HYBRID CHARGED SYSTEM SEARCH - PARTICLE SWARM OPTIMIZATION FOR DESIGN OF SINGLE-LAYER BARREL VAULT STRUCTURES

S. Talatahari* and Y. Jahani
Department of Civil Engineering, University of Tabriz, Tabriz, Iran

Received: 8 November 2014; Accepted: 6 February 2015

ABSTRACT

The barrel vaults are composed of member elements arranged on a cylindrical surface. This kind of structure is utilized to cover the long spans. In this paper, the hybrid charge system search and particle swarm optimization algorithm is improved and utilized to optimal design of single-layer barrel vault frames. Some modifications on parameter values are performed to enhance the performance of the hybrid algorithm. Comparison of the results with other meta-heuristic algorithms illustrates the efficiency of the hybrid CSS and PSO algorithm. Also some discussion on the loading conditions and group selecting are presented.

Keywords: Charged system search, particle swarm optimization, hybrid algorithms, optimal design, single layer barrel vault frame

1. INTRODUCTION

Space frames are usually arranged in an array of single, double, or multiple layers of intersecting members. A single-layer space frame that has the form of a curved surface is termed as braced vault, braced dome, or latticed shell. The barrel vaults, having the diagonal or hexagonal types of bracing, must have rigid joints to be stable and the influence of bending moments in their stress distribution is much more pronounced than the other types.

In the field of structural optimization, many meta-heuristic algorithms have been proposed in the last three decades. Although, there are many studies on optimization of structures using the current meta-heuristic algorithms; however, there are not many studies on optimization of space structures, and further studies on optimization of these spatial structures seems necessary. Kaveh et al. [1-3] are the first researchers who formulated the problem of optimum design of barrel vault structures. In Ref. [1], they optimized two single barrel vaults utilizing different Charged System Search (CSS)-based methods containing the standard CSS [4], improved CSS [5], a magnetic CSS (MCSS) [6], and its improved version

*E-mail address of the corresponding author: talatahari@tabrizu.ac.ir (S. Talatahari)
S. Talatahari and Y. Jahani

Also, Kaveh and Eftekhar [2] have performed optimal design of barrel vault frames using an improved Big Bang-Big Crunch algorithm, in which a single layer barrel vault is optimized under both symmetrical and unsymmetrical loading cases. In the other study by Kaveh et al. [3], some single and multiple layer barrel vaults are optimized via the CSS algorithms.

In the present study, optimal design of single layer barrel vault frames structures is performed and the aim is to optimize this kind of structures with a hybrid CSS and particle swarm optimization (PSO) method. This method utilizes some benefits of the PSO into the CSS. An improved variant of the hybrid method is presented and a comparing to the other CSS-based method as well as some aspects regarding to the problem statement are discussed.

2. OPTIMUM DESIGN OF BARREL VAULT STRUCTURES

The purpose of size optimization of barrel vault structures is to minimize the weight of the structure, \( W \), through finding the optimal cross-sectional areas \( A_i \) of members, in which all constraints exerted on the problem must be satisfied, simultaneously. Thus, the optimal design of barrel vault frame structures can be formulated as:

Find \( X = [x_1, x_2, x_3, ..., x_n] \)  

(1) 

to minimize \( Mer(X) = f_{penalty}(X) \times W(X) \)  

(2) 

The cost function is

\[
W(X) = \sum_{i=1}^{nm} \gamma_i x_i L_i
\]  

(3) 

where \( x_i \), \( \gamma_i \), and \( L_i \) are the area, material density and length of the steel section selected for member group \( i \), respectively. \( X \) is the vector containing the design variables; For the discrete optimum design problem, the variables \( x_j \) are selected from an allowable set of discrete values; \( n \) is the number of member groups. Here, the objective of finding the minimum weight structure is subjected to several design constraints, including strength and serviceability requirements.

