A New Method for Ranking in Perimeters of two Generalized Trapezoidal Fuzzy Numbers

S. Rezvani*

Received: 16 May 2012 ; Accepted: 10 August 2012

Abstract In this paper, we want to propose a new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers. A simpler and easier approach is proposed for the ranking of generalized trapezoidal fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

Keywords Generalized Trapezoidal Fuzzy Numbers, Ranking Method.

1 Introduction


In this paper, we want to propose a new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers. A simpler and easier approach is proposed for the ranking of generalized trapezoidal fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

* Corresponding Author. (E)
E-mail: salim_rezvani@yahoo.com (S. Rezvani)

S. Rezvani
Assistant prof, Department of Mathematics, Imam Khomeini Maritime University of Nowshahr, Nowshahr, Iran.
2 Preliminaries

Generally, a generalized fuzzy number $A$ is described as any fuzzy subset of the real line $R$, whose membership function $\mu_A$ satisfies the following conditions,

i) $\mu_A$ is a continuous mapping from $R$ to the closed interval $[0,1]$ ,

ii) $\mu_A(x) = 0$, $-\infty < x \leq a$ ,

iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a,b]$ ,

iv) $\mu_A(x) = w$, $b \leq x \leq c$ ,

v) $\mu_A(x) = R(x)$ is strictly increasing on $[c,d]$ ,

vi) $\mu_A(x) = 0$, $d \leq x < \infty$

where $0 \leq w \leq 1$ and $a,b,c$ and $d$ are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by $A = (a,b,c,d;w)$

A = $(a,b,c,d;w)$ is a fuzzy set of the real line $R$ whose membership function $\mu_A(x)$ is defined as

$$\mu_A(x) = \begin{cases} 
\frac{x-a}{b-a} & \text{if } a \leq x \leq b \\
w & \text{if } b \leq x \leq c \\
\frac{d-x}{d-c} & \text{if } c \leq x \leq d \\
0 & \text{otherwise}
\end{cases}$$

3 Proposed Approach

In this section some important results, that are useful for the proposed approach, are proved. Jiang Wen [16] proposed the concept of the method to calculate the degree of similarity between generalized fuzzy numbers. In this method, the horizontal center-of-gravity, the perimeter, the height and the area of the two fuzzy numbers are considered.

Suppose that $A_1 = (a_1,b_1,c_1,d_1;w_1)$ and $A_2 = (a_2,b_2,c_2,d_2;w_2)$ be the generalized trapezoidal fuzzy numbers. Where $0 \leq a_1 \leq b_1 \leq c_1 \leq d_1 \leq 1$ and $0 \leq a_2 \leq b_2 \leq c_2 \leq d_2 \leq 1$. Then the degree of similarity $S(A_1,A_2)$ between the generalized trapezoidal fuzzy numbers $A_1$ and $A_2$ is calculated as follows:

$$S(A_1,A_2) = [1 - |x_{A_1}^* - x_{A_2}^*|] \times [1 - |w_{A_1} - w_{A_2}|] \times \frac{\min(P(A_1),P(A_2)) + \min(A(A_1),A(A_2))}{\max(P(A_1),P(A_2)) + \max(A(A_1),A(A_2))}$$

(3)
where \( x^*_{A_1} \) and \( x^*_{A_2} \) are the horizontal center-of-gravity of the generalized trapezoidal fuzzy numbers \( A_1 \) and \( A_2 \) is calculated as follows:

\[
x^*_{A_i} = \frac{y^*_{A_i}(c_1+b_1)+(d_1+a_i)(w_{A_i}-y^*_{A_i})}{2w_{A_i}} \quad (4)
\]

\[
y^*_{A_i} = \begin{cases} 
\frac{w_{A_i}(\frac{c_1+b_1}{2}+2)}{6} & \text{if } a_i \neq d_i \text{ and } 0 < w_{A_i} \leq 1 \\
\frac{w_{A_i}}{2} & \text{if } a_i = d_i \text{ and } 0 < w_{A_i} \leq 1 
\end{cases} \quad (5)
\]

