (2002) A note on a hydromagnetic flow in a slowly rotating system in the presence of an inclined magnetic field Magnetohydrodynamics. S.K. Ghosh and I. Pop (2006) An analytical approach to MHD plasma behavior of a rotating environment in the presence of an inclined magnetic field as compared to excitation frequency. S.K. Ghosh, O. Anwar Beg and M. N. Arbab (2013) A study of unsteady rotating hydromagnetic free and forced convection in a channel subject to forced oscillation under an oblique magnetic field. M.N. Kherief and al. (2012) obtained numerical solutions for the velocity and temperature fields inside the enclosure, to determine the effects of the magnetic field strength and direction, the inclination of the enclosure on the transport phenomena. The results show that the dynamic and temperature fields are strongly affected by variations of the magnetic field intensity and the angle of inclination. Numerical simulations have been carried out considering different combinations of Grashof and Hartmann numbers. Ghosh and al. (2010) proposed transient hydromagnetic flow in a rotating channel permeated by an inclined magnetic field with magnetic induction and Maxwell displacement current effects. The previous studies of the laminar natural-convection flows in the presence of a magnetic field in enclosures have dealt with thermal boundary conditions involving mostly isothermal vertical walls and adiabatic horizontal walls and a transverse magnetic field. The present study considers laminar natural convection flows in the presence of a magnetic field in inclined rectangular enclosures heated from the left vertical wall and cooled from the top wall while the other walls are kept adiabatic. The boundary conditions considered have a practical importance in cooled ceiling applications. The object of the study is to obtain numerical solutions for the velocity and temperature fields inside the enclosure and to determine the effects of the magnetic field strength and direction, the aspect ratio and the inclination of the enclosure on the transport phenomena.

2. GEOMETRY AND MATHEMATICAL MODEL

The geometry considered is a rectangular enclosure...
having a length L and a width H, thus with an aspect ratio $A=L/H=4$, filled completely with a molten metal, the Prandtl number of which is $Pr=0.024$. Heated from one side and cooled from the adjacent side was considered, $(T_H>T_C)$. The other walls are supposed to be adiabatic. The inclination of the cavity was also considered, with a varying angle $\gamma$. The flow is subjected to the action of an external uniform and constant magnetic field.

MHD flow, likely to develop in this enclosure, is governed by the equations of continuity, momentum, energy conservation, the Ohm’s law and the conservation the electrical potential. The geometrical configuration is described in the Fig. 1.

![Fig. 1. The model problem.](image)

The governing equations are obtained using the following assumptions:
1. Joule heating is negligible.
2. Viscous dissipation is negligible.
3. The induced magnetic field is negligible because $Rm<<1$ (on the scale of the laboratory), Moreau (1991).
4. The liquid metal is not magnetized ($\mu_r=1$).
5. The liquid metal is incompressible and Newtonian.
6. The Boussinesq approximation holds.

The dimensionless governing equations for the conservation of mass, momentum, and energy, together with appropriate boundary conditions in the Cartesian coordinates system $(x, y)$, are written as follows:

$$\nabla \cdot \vec{V}_d = 0$$

$$\frac{\partial \vec{V}_d}{\partial t} + (\vec{V}_d \cdot \nabla) \vec{V}_d = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{V}_d + g \beta (T - T_0) e_y + F_e$$

$$\frac{\partial \theta}{\partial t} + (\vec{V}_d \cdot \nabla) \theta = \alpha \nabla^2 \theta$$

Where, $\vec{V}$ is the dimensional velocity vector, $p$ the dimensional pressure, $T$ is the dimensional temperature, $g$ is the gravitational acceleration, and $\rho$ the density, $\nu$ is the viscosity and $\alpha$ is the thermal diffusivity of the fluid, respectively.

The interaction between the magnetic field and convective flow involves an induced electric current:

$$\vec{j} = \sigma [-\nabla \phi + \vec{V} \times \vec{B}]$$

The divergence of Ohm’s law $\nabla \cdot \vec{j} = 0$ produces the equation of the electric potential:

$$\nabla^2 \phi = \nabla \cdot (\vec{V} \vec{\phi})$$

Whereas those of $\vec{F}$ have been obtained using the equation:

$$\vec{F} = \nabla \times \vec{B}$$

By neglecting the induced magnetic field, the dissipation and Joule heating, and the Boussinesq approximation is valid; and using $H/\alpha H^2/\nu$, $\rho_0 (a/H)^2$, $a B_0$, and $(T_H - T_C)$ as typical scales for lengths, velocities, time, pressure, potential, and temperature, respectively, the dimensionless governing equations for the conservation of mass, momentum and energy, together with appropriate boundary conditions in the Cartesian coordinates system $(x, y)$, are written as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

