Heat and Mass Transfer Effects on Unsteady MHD Natural Convection Flow of a Chemically Reactive and Radiating Fluid through a Porous Medium Past a Moving Vertical Plate with Arbitrary Ramped Temperature

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ABSTRACT

Investigation of unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, chemically reactive and optically thin radiating fluid past an exponentially accelerated moving vertical plate with arbitrary ramped temperature embedded in a fluid saturated porous medium is carried out. Exact solutions of momentum, energy and concentration equations are obtained in closed form by Laplace transform technique. The expressions for the shear stress, rate of heat transfer and rate of mass transfer at the plate for both ramped temperature and isothermal plates are derived. The numerical values of fluid velocity, fluid temperature and species concentration are displayed graphically whereas those of shear stress, rate of heat transfer and rate of mass transfer at the plate are presented in tabular form for various values of pertinent flow parameters. It is found that, for isothermal plate, the fluid temperature approaches steady state when \( t > 1.5 \). Consequently, the rate of heat transfer at isothermal plate approaches steady state when \( t > 1.5 \).

Keywords: Natural convection; Magnetic field; Chemical reaction; Radiation; Arbitrary ramped temperature.

NOMENCLATURE

- \( a \): surface acceleration parameter
- \( a' \): absorption coefficient
- \( B_0 \): uniform magnetic field
- \( C' \): species concentration
- \( c_p \): specific heat at constant pressure
- \( D_m \): chemical molecular diffusivity
- \( g \): acceleration due to gravity
- \( G_\theta \): thermal Grashof number
- \( G_s \): solutal Grashof number
- \( k \): thermal conductivity of the fluid
- \( K'_j \): permeability of porous medium
- \( K_1 \): permeability parameter
- \( K_2 \): chemical reaction coefficient
- \( K_s \): chemical reaction parameter
- \( M \): magnetic parameter
- \( N \): radiation parameter
- \( P_r \): Prandtl number
- \( q'_r \): radiating flux vector
- \( T' \): fluid temperature
- \( t_0 \): critical time for rampedness
- \( t_i \): non-dimensional critical time for rampedness
- \( U_o \): characteristic velocity
- \( a' \): fluid velocity in \( x' \) direction
- \( S_c \): Schmidt number
- \( \beta' \): coefficient of thermal expansion
- \( \beta' \): coefficient of expansion for species concentration
- \( \sigma \): electrical conductivity
- \( \rho \): fluid density
- \( \sigma' \): Stefan-Boltzmann constant
- \( \nu \): kinematic coefficient of viscosity
1. INTRODUCTION

Theoretical/experimental investigation of problems of unsteady hydromagnetic natural convection flow of an electrically conducting fluid within porous and non-porous media has received considerable attention of several researchers during past few decades due to its overwhelming and important applications in many areas of science and engineering which includes geophysics, astrophysics, electronics, aeronautics, metallurgy, chemical and petroleum engineering etc. Keeping in view the importance of this fluid flow, several researchers investigated unsteady hydromagnetic natural convection flow of an electrically conducting fluid past bodies with different geometries under different initial and boundary conditions. Mention may be made of the research studies of Gupta (1960), Pop (1969), Chamkha (1997, 2000), Helmy (1998), Kim (2000), Raptis et al. (2003), Makinde and Tshela (2014) and Seth et al. (2013, 2014).

Hydromagnetic natural convection flow of radiating and non-radiating fluid with heat and mass transfer in porous and non-porous media is studied by several researchers due to its varied and wide applications in astrophysics, geophysics, aeronautics, electronics, metallurgy, chemical and petroleum industries. Hydromagnetic natural convection flow of an electrically conducting fluid in a fluid saturated porous medium has also been successfully exploited in crystal formation.

Oreper and Szekely (1983) have found that the presence of a magnetic field can suppress natural convection currents and the strength of magnetic field is one of the important factors in reducing non-uniform composition thereby enhancing quality of the crystal. In addition to it, hydromagnetic problems with heat and mass transfer is of much significance in MHD flow-meters, MHD energy generators, MHD pumps, controlled thermo-nuclear reactors, MHD accelerators etc. Hossain and Mandal (1985) investigated mass transfer effects on unsteady hydromagnetic free convection flow past an accelerated vertical porous plate. Jha (1991) studied hydromagnetic free convection and mass transfer flow past a uniformly accelerated vertical plate through a porous medium when magnetic field is fixed with the moving plate.


Eldabe et al (2011) discussed unsteady MHD flow of a viscous and incompressible fluid with heat and mass transfer in a porous medium near a moving vertical plate with time-dependent velocity. Prakash et al. (2013) investigated diffusion thermo and radiation effect on unsteady MHD free convection flow through porous medium past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion.

In most of the chemical engineering processes, chemical reaction occurs between a foreign mass and the fluid. Chemical reactions can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. These processes take place in numerous industrial applications viz. polymer production, manufacturing of ceramics or glassware, food processing etc.


