

Original Article

## Calculations of Linac Photon Dose Distributions in Homogeneous Phantom Using Spline

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### Abstract

#### Introduction

Relative dose computation is a necessary step in radiation treatment planning. Therefore, finding an approach that is both fast and accurate seems to be necessary. The purpose of this work was to investigate the feasibility of natural cubic spline to reconstruct dose maps for linear accelerator radiation treatment fields in comparison with those of the simulation.

#### Materials and Methods

A natural cubic spline algorithm was used to reproduce dose calculations of linac radiation treatment fields resulting from GEANT4 application for tomographic emission (GATE) simulation. The spline algorithm was used to compute percent depth dose of radiation therapy fields for 6 MV X-rays, which were calculated by simulation of Elekta Compact Linac. It reconstructed 2-dimensional dose maps and created isodose distributions. This dose maps were evaluated and compared with the simulation, where the  $\gamma$ -index was used.

#### Results

A good agreement was found between the doses calculated from the simulation and the spline. In particular, an average  $\gamma$ -index passing rate of 0.24 was obtained for sample percent depth dose distributions, and an average  $\gamma$ -index passing rate of 0.20 was observed for sample dose profiles.

#### Conclusion

Natural cubic spline has been established to calculate dose maps from field characteristics. The feasibility and possibility of natural cubic spline to calculate dose maps for linac radiation therapy fields in a homogeneous phantom has been demonstrated.

**Keywords:**  $\gamma$ -index; Cubic Spline; Dose Calculation.

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## 1. Introduction

The most important purpose of external beam radiotherapy is to maximize tumor dose, while at the same time spare the adjacent healthy tissues as much as achievable. Hence, relative dose computation is an essential part of radiation treatment planning, which engages the use of analytical formulae, look-up tables, pencil beam, point dose convolution models, and Monte Carlo (MC) method [1-6]. There is a trade-off between speed and accuracy among these methods. The MC method has the greatest potential accuracy but is not normally used in clinical situations because it is time-consuming and complex to perform. However, other methods are faster and generally used in ordinary treatment planning systems (TPS) for computing dose distributions, although they are not so accurate (the best results are given by pencil beam and point kernels convolution). An overview of these methods is given by Ahnesjö and Aspradakis [1]. Therefore, establishing a new approach that is both fast and accurate seems to be necessary.

Splines are a type of interpolators composed of pieces of polynomial functions. These functions are defined on subintervals with a suitable degree of smoothness at places where the polynomial pieces are connected. Reasons for embracing of spline include its stability and calculation simplicity. Cubic spline is a popular one and has been recently used in many researches. Cubic spline has been capable of accurate prediction of the dome thickness of composite pressure vessels [7], has achieved better results by obtaining higher accuracy for low sampling frequency ECG data interpolation [8], and has indicated better performance in white matter fiber tracking than the two widely used tracking methods [9]. The purpose of this work was to suggest a new approach for dose calculation based on spline interpolation. Our code took in-line and cross-line profiles of some fields in given depths. After this phase, the model was ready to use and capable of giving accurate results. In addition, all calculations achieved by our code were obtained using an intelligent regression inside a grid of data, as this code can manage

other situations. Moreover, it is able to calculate dose maps for an unknown field with only given dimensions of that field.

## 2. Materials and Methods

Data were taken from MC numerical simulation. Then, spline model was built and could be used to generate dose distributions. Our final goal was to generate a treatment planning system.

### 2.1. Data calculation using MC code

GEANT4 application for tomographic emission (GATE) Monte Carlo package (version 6.1) was used to compute dose maps. Elekta Compact linear accelerator head was simulated and tuning of the primary electron beam characteristics was performed as recommended by Verhaegen and Seuntjens [10]. The mean electron beam energy of 6 MeV (FWHM energy equals to 3% of the mean energy) and a FWHM electron spot of 2 mm had the best fit to the measurements. The measurement data were taken using a 6MV Elekta Compact<sup>TM</sup> (Elekta, China) linear accelerator, a 50×50×50 cm<sup>3</sup> Scanditronix (Uppsala, Sweden) water phantom, and two Scanditronix (Uppsala, Sweden) diode detectors. The energy cut-off was set to 10 keV and 300 keV for photons and electrons, respectively.

