Extended Abstract

Rainfall –Runoff Prediction by Stochastic Models
Case Study: Watershed of Kardeh Dam

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Introduction
Due to the seasonal variabilities of the hydroclimatological data influenced by random variables, the stochastic models are among the best methods of prediction of these data.

The hydrological models represent an approximation of the real processes. The predicted values by these models are never completely certain but the important matter in the modeling process is to reach an acceptable agreement to the real records.

Objective
This research aimed the prediction of the rainfall-runoff for one or more time steps ahead in Kardeh basin located in north east of Iran. Changes in water storage in the dam and the downstream demand satisfaction were identified.

Methodology
For prediction of monthly rainfall-runoff values one or more time steps ahead, the state-space model of the second order was used. The input and output variables were separately modeled and the seasonal model of Box and Jenkins family (ARIMA) was used for description of both variables. The AIC and SBC and Root Mean Square methods were evaluated for different types of models.

Results and Discussion
Simultaneous analysis of mean monthly rainfall and runoff (input and output variables) was performed with the help of the state-space model which in general form is

\[ Z_{t+1} = FZ_t + G\epsilon_t \]

in which \( Z_t \) indicates the two variables vector of mean monthly rainfall and runoff. By the application of AIC model, the suitable model of state space was calculated as follows;

\[
\begin{bmatrix}
X_{t+1|t+1} \\
Y_{t+1|t+1}
\end{bmatrix} =
\begin{bmatrix}
0.049219 & 4.312719 \\
-0.00075 & 0.579179
\end{bmatrix}
\begin{bmatrix}
X_{t+1|t} \\
Y_{t+1|t}
\end{bmatrix} + \begin{bmatrix}
1 \\
0
\end{bmatrix} e_{t+1} + \begin{bmatrix}
0 \\
1
\end{bmatrix} \eta_{t+1}
\]

in which;

\[
\begin{bmatrix}
\text{var}\left( e_{t+1} \right) \\
\text{var}(\eta_{t+1})
\end{bmatrix} =
\begin{bmatrix}
799.681 & 5.99934 \\
5.99934 & 0.27653
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_t \\
Y_t
\end{bmatrix} =
\begin{bmatrix}
(1 - B^{12}) X_t - 28.6 \\
(1 - B^{12}) Y_t - 0.652
\end{bmatrix}
\]

Figs. 1 and 2 represent the observed and predicted mean monthly runoff and rainfall values based on the state-space models.

Analyzing the mean monthly rainfall and runoff values with respect to time series plot and the seasonal variability involved in the data, the application of the method of seasonal differencing used to reach the stationary position and to reach the final control in the system, and the input and output data revealed that the best seasonal fitted model was ARIMA(1,0,0)(1,1,1)12 for both variables. The input - output models were formulated as shown below:
For rainfall:

\[
(1 - 0.1486 B)(1 + 0.6343 B^{12})(1 - B^{12})Y_t = (1 - 0.5341 B^{24})e_t
\]

For runoff:

\[
(1 - 0.5568 B)(1 - 0.6346 B^{12})Y_r = (1 - 0.640 B^{12})e_t
\]

Figs. 3 and 4 represent the mean monthly observed and predicted runoff and rainfall values and their band limits by 95% degree of confidence based on seasonal Box and Jenkins models.

**Conclusion**

Descending order trend of predicted rainfall and runoff values by the fitted model of Box and Jenkins and the state space model achieved in this study indicated the reduction of storage of water in Kardeh dam and therefore presented the need for a management program for optimum use of this water in the system.

**Keywords:** State space models, Box and Jenkins seasonal models, stochastic models, Rainfall –Runoff prediction.

**References**