Chamkha et al. (2011) discussed the effects of Joule heating, chemical reaction and thermal radiation on unsteady hydromagnetic natural convection boundary layer flow with heat and mass transfer of a micro polar fluid from a semi-infinite heated vertical porous plate in the presence of a uniform transverse magnetic field. Bhattacharya and Layek (2012)
obtained similarity solution of MHD boundary layer flow with mass diffusion and chemical reaction over a porous flat plate with suction/blowing. Kishore et al. (2013) studied the effects of radiation and chemical reaction on unsteady hydromagnetic natural convection flow of a viscous fluid past over an exponentially accelerated vertical plate.

Mohamed et al. (2013) investigated unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible and electrically conducting and radiating fluid past through a porous medium near an impulsively moving hot vertical plate in the presence of homogeneous chemical reaction of first order and temperature dependent heat sink.

In fact, natural convection flows are generally modelled by the researchers under the consideration of uniform surface temperature or uniform surface heat flux. But, in many physical situations, the temperature of the bounding surface may require non-uniform or arbitrary wall conditions. Moreover, there may be step discontinuities in the surface temperature or ramped surface temperature. Keeping in view this fact, several researchers investigated natural convection from a vertical plate with step discontinuities in the surface temperature considering various aspects of the problem. Mention may be made of the research investigations of Hayday et al. (1967), Kelleher (1971), Kao (1975) and Lee and Yovanovich (1991).

In recent years, several researchers investigated unsteady hydromagnetic free convection flow past a vertical plate with ramped temperature considering various variations in the problem. Mention may be made of the research studies of Rajesh (2010), Samiulhaq et al. (2012), Das (2012), Nandkeolyar and Das (2013, 2014), Ahmed and Das (2013a, b), Nandkeolyar et al. (2013a, 2013b), Narañí and Sulaiman (2013), Das et al. (2014), Kundu et al. (2014). Recently, Seth et al. (2014, 2015a, b) considered hydromagnetic natural convection heat and mass transfer flow with Hall current past an infinite moving vertical plate with ramped temperature in a rotating medium considering different aspects of the problems.

However, in all these investigations, researchers have considered the time interval for rampedness in the plate temperature as fixed i.e. 0 < $\tau'$ ≤ $t_0$ (where $\tau'$ and $t_0$ being time and characteristic time respectively). Recently, Ahmed and Dutta (2014) investigated magnetohydrodynamic transient flow of a viscous, incompressible, electrically conducting and optically thin radiating fluid past an impulsively moving infinite vertical plate with temporary ramped wall temperature. They considered time $t_0$ as an arbitrary constant in place of characteristic time. But they have non-dimensionalized physical variables, namely, $y'$ and $r'$ in their model using $t_0$ . This is possible only when $t_0$ is considered as a characteristic time. With this non-dimensionalization process they have defined Reynolds number $R_e = \frac{U_0^2 L}{\nu}$, $U_0$, and $\nu$ being the uniform velocity of the plate at time $\tau'$ and kinematic coefficient of viscosity respectively) which is not correct. If $t_0$ is characteristic time then its dimension in terms of characteristic length $L$ is $\frac{L^2}{\nu}$ then $t_0$ is equal to $\frac{U_0^2 L}{\nu^2}$ which is equal to $R_e^2$. Due to this reason, the definition of Reynolds number also is not taken correctly. Also interval of ramped profile reduces to $0 < \tau' \leq 1$ ($\nu$ being non-dimensional time) only when $t_0$ is considered characteristic time. In our opinion $U_0^2 L/\nu$ is a non-dimensional arbitrary time constant which we have named as critical time for rampedness. This is true if $t_0$ is not a characteristic time. It is to be noted that time interval for ramped profiles varies from material to material depending upon the specific heat capacity of the material. Arbitrary ramped profiles appear in real world situation in building air-conditioning systems, fabrication of thin-film photovoltaic devices, phase transition in processing of materials, turbine blade heat transfer, heat exchangers etc.

Purpose of present investigation is to study unsteady hydromagnetic natural convection heat and mass transfer flow of an electrically conducting, viscous, incompressible, chemically reactive and optically thin heat radiating fluid past an exponentially accelerated moving vertical plate through fluid saturated porous medium with arbitrary ramped wall temperature. This problem is totally different from the research paper investigated by Ahmed and Dutta (2014).

In our paper, we have considered medium as porous, chemically reactive and optically thin radiating fluid when fluid flow is induced due to exponentially accelerated moving vertical plate through fluid saturated porous medium with arbitrary ramped wall temperature. We have approximated gradient of radiating flux vector i.e. $\frac{\partial Q}{\partial y'}$ by approximation suggested by Raptis (2011) in place of Cogley et al. (1968) which was adopted by Ahmed and Dutta (2014).

This research study may have strong bearings on numerous problems of practical interest where initial temperature profiles are of much significance in designing of so many hydromagnetic devices and in several industrial processes occurring at high temperatures where the effects of thermal radiation play a vital role in the fluid flow characteristics. We have compared numerical values of shear stress at the plate with those of Das et al. (2011) as a special case of our results which are in excellent agreement with the numerical values of Das et al. (2011).

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider unsteady magnetohydrodynamic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reactive and optically thin heat radiating
fluid through fluid saturated porous medium past an infinite moving vertical plate with arbitrary ramped temperature. Choose the coordinate system in such a way that \( x' \)-axis is along the length of the plate in the upward direction and \( y' \)-axis normal to plane of the plate in the fluid. The fluid is permeated by a uniform transverse magnetic field \( B_0 \) applied in a direction parallel to \( y' \)-axis. Initially i.e. at time \( t' \leq 0 \), both the fluid and plate are at rest and maintained at a uniform temperature \( T'_0 \). Also species concentration at the surface of the plate as well as at every point within the fluid is maintained at uniform concentration \( C'_w \).