$$\frac{\partial U}{\partial t} + (U \nabla) U + V \nabla V = -\frac{1}{\rho} \nabla p + \nu \nabla^2 U + \beta g \sin \gamma \frac{\partial \theta}{\partial Y}$$

$$\frac{\partial V}{\partial t} + (U \nabla) V + V \nabla V = \beta g \cos \gamma \frac{\partial \theta}{\partial X}$$

$$\frac{\partial \theta}{\partial t} + (U \nabla) \theta + \nu \nabla^2 \theta = \frac{1}{Pr \alpha} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)$$

Where: $Gr = g \beta (T_H - T_C) H^3/\alpha^2$ isthe Grashof number, $Ha = \frac{B_0 H \sqrt{\sigma \nu}}{\mu}$ the Hartmann number, and $P \tau = v/\alpha$ the Prandtl number.

The initial conditions that impose the fluid is:

- At $t>0$ the boundary conditions of the dimensionless quantities ($u, v$, and $\theta$) are:
  - At $X=0$ → $y=0$ and $X=A$ → $\partial \theta/\partial y=0$.
  - At $Y=1$ → $\theta=0$ and $Y=0$ → $\partial \theta/\partial y=0$.

### 3. NUMERICAL METHOD

The finite-volume method (1980), is used for the numerical resolution the system of transport eq. (13):

$$\frac{\partial \phi}{\partial t} + \frac{\partial (U \phi)}{\partial X} = \frac{\partial}{\partial X} \left( \Gamma \frac{\partial \phi}{\partial X} \right) + S_{\phi}$$

The discretized form is:

$$A_{ij} \phi_j = A_{ij} \phi_i + A_{ij} \phi_0 + A_{ij} \phi_N + A_{ij} \phi_S + b$$

The eq. (14) is solved using the SIMPLER algorithm (1980) the temporal derivative is discretized using the implicit scheme. Concerning
the spatial discretization, all the convective and the diffusive terms are discretized using the central differencing scheme.

**Numerical scheme**

The dimensionless governing equations were solved for stream function, vorticity and temperature using the centered differences. In this configuration, function $A(P)$ defined in the interfaces follows the following profile:

$$A(P) = 1 - 0.5 |P| \quad (i = e, w, n, s)$$

(15)

This diagram is stable for $|P| \leq 2$, which ensures of the plus coefficients in the equation of discretization.

**4. RESULTS AND DISCUSSION**

**a Code Validation in the Absence of a Magnetic Field**

In the absence of a magnetic field, the momentum equation, eq. (2)-(4) are solved after setting $F_x = F_y = 0$. The results are represented graphically in Figs. 3(a)-(d).

The flow structure is shown by the velocity vectors (Fig. 2(a)) and the velocity profiles (Figs. 2(a) and (b)). Fig. 2(a) shows that at the bottom of the cavity the flow is mainly longitudinal and is directed towards the hot wall (situated at $X = 0.0$) and at the top of the cavity the flow is directed towards the cold wall situated at $X = 4$. These boundary layers extend from the walls to the centre of the cavity, a behavior which is not common in ordinary fluids. From Figs. 2(a) and (b) one can notice that the $U$ and $V$ profiles are linear throughout the core region extending from $(X = 0.25$ to $X = 0.75)$and from $(Y = 0.25$ to $Y = 0.75)$. Comparison of Figs. 2(a) and 2(b) reveals the expected behavior that the flow in the vertical direction is fastest because of the buoyancy-induced acceleration experienced by fluid particles transported in this direction.

A preliminary validation of the numerical method can be done, at this stage, via theoretical estimation of the magnitude of the maximum velocity, which is approximately $30$ (Figs. 2(b) and (c)). One can, in effect, write, for large values of the Rayleigh number, that equilibrium exists between buoyancy forces and inertial forces, and has a value of $50$. This explains the noticed distortion of the isotherms shown in Fig. 2(d).

After that, we confronted our results with the results obtained by the references (1999) where the magnetic field is applied.