To determine the usefulness and effectiveness of this method, a simple water phantom is sufficient. Dose computation was performed for a source to skin distance (SSD) of 100 and 4×4, 5×5, 7×7, 10×10, 12×12, 20×20, 30×30, 35×35, and 40×40 cm<sup>2</sup> field sizes. A 50×50×30 cm<sup>3</sup> cube consisted of 0.5×0.25×0.3 cm<sup>3</sup> voxels was considered for dose calculation in a 60×60×60 cm<sup>3</sup> water phantom. The measurements are not always possible and simulation offers accurate data, so numerical simulation was chosen. A smoothing filter was applied to simulation results. Comparison of our new dose calculator with GATE results has been reported in this paper.

## 2.2. Spline

Spline interpolation uses a special type of piecewise polynomial interpolant, in which the approximation interval is divided into a collection of subintervals, and different approximating polynomials are constructed on each subinterval. The spline interpolation error can be small even when using low degree polynomials where high-degree polynomials can oscillate unpredictably, such that a negligible fluctuation over a small portion of the interval can make large fluctuations over the entire range. For spline interpolation, the most common polynomial used in each pair of nodes is the cubic polynomial, and such splines are called cubic spline interpolation. If there is a function  $f$  that is defined on  $[a, b]$  and this interval contains a group of nodes  $a = x_0 < x_1 < \dots < x_n = b$ , then a cubic spline interpolant  $P$  for  $f$  should satisfy the below conditions:

- (i)  $P_j(x)$  is a cubic polynomial on the subinterval  $[x_j, x_{j+1}]$  for each  $j = 0, 1, \dots, n-1$ ;
- (ii)  $P_j(x_j) = f(x_j)$  and  $P_j(x_{j+1}) = f(x_{j+1})$  for each  $j = 0, 1, \dots, n-1$ ;
- (iii)  $P_{j+1}(x_{j+1}) = P_j(x_{j+1})$  for each  $j = 0, 1, \dots, n-2$ ;
- (iv)  $P'_{j+1}(x_{j+1}) = P'_j(x_{j+1})$  for each  $j = 0, 1, \dots, n-2$ ;
- (v)  $P''_{j+1}(x_{j+1}) = P''_j(x_{j+1})$  for each  $j = 0, 1, \dots, n-2$ ;
- (vi)  $P''(x_0) = P''(x_n) = 0$  (for natural cubic spline)

The graph of a natural spline approximates the shape of a long flexible rod that is forced to go through the data points  $\{(x_0, f(x_0)), (x_1, f(x_1)), \dots, (x_n, f(x_n))\}$  [11].

To implement cubic spline, MATLAB (7.8.0, 2009) 'fit' command was used with a structure output. This output is similar to an arithmetic function that returns a number for each input.

## 2.3. Core code for dose calculation and output evaluation

A database consisting of in-line and cross-line profiles in 0, 1, 1.5, 2, 5, 10, 20, and 30 cm depths for  $4 \times 4$ ,  $6 \times 6$ ,  $10 \times 10$ ,  $15 \times 15$ ,  $20 \times 20$ ,  $25 \times 25$ ,  $30 \times 30$ , and  $40 \times 40$  cm<sup>2</sup> field sizes was created. These data were prepared by MC

simulation. Our core code takes the desired field size as input, and finds the larger and smaller field sizes in the database between which the desired field size is located. Next, the new profiles were constructed according to the larger and smaller profiles. If the database has the desired field size, the previous step will be ignored. Now, using the in-line and cross-line profiles, dose distribution at any location in the transverse plane at the depth profiles was calculated. In the next step, MATLAB 'fit' command was applied to corresponding points on consecutive planes at different depths. After that, the code calculated dose distribution in a 3-dimensional space.

One of the best numerical evaluation method to compare the measured and calculated dose distribution values is the Low's  $\gamma$ -index [12]. A MATLAB code was written based on this index and dose difference and distance-to-agreement criteria were set to 2% and 2 mm, respectively.

## 3. Results

### 3.1. Simulation commissioning

Before using the simulation results, the model should be authenticated. Percent depth dose (PDD) comparison between measurement and simulation for  $10 \times 10$  cm<sup>2</sup> field size is shown in Figure 1. PDD values were normalized to depth of maximum dose ( $d_{max}$ ). Dose profile comparison at depth of 10 cm for the  $10 \times 10$  cm<sup>2</sup> field size and the related  $\gamma$ -index are shown in Figure 2. To evaluate  $\gamma$ -index, dose difference and distance-to-agreement criteria were set to 2% and 2 mm, respectively.