At time \( t' > 0 \), plate is exponentially accelerated with velocity \( U_0e^{v't} \) in \( x' \)- direction (\( U_0 \) being characteristic velocity). Temperature of the plate is raised or lowered to \( T'_0 + (T'_w - T'_0)t'/t_0 \) when \( 0 < t' \leq t_0 \) (\( t_0 \) being critical time for rampedness) and uniform temperature \( T'_0 \) when \( t' > t_0 \). Also at time \( t' > 0 \), species concentration at the surface of the plate is raised to uniform species concentration \( C'_w \) and is maintained thereafter. The fluid is considered as a gray, emitting-absorbing radiation but non scattering medium. It is assumed that there exists a homogeneous chemical reaction of first order with constant rate \( K'_1 \) between diffusing species and the fluid. Physical model of the problem is presented in Fig. 1.

![Physical model of the problem.](image)

Fluid is metallic liquid or partially ionized whose magnetic Reynolds number is very small. Hence, the induced magnetic field generated by fluid motion is negligible in comparison to the applied one (Crammer and Pai, 1973). Thus, the magnetic field \( \vec{B} = (0, B_0, 0) \). Since, no external electric field is applied into the flow-field, the effect of polarization of fluid becomes negligible (Meyer, 1958). With these assumptions, the governing equations for unsteady magnetohydrodynamic natural convection flow of an electrically conducting, viscous, incompressible, chemically reactive and optically thin heat radiating fluid through fluid saturated porous medium are given by

\[
\begin{align*}
\ddot{u}' + v\dot{u}' - \frac{\sigma B_0^2}{\rho} \left( \frac{\sigma B_0^2}{\rho} \right) u' &= \frac{\Gamma - T'}{K_1'} + g\beta'(T' - T'_0) \\
\dot{T}' &= \frac{k}{\rho c_p} \frac{\partial T'}{\partial y'} - \frac{1}{\rho c_p} \frac{\partial q_j}{\partial y'} \\
\dot{C}' &= D_2 \frac{\partial^2 C'}{\partial y'^2} - K'_2 (C' - C'_w)
\end{align*}
\]

where \( u' \), \( T' \), \( K'_1 \), \( \beta \), \( \rho \), \( q_j \), \( v \), \( \sigma \), \( \beta \), and \( \beta' \) are, respectively, fluid velocity in \( x' \)- direction, fluid temperature, permeability of porous medium, chemical reaction coefficient, thermal conductivity, specific heat at constant pressure, species concentration, chemical molecular diffusivity, radiating flux vector, kinematic coefficient of viscosity, electrical conductivity, fluid density, acceleration due to gravity, coefficient of thermal expansion and coefficient of expansion for species concentration.

Appropriate initial and boundary conditions for the fluid flow problem are given by

\[
\begin{align*}
&u' = 0, T' = T'_0, C' = C'_w \quad \text{for } t' > 0, t' \leq 0, \quad (4a) \\
&u' = U_0e^{v't}, C' = C'_w \quad \text{at } y' = 0 \quad \text{for } t' > 0, \quad (4b) \\
&T' = T'_0 + (T'_w - T'_0)t'/t_0 \quad \text{at } y' = 0 \quad \text{for } 0 < t' \leq t_0, \quad (4c) \\
&T' = T'_w \quad \text{at } y' = 0 \quad \text{for } t' > t_0, \quad (4d) \\
&C' \to C'_w \quad \text{as } y' \to \infty \quad \text{for } t' > 0. \quad (4e)
\end{align*}
\]

For an optically thin gray fluid the local radiant absorption (Raptis, 2011) is expressed as

\[
\frac{\partial q_j}{\partial y'} = -4\alpha' \sigma' (T'_w - T'),
\]

where \( \alpha' \) is absorption coefficient and \( \sigma' \) is Stefan-Boltzmann constant.

It is assumed that the temperature difference between the fluid in the boundary layer and free stream is sufficiently small so that \( T'' \) may be expressed as a linear function of \( T' \). This is accomplished by expanding \( T'' \) in a Taylor series about free stream temperature \( T'_w \). Neglecting second and higher order terms, \( T'' \) is expressed as \( T'' \approx 4T'_w T' - 3T'_w^2 \).