![Fig. 2. (a) Velocity vector plot. (b) Distribution of vertical Velocity. (c) Distribution of horizontal Velocity. (d) Isotherms for $Ha=0$ and $Ra=10^5$. (1997)](image)

**Fig. 3. Validation of the results of the current functions obtained for various values of Hartmann with that from Ben Hadid : a- $Ha=0$; b- $Ha=5$; c- $Ha=10$; d- $Ha=100$, $Gr=8x10^{-2}$, $Pr=0$, 01, $A=4$.**

In order to give a better insight into the physics behind the change in flow pattern, sketches of the current path corresponding to $Ha= 25$ and $Ha=50$ are given, respectively, in Figs. 4(a) and (b).

![Fig. 4. (a) Current path for $Ha= 25$. (b) Current path for $Ha=50$.](image)

The Lorentz forces produced by the interaction between these currents and the applied vertical field are given, respectively, in Figs. 5(a) and (b). As can be noticed from Figs. 4, the flowing fluid generates under the action of the magnetic field, currents which are positive in the neighborhood of the top wall and negative in the neighborhood of the bottom wall. This difference in sign is due to the
different directions of the fluid in contact with the top and bottom walls. Because of this difference in sign, the Lorentz force acting on the top layers of the fluid is negative (i.e. a retarding force) and that acting on the bottom layers positive (i.e. also a retarding force since the fluid flows in the negative). When the value of $Ha$ is increased, the magnitude of Lorentz forces increases (Fig. 5) and therefore reduces the magnitude of the velocity. This provokes the damping of the flow.

![Fig. 5. (a) Component Fx of the Lorentz force for Ra=800 and Ha=25. (b) Component Fx of the Lorentz force for Ra=800 and Ha=50.](image)

(b) Test and Choice the Grid

We go tested the effect of various kinds of grid to see the behavior of the Nusselt number average and the current function in the enclosure and this by fixing the following parameters: $Pr = 0.024$, $Gr = 5000$, $\alpha = 0$ and $Ha = 30$. With a dimensional step: $\Delta t=0.0001$.

<table>
<thead>
<tr>
<th>Grid</th>
<th>$Nu_{1=0}$</th>
<th>$Nu_{r=1}$</th>
<th>$\Psi_{\text{min}}$</th>
<th>$\Psi_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>82x32</td>
<td>0.2501304</td>
<td>0.2500866</td>
<td>1.794190</td>
<td>2.440E-07</td>
</tr>
<tr>
<td>96x42</td>
<td>0.2501372</td>
<td>0.2500819</td>
<td>1.790469</td>
<td>6.619E-07</td>
</tr>
<tr>
<td>112x42</td>
<td>0.2501345</td>
<td>0.2500816</td>
<td>1.789955</td>
<td>6.487E-07</td>
</tr>
</tbody>
</table>

The grid used for $Ha = 0$ was chosen after performing grid independency tests. The computed average Nusselt numbers for grids finer than 112x42 only differ hence the choice of this grid fig.6. Convergence of the numerical solution was obtained when the mass, momentum and energy residuals are below $10^{-4}$.

![Fig. 6. Grid employed for the rectangular enclosure A=4.](image)

c Discussion

Streamlines and isotherms for ($Ha = 0$) are presented herein Figs. 7–8 for various enclosure inclinations. When the enclosures tilted 45° in the clockwise direction ($\gamma=45^\circ$), the hot and cold walls form the upper left and right walls of the enclosure while the adiabatic walls constitute the lower walls. On the other hand, the hot and cold walls form the lower and upper left walls of the enclosure while the lower and upper right walls are adiabatic when the enclosure is tilted 45° in the counterclockwise direction ($\gamma=-45^\circ$). The buoyancy force which ascends the fluid particles heated near the hot acts parallel to the hot wall when the enclosures are not inclined but forces them toward to and away from the hot wall when the enclosure is inclined in the clockwise and counterclockwise directions, respectively. Therefore, while the streamlines form a single eddy with clockwise rotation for $\gamma=0^\circ$ and $-45^\circ$, a pair of counter-rotating eddies are formed for $\gamma=45^\circ$.

![Fig. 7. Evolution of the current function for various orientation angle of the inclination. a) $\gamma=0^\circ$, b) $\gamma=45^\circ$, c) $\gamma=-45^\circ$. And $Ha=0$, $Gr=10000$, $\alpha=0^\circ$.](image)

Circulation is weak due to relatively low Grashof number and the isotherms are spread almost radially between the hot and cold walls resembling heat transfer by pure conduction only.