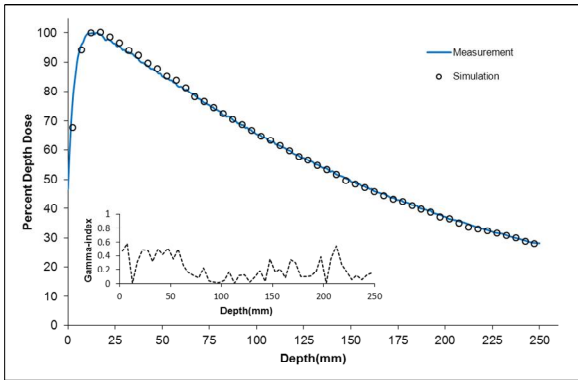


Figure 1. Calculated and measured PDD curves and related  $\gamma$ -index for  $10 \times 10 \text{ cm}^2$  field size.

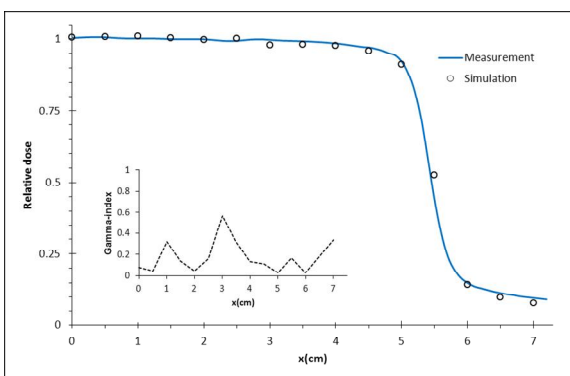


Figure 2. Calculated and measured dose profiles and related  $\gamma$ -index, at 10 cm depth and  $10 \times 10 \text{ cm}^2$  field size.

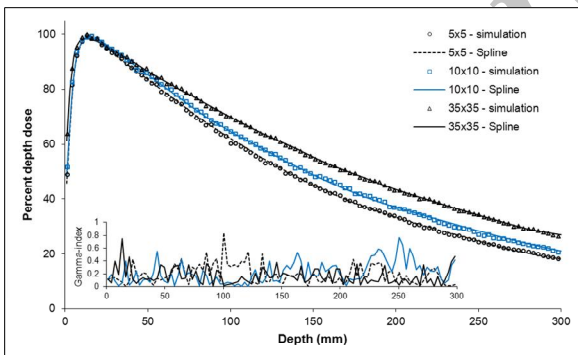


Figure 3. Comparison of central axis PDD distributions using GATE and spline for  $5 \times 5$ ,  $10 \times 10$  and  $35 \times 35 \text{ cm}^2$  field sizes.

### 3.2. Percent depth dose

The central axis PDD distributions are shown in Figure 3 for  $5 \times 5$ ,  $10 \times 10$ , and  $35 \times 35 \text{ cm}^2$  field sizes. As the  $\gamma$ -index shows, spline results are good for all regions. The PDD provided by spline was used neither during training nor in the test step.

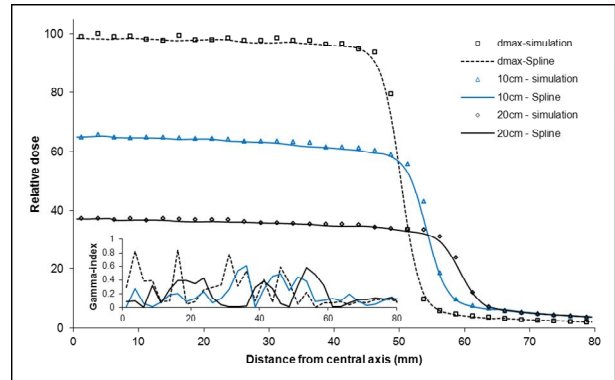


Figure 4. Comparison of relative profile values of spline and GATE results at depths of  $d_{max}$ , 10 cm and 20 cm for  $10 \times 10 \text{ cm}^2$  field size in a water phantom.

### 3.3. Dose profiles

Figure 4 shows comparison of spline and GATE dose profile values for a  $10 \times 10 \text{ cm}^2$  field size at depths of  $d_{max}$ , 10 cm and 20 cm in a water phantom. As can be seen in this Figure,  $\gamma$ -index did not exceed 0.8 in all of the regions and there are only small differences between the spline points and the GATE values, mainly near the penumbra. These discrepancies are due to the nature of the penumbra region of the beam, where large changes in dose arise if there are small spatial uncertainties.