Using equations (5) and (6), in equation (2), we obtain

\[
\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial T'}{\partial y'} - \frac{16\alpha' \sigma' T'_w}{\rho c_p} (T' - T'_w).\]

In order to represent equations (1), (3) and (7) along with initial and boundary conditions (4a) to (4e) in non-dimensional form, we are introducing the following non-dimensional variables and parameters
\[ y = \frac{U_0}{v}y', t = \frac{U_0^2}{v}t', u = \frac{u'}{v}, T = \frac{T - T_0}{T_0' - T_0'}, C = \frac{C - C_0}{C_0'}, a = \frac{\alpha'}{C_0'}, G = \frac{\nu\beta\beta'(C_0' - C_0)}{U_0^2}, G_r = \frac{\nu\beta\beta'(C_0' - C_0)}{U_0^2}, K_r = K_r' \sqrt{\nu}, K_r = \frac{\nu K_r'}{U_0^2}, M = \frac{\sigma \beta'}{\rho v^2}, N = \frac{16\alpha\nu\sigma T_0'}{U_0^2 \nu v^2}, P_r = \frac{\nu v^2}{k}, S_r = \frac{v}{D_u} \text{ and } t = \frac{U_0^2}{v}t \]

where \( a, G_r, G_r', K_r, K_r', M, N, P_r, S_r \) and \( t \) are, respectively, surface acceleration parameter, thermal Grashof number, solutal Grashof number, permeability parameter, reaction parameter, magnetic parameter, radiation parameter, Prandtl number, Schmidt number and non-dimensional fixed time which varies from material to material of plate depending on specific capacity of material which we have named as non-dimensional critical time for rampedness.

The equations (1), (3) and (7), in non-dimensional form, reduce to

\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{K_r} + G_r T + G_r C, \]
\[ \frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} - NT, \]
\[ \frac{\partial C}{\partial t} = \frac{1}{S_r} \frac{\partial^2 C}{\partial y^2} - K_r C, \]

Initial and boundary conditions (4a) to (4e) in non-dimensional form, are presented below:

\[ u = 0, T = 0, C = 0 \text{ for } y \geq 0 \text{ and } t \leq 0, \]
\[ u = e^{\alpha t'}, C = 1 \text{ at } y = 0 \text{ for } t > 0, \]
\[ T = \frac{L}{t_t} \text{ at } y = 0 \text{ for } 0 < t \leq t_t, \]
\[ T = 1 \text{ at } y = 0 \text{ for } t > t_t, \]
\[ u \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty, t > 0. \]

Exact solution for fluid velocity \( u(y,t) \), fluid temperature \( T(y,t) \) and species concentration \( C(y,t) \) is obtained by solving equations (9) to (11) subject to the initial and boundary conditions (12a) to (12c) using Laplace transform technique and are presented in the following simplified form after simplification:

\[ u(y,0) = \frac{e^{\alpha t}}{2} f_1(y,\lambda,\lambda,t) - \frac{1}{2\xi} \left[ e^{\xi y} \left\{ f_1(y,\lambda,\lambda,t) - f_2(y,\lambda,\lambda,K_r,S) t \right\} - f_3(y,\lambda,\lambda,K_r,S) t^2 \right] + f_2(y,0,K_r,S) t + G(y,t) + H(t-t_t)G(y,t-t_t), \]

\[ T(y,t) = \frac{1}{t_t} \left[ \frac{P_r(y,t) - H(t-t_t)}{t_t P_r(y,t-t_t)} \right], \]

\[ C(y,t) = \frac{1}{2} e^{\xi y} \left[ \frac{S}{2\sqrt{\xi}} + \sqrt{\xi M} \right] + e^{-\xi y} \left[ \frac{S}{2\sqrt{\xi}} - \sqrt{\xi M} \right], \]

where

\[ G(y,t) = \frac{1}{2} \left[ \frac{\lambda}{2\lambda_1} \right] e^{\lambda y} \left\{ f_1(y,P,N,\lambda_1,t) - f_2(y,1,\lambda_1,\lambda_1,t) - \lambda_2 \left[ f_1(y,P,N,\lambda_1,t) + f_2(y,1,\lambda_1,\lambda_1,t) \right] \right\}, \]

\[ P_r(y,t) = \frac{1}{2} \left[ \left( 1 + \frac{P}{N^\lambda_1} \right) e^{\lambda_1 y} \left\{ \frac{S}{2\sqrt{\lambda_1}} + \sqrt{\lambda_1 M} \right\} - \frac{P}{N^\lambda_1} e^{\lambda_1 y} \left\{ \frac{S}{2\sqrt{\lambda_1}} - \sqrt{\lambda_1 M} \right\} \right], \]

\[ \lambda_1 = \frac{K_r S_R - \lambda_1}{1 - S_R}, \lambda_2 = \frac{P}{N^\lambda_1 - \lambda_1} - \frac{1 - P}{N^\lambda_1}. \]

The expressions for \( f_i(i=1,2,3,4) \) are provided in Appendix.

2.1 Solution for Unit Prandtl and Unit Schmidt Number

Solution (13) for fluid velocity \( u(y,t) \) is not valid for unit Prandtl number and unit Schmidt number. Since Prandtl number is a measure of relative strength of thermal diffusivity to molecular (mass) diffusivity of fluid, the case \( P_r = 1 \) corresponds to those fluids for which the viscous, thermal and concentration boundary layer thicknesses are same order of magnitude. There are some fluids of practical significance which belong to this category (Chen, 2004). Setting \( P_r = 1 \) and \( S_r = 1 \) in equations (10) and (11) and following the same procedure adopted earlier, exact solution for fluid velocity \( u(y,t) \), fluid temperature \( T(y,t) \) and fluid concentration \( C(y,t) \) is obtained and presented in the following simplified form:

\[ u(y,0) = \frac{1}{2} e^{\alpha t} f_1(y,\lambda,1,a,0) + \frac{\lambda_1}{2} f_1(y,\lambda_0,0,0), \]
\[ -f_1(y,K_r,0,t) + G(y,t) - H(t-t_t) \times G(y,t-t_t), \]
\[ T(y,t) = \frac{1}{t_t} \left[ \frac{P_r(y,t) - H(t-t_t)}{t_t P_r(y,t-t_t)} \right]. \]
where
\[ \alpha_i = \frac{G_i}{(N - \lambda_i)} \quad \text{and} \quad \gamma_i = \frac{G_i}{K_i - \lambda_i}. \]
The expression for \( f_i \) is provided in Appendix.