The penalty function is

\[
f_{penalty}(X) = \left(1 + \varepsilon_i \sum_{j=1}^{np} U_j^k \right)^{\varepsilon_1}
\]  

(4)
where \( np \) is the number of multiple loading conditions. In this paper \( \varepsilon_1 \) is taken as unity and \( \varepsilon_2 \) is set to 1.5 in the first iterations of the search process, but gradually it is increased to 3, \(^{[7]}\). \( \nu^i \) is the summation of penalties for all imposed constraints for \( k \)th charged particle which is mathematically expressed as:

\[
\nu = \sum_{i=1}^{nm} \max \left( \nu^d_i, 0 \right) + \sum_{i=1}^{nm} \left( \max \left( \nu^s_i, 0 \right) + \max \left( \nu^t_i, 0 \right) \right)
\]

(5)

where \( \nu^d_i, \nu^s_i, \nu^t_i \) are the summation of displacement, shear and interaction formula penalties, calculated by Eqs. (6) through (8), respectively.

Displacement constraint:

\[
\nu^d_i = \left| \frac{\delta_i}{\delta_{i0}} \right| - 1 \leq 0 \quad i=1,2,\ldots,nn
\]

(6)

Shear constraint, for both major and minor axis, \(^{[8]}\):

\[
\nu^s_i = \frac{V_u}{\phi V_n} - 1 \leq 0 \quad i=1,2,\ldots,nm
\]

(7)

Constraints corresponding to interaction of flexure and axial force \(^{[8]}\):

\[
\nu^t_i = \begin{cases} 
\frac{P_u}{\phi n P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_M M_{ux}} + \frac{M_{uy}}{\phi_M M_{uy}} \right) - 1 \leq 0 & \text{for} \quad \frac{P_u}{\phi n P_n} \geq 0.2 \\
\frac{P_n}{2\phi n P_n} + \frac{M_{ux}}{\phi_M M_{ux}} + \frac{M_{uy}}{\phi_M M_{uy}} - 1 \leq 0 & \text{for} \quad \frac{P_u}{\phi n P_n} < 0.2 
\end{cases}
\]

(8)

where \( nm \) is the number of nodes; \( \delta_i, \bar{\delta}_i \) are the displacement of the joints and the allowable displacement, respectively; \( nm \) is the number of members; \( V_u \) is the required shear strength; \( V_n \) is the nominal shear strength which is defined by the LRFD Specification, \(^{[8]}\); \( \phi_n \) is the shear resistance factor (\( \phi_n = 0.9 \)); \( P_u \) is the required strength (tension or compression); \( P_n \) is the nominal axial strength; \( \phi_c \) is the resistance factor (\( \phi_c = 0.9 \) for tension, \( \phi_c = 0.85 \) for compression); \( M_u \) is the required flexural strength; i.e., the moment due to the total factored load (Subscript \( x \) or \( y \) denotes the axis about which bending occurs.); \( M_n \) is the nominal
flexural strength determined in accordance with the appropriate equations in Chapter F of the LRFD Specification, [8] and $\phi_b$ is the flexural resistance reduction factor ($\phi_b = 0.9$).

### 3. NOMINAL STRENGTHS

Based on AISC-LRFD [8] specification, the nominal tensile strength of a member is equal to:

$$P_n = F_y A_g$$  \hspace{1cm} (9)

where $A_g$ is the gross section of the member.

The nominal compressive strength of a member is the smallest value obtained from the limit states of flexural buckling, torsional buckling, and flexural–torsional buckling. For members with compact and/or non-compact elements, the nominal compressive strength of the member for the limit state of flexural buckling is as follows:

$$P_n = F_{cr} A_g$$  \hspace{1cm} (10)

where $F_{cr}$ is the critical stress based on flexural buckling of the member, calculated as:

For $\lambda = \frac{Kl}{\pi r \sqrt{E}} \leq 1.5$, $F_{cr} = \left(0.658 \lambda^2\right) F_y$  \hspace{1cm} (11)

For $\lambda = \frac{Kl}{\pi r \sqrt{E}} > 1.5$, $F_{cr} = \left[ \frac{0.877}{\lambda^2} \right] F_y$  \hspace{1cm} (12)

In the above equations, $l$ is the laterally unbraced length of the member; $K$ is the effective length factor; $r$ is the governing radius of gyration about the axis of buckling and $E$ is the modulus of elasticity.