\( P(A_i) \) and \( P(A_2) \) are the perimeters of two generalized trapezoidal fuzzy numbers which are calculated as follows:

\[
P(A_i) = \sqrt{(a_i-b_i)^2 + w_{A_i}^2} + \sqrt{(d_i-c_i)^2 + w_{A_i}^2} + (c_1-b_1) + (d_1-a_i) \quad (6)
\]

\[
P(A_2) = \sqrt{(a_2-b_2)^2 + w_{A_2}^2} + \sqrt{(c_2-d_2)^2 + w_{A_2}^2} + (c_1-b_1) + (d_1-a_2) \quad (7)
\]

\( A(A_i) \) and \( A(A_2) \) are the areas of two generalized trapezoidal fuzzy numbers which are calculated as follows:

\[
A(A_i) = \frac{1}{2}w_{A_i}(c_1-b_1+d_1-a_i) \quad (8)
\]

\[
A(A_2) = \frac{1}{2}w_{A_2}(c_2-b_2+d_2-a_2) \quad (9)
\]

The larger the value of \( S(A_1,A_2) \), the more the similarity measure between two generalized trapezoidal fuzzy numbers \( A_1 \) and \( A_2 \).

**Theorem 1.** Let \( A_1=(a_1,b_1,c_1,d_1;w_1) \) and \( A_2=(a_2,b_2,c_2,d_2;w_2) \) be the generalized trapezoidal fuzzy numbers. Where \( 0 \leq a_1 \leq b_1 \leq c_1 \leq d_1 \leq 1 \) and \( 0 \leq a_2 \leq b_2 \leq c_2 \leq d_2 \leq 1 \), and \( P(A_1) \) and \( P(A_2) \) are the areas of two generalized trapezoidal fuzzy numbers. Then

i) If \( P(A_1) < P(A_2) \), then \( A_1 < A_2 \).

ii) If \( P(A_1) > P(A_2) \), then \( A_1 > A_2 \).

iii) If \( P(A_1) \sim P(A_2) \), then \( A_1 \sim A_2 \).
4 Results

**Example 1.** Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 7)$ be two generalized trapezoidal fuzzy number, then

$$P(A) = \sqrt{(a_1 - b_1)^2 + (d_1 - c_1)^2 + (d_1 - d_1) + (d_1 - a_1)}$$

$$= \sqrt{(-0.2)^2 + (0.35)^2 + (-0.2)^2 + (0.35)^2 + (0.2) + (0.6)} = 1.61$$

and

$$P(B) = \sqrt{(a_1 - b_1)^2 + (d_1 - c_1)^2 + (d_1 - d_1) + (d_1 - a_1)}$$

$$= \sqrt{(-0.1)^2 + (0.7)^2 + (-0.1)^2 + (0.7)^2 + (0.1) + (0.3)} = 1.81$$

so

$$P(A) < P(B) \Rightarrow A < B.$$  

**Example 2.** Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$P(A) = \sqrt{(a_1 - b_1)^2 + (d_1 - c_1)^2 + (d_1 - d_1) + (d_1 - a_1)}$$

$$= \sqrt{(-0.1)^2 + 1 + (-0.1)^2 + 1 + (0.2) + (0.4)} = 2.61$$

and

$$P(B) = \sqrt{(a_1 - b_1)^2 + (d_1 - c_1)^2 + (d_1 - d_1) + (d_1 - a_1)}$$

$$= \sqrt{(-0.2)^2 + 1 + (-0.2)^2 + 1 + 0 + (0.4)} = 2.44$$

so

$$P(A) > P(B) \Rightarrow A > B.$$  

**Example 3.** Let $A = (0.1, 0.2, 0.4, .5; 1)$ and $B = (1, 1, 1, 1; 1)$ be two generalized trapezoidal fuzzy number, then

$$P(A) = \sqrt{(a_1 - b_1)^2 + (d_1 - c_1)^2 + (d_1 - d_1) + (d_1 - a_1)}$$

$$= \sqrt{(-0.1)^2 + 1 + (-0.1)^2 + 1 + (0.2) + (0.4)} = 2.61$$

and
\[ P(B) = \sqrt{(a_1 - b_1)^2 + w_{A_1}^2} + \sqrt{(d_1 - c_1)^2 + w_{A_1}^2} + (c_1 - b_1) + (d_1 - a_1) \]
\[ = \sqrt{0 + 1} + \sqrt{0 + 1} + 0 + 0 = 2 \]

so

\[ P(A) > P(B) \Rightarrow A > B. \]