### 2.2 Solution when Fluid is in Contact to Isothermal Plate

Solution (13) to (15) is the solution for fluid velocity, fluid temperature and species concentration for natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible, chemically reactive and optically thin radiating fluid past an exponentially accelerated moving vertical plate with arbitrary ramped temperature. In order to emphasize the effects of ramped temperature distribution within the plate on fluid flow, it may be worthwhile to compare such flow with the one near an exponentially accelerated moving vertical plate with uniform temperature. Keeping in view the assumptions made earlier, the solutions for fluid velocity and fluid temperature for the flow past an exponentially accelerated moving isothermal vertical plate is obtained and presented in the following form:

\[ u(y,t) = \left( \frac{e^{\alpha y}}{2} \right) f_i(y, \lambda_i, a_i) - \frac{1}{2 \lambda_i^2} \left[ \int f_i(y, \lambda_i, t) \right] \]

\[ - f_i(y, \lambda_i, N, P, t) \right] [f_i(y, \lambda_i, 0, t) + \]

\[ \left\{ f_i(y, \lambda_i, \lambda_i, t) - f_i(y, \lambda_i, K_i, P, t) \right\} \]

\[ - \left[ f_i(y, \lambda_i, 0, t) - f_i(y, \lambda_i, K_i, P, t) \right], \]

\[ T(y,t) = \left( \frac{1}{2} \right) e^{\gamma y} \text{erfc} \left( \frac{y}{2 \sqrt{P} t} - \sqrt{N t} \right) + e^{-\gamma y} \times \text{erfc} \left( \frac{y}{2 \sqrt{P} t} - \sqrt{N t} \right). \]

### 2.3 Shear Stress and Rate of Heat Transfer at the Plate

The expressions for shear stress \( \tau \) and rate of heat transfer \( N \alpha \) at the plate are obtained and presented in the following form for both ramped temperature and isothermal plates.

(i) For ramped temperature plate:

\[ \tau = \frac{\partial u}{\partial y} \bigg|_{y=0} \]

\[ = e^{\alpha y} f_i \left( \lambda_i, a_i, t \right) - \frac{1}{2 \lambda_i^2} \left[ \int f_i(y, \lambda_i, 0, t) + \left\{ f_i(y, \lambda_i, \lambda_i, t) - f_i(y, \lambda_i, K_i, P, t) \right\} \right] \]

\[ - \left[ f_i(y, \lambda_i, 0, t) - f_i(y, \lambda_i, K_i, P, t) \right], \]

\[ T(y,t) = \left( \frac{1}{2} \right) e^{\gamma y} \text{erfc} \left( \frac{y}{2 \sqrt{P} t} - \sqrt{N t} \right) + e^{-\gamma y} \times \text{erfc} \left( \frac{y}{2 \sqrt{P} t} - \sqrt{N t} \right). \]

(ii) For isothermal plate:

\[ \tau = \frac{\partial u}{\partial y} \bigg|_{y=0} \]

\[ = -f_i \left( \lambda_i, a_i, t \right) + \frac{1}{2 \lambda_i^2} \left[ \int f_i(y, \lambda_i, 0, t) + \left\{ f_i(y, \lambda_i, \lambda_i, t) - f_i(y, \lambda_i, K_i, P, t) \right\} \right] \]

\[ - \left[ f_i(y, \lambda_i, 0, t) - f_i(y, \lambda_i, K_i, P, t) \right], \]

\[ T(y,t) = \frac{1}{2} e^{\gamma y} \text{erfc} \left( \frac{y}{2 \sqrt{P} t} - \sqrt{N t} \right) \]

\[ + e^{-\gamma y} \times \text{erfc} \left( \frac{y}{2 \sqrt{P} t} - \sqrt{N t} \right). \]

### 2.4 Rate of Mass Transfer

The expressions for rate of mass transfer \( S \) at the plate is obtained and presented in the following form:

\[ S = \frac{\partial C_i}{\partial y} \bigg|_{y=0} = -e^{\alpha y} \sqrt{\frac{P}{\pi t}} + \sqrt{K_i N t} \text{erfc} \left( \sqrt{N t} \right) - 1. \]

### 3. VALIDATION OF RESULT

In order to validate our numerical results, we have compared the numerical values of shear stress at the plate of a special case of our numerical results with those of Das et al. (2011), for various values of \( K_i \) and \( N \), taking \( M = 0 \), \( t_i = 1 \), \( a = 0 \), \( t = 1 \), \( G_i = 10 \), \( G = 0 \), \( K_i = 0 \) and \( P = 0.71 \). Our numerical results are in excellent agreement with those of Das et al (2011) which is clearly evident from Tables 1 and 2.
Table 1 Shear stress $\tau$ at the ramped temperature plate when $M = 0$, $t_1 = 1$, $a = 0$, $\alpha = 1$, $K_1 = 0$, $G_r = 10$, $G_S = 0$ and $P_r = 0.71$.