![Fig. 8. Evolution of the Isotherm for various orientation angle of the inclination. a) $\gamma=0^\circ$, b) $\gamma=45^\circ$, c) $\gamma=-45^\circ$. And $Ha=0$, $Gr=10000$, $\alpha=0^\circ$.](image)

Streamlines and isotherms in a tall rectangular enclosure with the presence of a strong magnetic field ($Ha = 100$) for various enclosure inclinations, magnetic field directions and Grashof numbers are
near the lower hot wall as the Grashof number increases up to $10^5$ and $10^7$. This result is stronger when the magnetic field is applied normally to the cold wall ($\alpha=90^\circ$) and when the enclosure is tilted $45^\circ$ in the clockwise direction ($\gamma=45^\circ$) or not tilted ($\gamma=0^\circ$). The flow field displays a very complex pattern for the case of $\gamma=45^\circ$. There exist a pair of counter-rotating when the magnetic field is applied in the clockwise direction ($\alpha=90^\circ$ or $\alpha=0^\circ$) or when the enclosure is not tilted ($\gamma=0^\circ$). The lower eddy encloses two loops with a stagnation point between them for $Gr = 10^5$. When the magnetic field is applied in the clockwise direction ($\gamma=45^\circ$), the lower eddy is observed to extend to the upper hot wall, and the upper eddy is pushed toward the upper adiabatic wall.

The loops within the lower eddy are more visible in this case. As the magnetic field is further...
rotated counterclockwise and applied normal to the cold wall, the lower eddy grows along the hot wall with the loops disappearing and the dividing streamline is almost parallel to the hot wall. Circulation of eddies increases as the Grashof number increases and the loops within the lower eddy disappear when the magnetic field is applied X-direction to the hot wall. When the magnetic field is applied Y-direction, the upper eddy grows substantially and reaches the hot wall dividing the lower eddy into multiple eddies.

However, when the magnetic field is applied X-direction to the cold wall, a pair of counter-rotating eddies are formed with the dividing streamline almost parallel to the hot wall. The isotherms are almost equally spaced between the hot and cold walls for \( Gr = 10^3 \) but the increased circulation and the formation of multiple eddies cause a kinky behavior in the isotherms for \( Gr = 10^5 \) and \( Gr = 10^7 \).

We notice that for a small number of Gr, the flow generates very weak velocity gradients, when the Gr number increases; the flow induced by the increasing buoyancy forces becomes animated. Significant velocity gradients are then localized near the walls, resulting in the production of multiple eddies. This is well illustrated in Figs. 11-a, b and c.

Concerning the horizontal normalized velocity profiles, they are shown in Figs. 11 of the X-

Fig. 10. Evolution of the Isotherm for various Grashof numbers according to a Hartmann number, orientation angle of the magnetic field and the inclination angle.
direction magnetic field and Y-direction magnetic field.

It is clear from the results that as \( Ha \) are increased; the velocity components tend to diminish. In fact, for \( Ha=100 \), their values are practically equal to zero in the major part of the cavity except near the end walls. It is clear that the use of a magnetic field can strongly decrease the flow intensity, but cannot completely inhibit fluid motion.

The enclosure slope has a strong effect on the flow and the heat transfer behavior. A single cell is obtained; it appears to be completely stable, symmetrical and fills all the enclosure, the effect of inclination on the velocity diminishes in the presence of the magnetic field.

Average Nusselt numbers are listed in Table 2. Average Nusselt number increases naturally with Grashof number and it is substantially reduced by the magnetic field. The magnetic field applied Y-direction to the hot wall is more effective reducing the convection and therefore the heat transfer for enclosure and the magnetic field applied Y-direction to the cold wall is more effective reducing the convection for shallow enclosures. The average Nusselt number is slightly reduced by the counterclockwise inclination in the enclosure. The effect of inclination on the average Nusselt number diminishes in the presence of the magnetic field.

5. CONCLUSIONS

The present study considers laminar natural-convection flow in the presence of a magnetic field in an inclined rectangular enclosure heated from the left vertical wall and cooled from the top wall while the other walls are kept adiabatic. The flow characteristics and the convection heat transfer inside the tilted enclosure, depend strongly upon the strength and direction of the magnetic field and the inclination of the enclosure. Circulation and convection become stronger with increasing Grashof numbers but they are significantly
suppressed by the presence of a strong magnetic field. As a result, formation of multiple eddies of counterclockwise inclination greatly influences the temperature field.

The local Nusselt number increases considerably with Grashof number since the circulation becomes stronger.

The magnetic field significantly reduces the local Nusselt number by suppressing the convection currents.

REFERENCES