### 4. DESIGN LOADS

According to ANSI-A58.1 and ASCE/SEI 7-10 codes [9,10], there are some specific considerations for loading conditions of arched roofs such as barrel vault structures. In this study, the load conditions are taken from Ref. [1].

#### 4.1 Dead Load

The design dead load is established on the basis of the actual loads that may be expected to act on the structure of constant magnitude. The weight of various accessories, cladding,
supported lighting, heat and ventilation equipment, and the weight of space frame comprise the total dead load. In this study, a uniform dead load of 100 kg/m² is considered for estimated weight of sheeting, space frame, and nodes of barrel vault structures.

### 4.2 Snow Load

The snow load for arched roofs is calculated according to mentioned codes. Snow loads acting on a sloping surface shall be assumed to act on the horizontal projection of that surface. The sloped roof (balanced) snow load, $P_s$, shall be obtained by multiplying the flat roof snow load, $P_f$, by the roof slope factor, $C_s$, as follows:

$$P_s = C_s P_f$$  \hspace{1cm} (13)

where $C_s$ is

$$C_s = \begin{cases} 
1.0 & \alpha < 15^\circ \\
1.0 - \frac{\alpha - 15^\circ}{60^\circ} & 15^\circ < \alpha < 60^\circ \\
0.25 & \alpha > 60^\circ 
\end{cases}$$  \hspace{1cm} (14)

The distribution in arched roofs is shown in Fig. 1. In this paper, the flat roof snow load ($P_f$) is set to 150 kg/m².

![Figure 1. C_s distribution in arched roofs, [1]](image)

### 4.3 Wind Load

For the wind load in arched roofs, different loads are applied in the windward quarter, center half and leeward quarter of the roof (Fig. 2) which are calculated based on ANSI and ASCE codes [9,10] as:

$$P = q G_h C_p$$  \hspace{1cm} (15)

where $q$ is the wind velocity pressure, $G_h$ is the gust-effect factor and $C_p$ is the external
pressure coefficient.

Figure 2. Wind pressure on an arched roof, [1]

5. A REVIEW ON CHARGED SYSTEM SEARCH AND PARTICLE SWARM OPTIMIZATION

Since the hybrid algorithm is based on the CSS and PSO, here a brief review on these algorithms is described in the following subsections and then the hybrid algorithm will be developed in the next section.

5.1 Charged system search
The CSS algorithm contains a number of CPs where each one is treated as a charged sphere and can insert an electric force to the others, [4]. The pseudo-code for the CSS algorithm is summarized as follows:

Step 1: initialization. The magnitude of the charge for each CP is defined as:

\[ q_i = \frac{W_i - W_{\text{worst}}}{W_{\text{best}} - W_{\text{worst}}}, \ i=1,2,\ldots,N \]  \hspace{1cm} (16)

where \( W_{\text{best}} \) and \( W_{\text{worst}} \) are the best and the worst objective function values among all of the particles; \( W_i \) represents the fitness of the agent \( i \); and \( N \) is the total number of CPs. The separation distance \( r_{ij} \) between any two CP is defined as follows:

\[ r_{ij} = \frac{\| X_i - X_j \|}{\| (X_i + X_j)/2 - X_{\text{best}} \| + \varepsilon} \]  \hspace{1cm} (17)

where \( X_i \) and \( X_j \) are the positions of the \( i \)th and \( j \)th CPs, respectively; \( X_{\text{best}} \) is the position of the best current CP; and \( \varepsilon \) is a small positive number. The initial positions of CPs are determined randomly and the initial velocities of CPs are assumed to be zero.

Step 2: CM creation. A number of the best CPs and the values of their corresponding objective functions are saved in the charged memory (CM).