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<th>35</th>
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<td></td>
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<tr>
<td>Present Value</td>
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<td>3.93632</td>
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<td></td>
<td>0.045</td>
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<td>3.66324</td>
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<tr>
<td></td>
<td>0.050</td>
<td>3.34675</td>
<td>3.39462</td>
</tr>
</tbody>
</table>

Table 2 Shear stress $\tau$ at isothermal plate when $M = 0$, $t_1 = 1$, $a = 0$, $\alpha = 1$, $K_1 = 0$, $G_r = 10$, $G_S = 0$ and $P_r = 0.71$.

<table>
<thead>
<tr>
<th>$K_1$ ↓ $N$ →</th>
<th>25</th>
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<td>Present Value</td>
<td>0.040</td>
<td>3.91459</td>
<td>3.95998</td>
</tr>
<tr>
<td></td>
<td>0.045</td>
<td>3.59386</td>
<td>3.64215</td>
</tr>
<tr>
<td></td>
<td>0.050</td>
<td>3.32075</td>
<td>3.37170</td>
</tr>
</tbody>
</table>

4. RESULTS AND DISCUSSION

In order to analyze the effects of critical time for rampedness, surface acceleration parameter, time, radiation, permeability of porous medium, mass diffusion, solutal buoyancy force and chemical reaction on the flow-field, the numerical values of fluid velocity within boundary layer region, computed from the analytical solutions (13) and (19), are displayed graphically versus boundary layer coordinate $y$ for various values of dimensionless fixed time $t_1$, $a$, time $t$, radiation parameter $N$, permeability parameter $K_1$, Schmidith number $S_r$, solutal Grashof number $G_r$ and chemical reaction parameter $K_2$ in figures 2 to 9 taking magnetic parameter $M = 4$, Prandtl number $P_r = 0.71$ (ionized air) and thermal Grashof number $G_S = 20$. It is observed from figures 2 to 9 that, for both ramped temperature and isothermal plates, fluid velocity $u$ attains a distinctive maximum value in the vicinity of the plate and then decreases properly on increasing boundary layer coordinate $y$ to approach free stream value.

This is due to fact that thermal and solutal buoyancy forces have significant role on the fluid flow in the region near the plate and its effect is nullified in the free stream. Also, it is evident from figures 3 to 9 that fluid velocity is faster in case of isothermal plate than that of ramped temperature plate. This may be due to reason that in the case of ramped temperature plate, plate temperature $T$ increases with respect to time $t$ and attains value $T = 1$ when $t > t_1$, whereas in case of isothermal plate, plate temperature is $T = 1$ for every value of time $t$ i.e. $t > 0$. This means that plate temperature is cooler up to the critical time for rampedness $t_1$ in case of ramped temperature plate than that for isothermal plate. Fig. 2 depicts the effect of critical time for rampedness on fluid flow in the boundary layer region for ramped temperature plate. It is noticed from Fig. 2 that $u$ decreases on increasing $t_1$. This implies that, for ramped temperature plate, fluid flow is getting accelerated in the boundary layer region with the increase in critical time for rampedness.

Fig. 2. Velocity profiles when $a = 0.4$, $t = 1.2$, $N = 2$, $K_1 = 0.2$, $S_r = 0.6$, $G_r = 7$ and $K_2 = 0.5$.

Fig. 3 illustrates the effect of surface acceleration parameter on fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 3 that $u$ increases on increasing $a$ for both ramped temperature and isothermal plates. This implies that, for both ramped temperature and isothermal plates, fluid flow is getting accelerated in the boundary layer region with the increase in surface acceleration parameter.

Fig. 4 depicts the effect of time on fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 4 that $u$ increases on increasing $t$ for both ramped temperature and isothermal plates. This implies that fluid flow in the boundary layer region is getting accelerated with the progress of time for both ramped temperature and isothermal plates.

Fig. 3. Velocity profiles when $t_1 = 2$, $t = 1.2$, $N = 2$, $K_1 = 0.2$, $S_r = 0.6$, $G_r = 7$ and $K_2 = 0.5$.
Fig. 4. Velocity profiles when 

\[ t_1 = 2, \ a = 0.4, N = 2, \]
\[ K_i = 0.2, S_c = 0.6, G_c = 7 \text{ and } K_j = 0.5. \]

Fig. 5 illustrates the effect of radiation on fluid velocity for both ramped temperature and isothermal plates.

Fig. 5. Velocity profiles when 

\[ t_1 = 2, \ a = 0.4, t = 1.2, \]
\[ K_i = 0.2, S_c = 0.6, G_c = 7 \text{ and } K_j = 0.5. \]

It is noticed from Fig. 5 that, for both ramped temperature and isothermal plates, \( u \) decreases on increasing \( N \). This implies that radiation has a tendency to decelerate the fluid flow in the boundary layer region for both ramped temperature and isothermal plates for optically thin radiating fluid. This is due to the fact that fluid temperature is getting reduced due to thermal radiation and fluid flow in the boundary layer region is getting retarded.