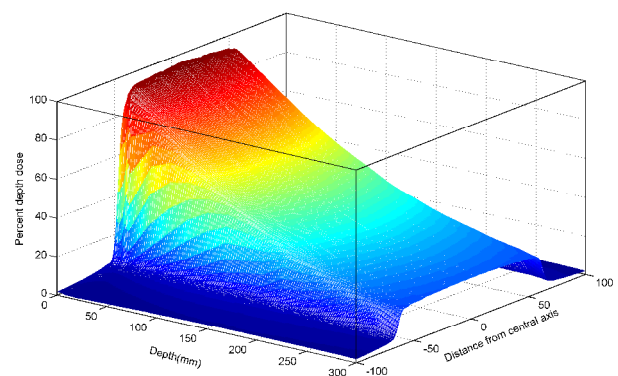


Figure 5. Three-dimensional illustration with dose plotted on the Z-axis against position in the X-Y plane of a  $10 \times 10 \text{ cm}^2$  field size.

### 3.4. 2D and isodose curve plots

Using spline, dose points were computed on a two-dimensional grid with a spacing of 1 mm. Figure 5 shows a sample three-dimensional plot of a  $10 \times 10 \text{ cm}^2$  field size and shows a smooth interpolation between dose values

from different positions. Isodose plot for  $10 \times 10 \text{ cm}^2$  field size was then generated, and this is shown in Figures 6.

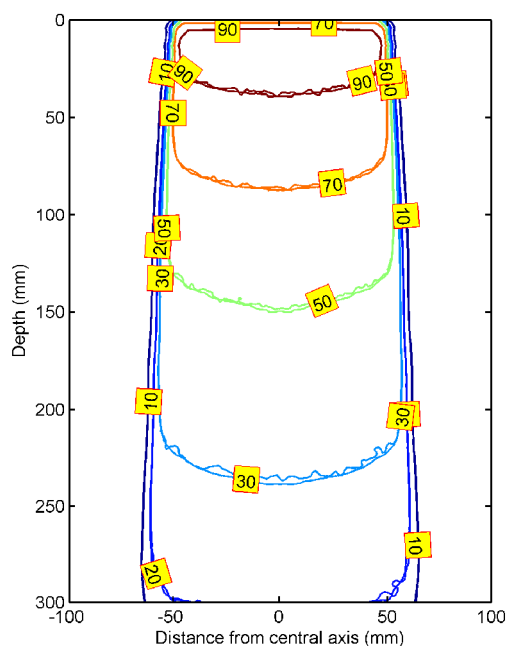


Figure 6. Isodose comparison of spline and GATE simulation for a  $10 \times 10 \text{ cm}^2$  field size. The curves with turbulence are related to simulation.

#### 4. Discussion

In this work, spline was used for dose calculation in radiation therapy. The main purpose of this study was to find the generalizability and interpolation ability of spline.

Since the field size can affect the beam homogeneity, the data of different field sizes were simulated [13]. Spline was trained with a subset of the total dataset and was examined with the test data. The test results showed the capability of generalization and verified performance accuracy of spline. The ability of the spline for nonlinear interpolation is indicated by the PDD distributions (Figure 3).

There was a very good agreement between spline and GATE to calculate the central axis depth dose distribution (Figure 3). In addition, the spline performed well in the central portion of the beam as well as in the penumbral region

(Figure 4). The International Commission on Radiation Units and Measurements (ICRU) has proposed the acceptability criteria for demonstration of a dose distribution of  $\pm 2\%$  dose or  $\pm 2 \text{ mm}$  in high dose gradient regions [14]. These limits were applied to Low's  $\gamma$ -index MATLAB code (2% in dose difference and 2 mm in distance-to-agreement) and the results verified accuracy of the spline.

Outside the penumbra, the dose is mostly due to scattered radiation from the main beam and varies slowly with position. However, to protect critical organs, accuracy in this region is essential [15]. Spline modeling for this region was very good ( $\gamma$ -index  $< 0.2$ ), even up to 200% of beam edge distance.

This study showed that spline could be used for dose computation, especially in utilizing smaller datasets for training and insensitivity to noise in the data.

#### 5. Conclusions

This work was a pre-study for radiotherapy dose calculations by the spline. This method is better than other correction-based algorithms for three reasons: (1) It is insensitive to noise within the data, (2) It needs small database for train and test, and (3) It takes a short time to train. The results obtained in this study have confirmed the accuracy of this method.

Our ultimate goal is to provide a new dose calculator for treatment planning systems. In this context, our current study is focused on calculating the effects of phantom heterogeneities. Initial results are very encouraging in that domain, but further studies are needed.

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