**Example 4.** Let \( A = (-0.5, -0.3, -0.3, -0.1; 1) \) and \( B = (0.1, 0.3, 0.3, 0.5; 1) \) be two generalized trapezoidal fuzzy number, then

\[ P(A) = \sqrt{(-0.2)^2 + 1} + \sqrt{(-0.2)^2 + 1} + 0 + (0.4) = 2.44 \]

and

\[ P(B) = \sqrt{(-0.2)^2 + 1} + \sqrt{(-0.2)^2 + 1} + 0 + (0.4) = 2.44 \]

so

\[ P(A) \sim P(B) \Rightarrow A \sim B. \]

**Example 5.** Let \( A = (0.3, 0.5, 0.5, 1; 1) \) and \( B = (0.1, 0.6, 0.6, 0.8; 1) \) be two generalized trapezoidal fuzzy number, then

\[ P(A) = \sqrt{(-0.2)^2 + 1} + \sqrt{(-0.5)^2 + 1} + 0 + (0.7) = 2.84 \]

and

\[ P(B) = \sqrt{(-0.5)^2 + 1} + \sqrt{(-0.2)^2 + 1} + 0 + (0.7) = 2.84 \]

so

\[ P(A) \sim P(B) \Rightarrow A \sim B. \]

**Example 6.** Let \( A = (0, 0.4, 0.6, 0.8; 1) \) and \( B = (0.2, 0.5, 0.5, 0.9; 1) \) and \( C = (0.1, 0.6, 0.7, 0.8; 1) \) be two generalized trapezoidal fuzzy number, then
\[ P(A) = \sqrt{(a_i - b_i)^2 + w_{A_i}^2} + \sqrt{(d_i - c_i)^2 + w_{A_i}^2} + (c_i - b_i) + (d_i - a_i) \]
\[ = \sqrt{(-0.4)^2} + 1 + \sqrt{(-0.2)^2} + 1 + (0.2) + (0.8) = 3.1 \]

and

\[ P(B) = \sqrt{(a_i - b_i)^2 + w_{A_i}^2} + \sqrt{(d_i - c_i)^2 + w_{A_i}^2} + (c_i - b_i) + (d_i - a_i) \]
\[ = \sqrt{(-0.3)^2} + 1 + \sqrt{(-0.4)^2} + 1 + 0 + (0.7) = 2.82 \]

and

\[ P(C) = \sqrt{(a_i - b_i)^2 + w_{A_i}^2} + \sqrt{(d_i - c_i)^2 + w_{A_i}^2} + (c_i - b_i) + (d_i - a_i) \]
\[ = \sqrt{(-0.5)^2} + 1 + \sqrt{(-0.1)^2} + 1 + (0.1) + (0.7) = 2.92 \]

so

\[ P(A) > P(C) > P(B) \Rightarrow A > C > B. \]

**Example 7.** Let \( A = (0.1, 0.2, 0.4, 0.5; 1) \) and \( B = (-2, 0, 0, 2; 1) \) be two generalized trapezoidal fuzzy number, then

\[ P(A) = \sqrt{(a_i - b_i)^2 + w_{A_i}^2} + \sqrt{(d_i - c_i)^2 + w_{A_i}^2} + (c_i - b_i) + (d_i - a_i) \]
\[ = \sqrt{(-0.1)^2} + 1 + \sqrt{(-0.1)^2} + 1 + (0.2) + (0.4) = 2.61 \]

and

\[ P(A) = \sqrt{(a_i - b_i)^2 + w_{A_i}^2} + \sqrt{(d_i - c_i)^2 + w_{A_i}^2} + (c_i - b_i) + (d_i - a_i) \]
\[ = \sqrt{(-2)^2} + 1 + \sqrt{(-2)^2} + 1 + 0 + (4) = 8.47 \]

so

\[ P(A) < P(B) \Rightarrow A < B. \]

It is clear from Table 1 that the results of the proposed approach are same as obtained by using the existing approach (Chen and Chen, 2009). The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.
A New Method for Ranking in Perimeters of …

Table 1 A comparison of the ranking results for different approaches

<table>
<thead>
<tr>
<th>Approaches</th>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
<th>Example 4</th>
<th>Example 5</th>
<th>Example 6</th>
<th>Example 7</th>
</tr>
</thead>
</table>

References