Fig. 6 reveals the effect of permeability of porous medium on fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 6 that, for both ramped temperature and isothermal plates, \( u \) increases on increasing \( K_i \). An increase in permeability of medium implies that there is a decrease in the resistance of the porous medium which in turn accelerate fluid flow in boundary layer region for both ramped temperature and isothermal plates.

Fig. 6. Velocity profiles when 

\[ t_1 = 2, \ a = 0.4, t = 1.2, \]
\[ N = 2, S_c = 0.6, G_c = 7 \text{ and } K_j = 0.5. \]

Fig. 7. Velocity profiles when 

\[ t_1 = 2, \ a = 0.4, t = 1.2, \]
\[ N = 2, K_i = 0.2, S_c = 0.6, G_c = 7 \text{ and } K_j = 0.5. \]

Fig. 7 depicts the effects of mass diffusion on the fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 7 that, for both ramped temperature and isothermal plates, fluid velocity \( u \) decreases on increasing \( S_c \). \( S_c \) represents the ratio of momentum diffusivity and molecular (mass) diffusivity. \( S_c \) decreases on increasing mass diffusivity. This implies that mass diffusion tends to accelerate the fluid flow in the boundary layer region for both ramped temperature and isothermal plates.

Fig. 7. Velocity profiles when 

\[ t_1 = 2, \ a = 0.4, t = 1.2, \]
\[ N = 2, K_i = 0.2, S_c = 0.6, G_c = 7 \text{ and } K_j = 0.5. \]

Fig. 8. Velocity profiles when 

\[ t_1 = 2, \ a = 0.4, t = 1.2, \]
\[ N = 2, K_i = 0.2, S_c = 0.6 \text{ and } K_j = 0.5. \]

Fig. 8 reveals the effects of solutal buoyancy force on the fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 8 that, for both ramped temperature and isothermal plates, fluid velocity \( u \) increases on increasing \( G_c \). \( G_c \) signifies the relative strength of solutal buoyancy force to viscous force. \( G_c \) increases when solutual buoyancy
force increases. This implies that solutal buoyancy force tends to accelerate the fluid flow in the boundary layer region for both ramped temperature and isothermal plates.

Fig. 9 exhibits the effect of chemical reaction on the fluid flow in the boundary layer region for both ramped temperature and isothermal plates. It is noticed from Fig. 9 that \( u \) decreases on increasing \( K_r \) for both ramped temperature and isothermal plates. This implies that chemical reaction tends to decelerate fluid flow in the boundary layer region for both ramped temperature and isothermal plates.

\[
\text{Fig. 9. Velocity profiles when } t_1 = 2, \ \alpha = 0.4, \ t = 1.2, \ N = 2, \ K_r = 0.2, \ S_r = 0.6 \ \text{and } G_r = 7.
\]

The numerical values of fluid temperature \( T \), computed form analytical solutions (14) and (20), are depicted graphically in Figs. 10 to 12 for various values of critical time for rampedness \( t_1 \), time \( t \) and radiation parameter \( N \) taking \( t_1 = 0.71 \). It is observed from Figs. 10 to 12 that fluid temperature \( T \) is maximum at the surface of the plate and decreases properly on increasing boundary layer co-ordinate \( y \) to approach free stream value. It is also noticed from Figs. 11 and 12 that fluid temperature is higher in case of isothermal plate than that of ramped temperature plate. Fig. 10 depicts the effect of critical time for rampedness on fluid temperature for ramped temperature plate. It is noticed from Fig. 10 that \( T \) decreases on increasing \( t_1 \) for ramped temperature plate. An increase in critical time for rampedness implies that there is a decrease in fluid temperature in the boundary layer region for ramped temperature plate.

\[
\text{Fig. 10. Temperature profiles when } t_1 = 1.2 \ \text{and } N = 2.
\]

Figs. 11 and 12 demonstrate the influence of time and radiation on fluid temperature \( T \) for both ramped temperature and isothermal plates. It is noticed from Figs. 11 and 12 that fluid temperature \( T \) increases on increasing \( t \) whereas it decreases on increasing \( N \) for both ramped temperature and isothermal plates. This implies that radiation has a tendency to reduce fluid temperature in the boundary layer region for both ramped temperature and isothermal plates. Fluid temperature is getting enhanced in the boundary layer region with the progress of time for both ramped temperature and isothermal plates. It is interesting to note from Fig. 11 that fluid temperature at isothermal plate attains steady state when \( t > 1.5 \).

\[
\text{Fig. 11. Temperature profiles when } t_1 = 2a \ \text{and } N = 2.
\]

The numerical values of species concentration \( C \), computed form analytical solution (15), is depicted graphically in Figs. 13 and 14 for various values of \( K_s \) and \( S_r \) taking \( t_1 = 1.2 \). It is observed from Figs. 13 and 14 that species concentration \( C \) is maximum at the surface of the plate and it decreases properly on increasing boundary layer co-ordinate \( y \) to approach free stream value. It is noticed from Figs. 13 and 14 that \( C \) decreases on increasing either \( K_s \) or \( S_r \). This implies that chemical reaction tends to reduce species concentration whereas mass diffusion has a reverse effect on it.

\[
\text{Fig. 12. Temperature profiles when } t_1 = 2 \ \text{and } N = 2.
\]
The numerical values of shear stress $\tau$ at the plate, computed from analytical expressions (21) and (23), are presented in tabular form in Tables 3 to 6 for various values of $S_t$, $t_1$, $a$, $t$, $N$, $K_2$, $K_1$ and $G_t$ taking $M = 4, G_r = 20$ and $P_r = 0.71$ (ionized air).