Step 3: The forces determination. The probability of moving each CP towards the others is determined using the following function:
\[ p_{ij} = \begin{cases} 1 & \frac{W_i - W_{best}}{W_j - W_i} > \text{rand} \text{ or } W_j > W_i \\ 0 & \text{otherwise} \end{cases} \] (18)

Then, the resultant force vector for each CP is calculated as:

\[ F_j = q_j \sum_{i \neq j} \left( \frac{q_i}{d_{ij}^2} + \frac{q_i}{d_{ij}^2} \right) p_{ij} (X_i - X_j), \quad \begin{cases} j = 1, 2, ..., N \\ i_1 = 1, i_2 = 0 \Leftarrow r_{ij} < a \\ i_1 = 0, i_2 = 1 \Leftarrow r_{ij} \geq a \end{cases} \] (19)

where \( F_j \) is resultant force acting on the \( j \)th CP; \( X_i \) and \( X_j \) are the positions of the \( i \)th and \( j \)th CPs, respectively.

Step 4: Solution construction. Each CP moves to the new position as:

\[ X_{j, new} = \text{rand}_1 \cdot k_a \cdot F_j + \text{rand}_2 \cdot k_v \cdot V_{j, old} + X_{j, old} \] (20)

\[ V_{j, new} = \frac{X_{j, new} - X_{j, old}}{\Delta t} \] (21)

where \( k_a \) and \( k_v \) are the acceleration and the velocity coefficients, respectively; and \( \text{rand}_1, \text{rand}_2 \) are two random numbers.

Step 5: CM updating. The better new vectors are included to the CM and the worst ones are excluded from the CM.

Step 6: Terminating criterion control. Steps 3-5 are repeated until a terminating criterion is satisfied.

5.2. Particle swarm optimization

The PSO is based on a metaphor of social interaction such as bird flocking and fish schooling, and is developed by Eberhart and Kennedy, [11]. The PSO simulates a commonly observed social behavior, where particles of a group (swarm) tend to follow the lead of the best of the group. In other words, the particles fly through the search space and their positions are updated based on the best positions of individual particles denoted by \( P_i^e \) and the best position among all particles in the search space represented by \( P_g^e \).

The procedure of the PSO is reviewed as follows:

Step 1: Initialization. An array of particles and their associated velocities are initialized with random positions.

Step 2: Local and global best creation. The initial particles are considered as the first local best and the best of them corresponding to the minimum fitness function will be the first global best.

Step 3: Solution construction. The velocity and location of each particle are changed to the new position using the following equations:
where $X_i^k$ and $V_i^k$ are the position and velocity for the $i$th particle at iteration $k$; $\omega$ is an inertia weight to control the influence of the previous velocity; $r_1$, and $r_2$ are two random numbers; $c_1$ and $c_2$ are two constants; $P_i^k$ is the best position of the $i$th particle up to the current iteration; $P_g^k$ is the so-far best position among all particles in the swarm and the sign “$\circ$” denotes element-by-element multiplication.

Step 4: Local best updating. The objective function of the particles is evaluated and $P_i^k$ is updated according to the best current value of the fitness function.

Step 5: Global best updating. The current global minimum objective function value among the current positions is determined and thus $P_g^k$ is updated if the new position is better than the previous one.

Step 6: Terminating criterion control. Steps 3-5 are repeated until a terminating criterion is satisfied.

It should be noted that for improving the performance of the PSO algorithm, equation (23) can be modified as

$$V_i^{k+1} = \omega V_i^k + c_1 r_1 \circ (P_i^k - X_i^k) + c_2 r_2 \circ (P_g^k - X_i^k) + \sum_{j=1}^{ne} c_j r_j \circ (R_j^k - X_i^k)$$  \hspace{1cm} (24)$$

where $c_j$ is a constant and $r_j$ is a random vector. $ne$ denotes the number of extra terms considered in the algorithm, and $R_j^k$ is a position of an agent defined based on the type of the algorithm being used.

6. A HYBRID CHARGED SYSTEM SEARCH - PARTICLE SWARM OPTIMIZATION

The hybrid CSS-PSO algorithm was presented by Kaveh and Talatahari [12] in which the location of the global and local best CPs are utilized to improve the searching process. In other words in the CSS-PSO, the advantage of the PSO consisting of utilizing the local best and the global best is added to the CSS algorithm.