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Shear stress $\tau$ at the plate when $a = 0.4$, $t = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 2, K_0 = 0.5, K_1 = 0.2$ and $G_t = 7$.</td>
<td></td>
</tr>
<tr>
<td>$S_t \downarrow t_1 \rightarrow$</td>
<td>1.8</td>
</tr>
<tr>
<td>Ramped Temperature $(-\tau)$</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
</tr>
</tbody>
</table>

It is observed from Table 3 that shear stress $\tau$ at the plate increases on increasing either $S_t$ or $t_1$ for ramped temperature plate. This implies that mass diffusion tends to reduce shear stress at ramped temperature plate. An increase in critical time for rampedness implies that there is an enhancement in shear stress at ramped temperature plate whereas there is a reduction in the shear stress at isothermal plate.

It is analyzed from Table 5 that, for ramped temperature plate, $\tau$ increases on increasing either $N$ or $K_2$ whereas it decreases on increasing for isothermal plate. This implies that radiation and chemical reaction tend to enhance the shear stress at ramped temperature plate whereas it have reverse effect on the shear stress at isothermal plate. It is revealed from Table 6 that $\tau$ decreases on increasing either $K_1$ or $G_r$ for ramped temperature plate whereas it increases on increasing either $K_1$ or $G_r$ for isothermal plate. This implies that permeability of porous medium and solutal buoyancy force tend to reduce the shear stress at ramped temperature plate whereas these agencies have reverse effect on the shear stress at isothermal plate.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Shear stress $\tau$ at the plate when $S_t = 0.6$, $t_1 = 2$,</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 2, K_0 = 0.5, K_1 = 0.2$ and $G_t = 7$.</td>
<td></td>
</tr>
<tr>
<td>$a \downarrow t \rightarrow$</td>
<td>1</td>
</tr>
<tr>
<td>Ramped Temperature $(-\tau)$</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Shear stress $\tau$ at the plate when $a = 0.4$, $t = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 2, K_0 = 0.5, K_1 = 0.2$ and $G_t = 7$.</td>
<td></td>
</tr>
<tr>
<td>$K_2 \downarrow t_1 \rightarrow$</td>
<td>0.5</td>
</tr>
<tr>
<td>Ramped Temperature $(-\tau)$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Shear stress $\tau$ at the plate when $a = 0.4$, $t = 1.2$, $N = 2$ and $K_1 = 0.5$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1 \downarrow G_r \rightarrow$</td>
<td>3</td>
</tr>
<tr>
<td>Ramped Temperature $(-\tau)$</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
</tr>
</tbody>
</table>

The numerical values of rate of heat transfer at the
Investigation of unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, chemically reactive and optically thin radiating fluid past an exponentially accelerated moving vertical plate with variable ramped temperature embedded in a fluid saturated porous medium is carried out. Significant finding are as follows:

- Critical time for rampedness tends to reduce fluid fluid flow as well as fluid temperature in boundary layer region.
- Surface acceleration parameter, permeability of porous medium, mass diffusion, solutal buoyancy force and time tend to accelerate fluid flow in boundary layer region whereas radiation and chemical reaction have reverse effect on it. Radiation tends to reduce fluid temperature and fluid temperature is getting enhanced with progress of time. It approaches steady state when \( t > 1.5 \) for isothermal plate.
- Chemical reaction tends to reduce species concentration whereas mass diffusion has a reverse effect on it.
- We have compared numerical values of shear stress at the plate of Das et al. (2011) as a special case of our result which are in excellent agreement with the numerical values of Das et al. (2011).
- Critical time for rampedness tends to enhance shear stress at the plate whereas mass diffusion has a reverse effect on it for ramped temperature plate.
- Chemical reaction tends to enhance rate of mass transfer at the plate whereas mass diffusion has a reverse effect on it.

5. CONCLUSIONS

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Ibrahim, F. S., A. M. Elaiw and A. A. Bakr (2008). Effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and


Rajesh, V. and S. V. K. Varma (2009). Radiation and mass transfer effects on MHD free convection flow past an exponentially


**Appendix**
\[ f_{ij}(c_i, c_j, c_k) = e^{\frac{-(c_i + c_j) b_{ij}}{\sqrt{c_j \pi}}} + \sqrt{c_i + c_j} \times \\
\left( \text{erfc} \left( \sqrt{c_i + c_j} c_k \right) - 1 \right) \]

\[ f_{23}(c_i, c_j, c_k, c_l, c_m) = e^{\pi \left[ -\sqrt{c_i + c_j} \left( \text{erfc} \left( \sqrt{c_i + c_j} c_k \right) - 1 \right) \right.} \\
+ \sqrt{c_i + c_j} \left[ \text{erfc} \left( \sqrt{c_i + c_j} c_k \right) - 1 \right] \]