6.1. Hybrid CSS-PSO Method

The CM for the hybrid algorithm is treated as the local best in the PSO, and the CM updating process is defined as follows, [12]:

$$X_i^{k+1} = X_i^k + V_i^{k+1}$$
$$V_i^{k+1} = \omega V_i^k + c_1 r_1 \circ (P_i^k - X_i^k) + c_2 r_2 \circ (P_g^k - X_i^k)$$  \hspace{1cm} (22)$$

(23)
HYBRID CHARGED SYSTEM SEARCH - PARTICLE SWARM OPTIMIZATION...

\[ CM_{i,\text{new}} = \begin{cases} CM_{i,\text{old}} & W(X_{i,\text{new}}) \geq W(CM_{i,\text{old}}) \\ X_{i,\text{new}} & W(X_{i,\text{new}}) < W(CM_{i,\text{old}}) \end{cases} \] (25)

If the coefficient \( k_j \) is defined as:

\[ k_j = \left( \frac{q_i}{a} r_{ij} i_1 + \frac{q_i}{r_{ij}^2} i_2 \right) ar_{ij} p_{ij} \] (26)

Then the equation (13) can be simplified as

\[ F_j = \sum_{i \in S_1} k_i \left( CM_{i,\text{old}} - X_j \right) + \sum_{i \in S_2} k_i \left( X_i - X_j \right) \] (27)

Here, \( ar_{ij} \) determines the kind of force and is defined as

\[ ar_{ij} = \begin{cases} +1 & \text{rand} < 0.8 \\ -1 & \text{otherwise} \end{cases} \] (28)

where \( \text{rand} \) represents a random number.

In Ref. [12], four variant hybrid methods were proposed as the CSS-PSO algorithms. Here, the best one is selected and utilized. In the selected method, not only the global and local best agents from the CM but some other stored points are utilized. In addition, some of the locations of the current agents are also utilized to determine the resultant force. The corresponding equation can be expressed as:

\[ F_j = k_1 \left( CM_{g,\text{old}} - X_j \right) + k_2 \left( CM_{j,\text{old}} - X_j \right) + \sum_{i \in S_1} k_i \left( CM_{i,\text{old}} - X_j \right) + \sum_{i \in S_2} k_i \left( X_i - X_j \right) \] (29)

Where \( S_1 \) and \( S_2 \) are defined as follows:

\[ S_1 = \{ t_1, t_2, \ldots, t_n \mid q(t) > q(j), \quad j=1,2,\ldots,N, \quad j \neq i,g \} \] (30)

\[ S_2 = S - S_1 \] (31)

in which \( S_1 \) determines the set of agents from CM utilized in equation (29). \( N \) denotes the number of agents in the CM. \( S \) is utilized as a set of all agents’ number and thus \( S_2 \) will be the set of current agents used for directing the agent \( j \). These equations clarify that for each agent one of its locations, namely its local best or its current location, are certainly utilized. For this formulation, in the primary iterations \( n \) is set to zero then it is increased linearly to \( N \) in the last iterations.
6.2. Parameter improvement for the CSS-PSO method

The CSS-PSO method utilizes harmony search-based approach for position correction of CPs. This method needs some parameters such as CMCR (Charge Memory Considering Rate) and PAR (Pitch Adjustment Rate) parameters that help the algorithm to find globally and locally improved solutions, respectively [13]. $\text{PAR}$ and $bw$ in this step are very important parameters in fine-tuning of optimized solution vectors, and can be potentially useful in adjusting convergence rate of algorithm to optimal solution [14]. In the standard hybrid algorithm, the fixed values were used for these parameters. Here, to improve the performance of this step of the algorithm and eliminate the drawbacks lies with fixed values of $\text{PAR}$ and $bw$, they change dynamically with iteration number as follow [14]:

$$PAR_i = PAR_{\text{min}} + \frac{PAR_{\text{max}} - PAR_{\text{min}}}{i_{\text{max}}} \cdot i$$

(32)

$$bw_i = bw_{\text{max}} \cdot e^{-c}, \quad c = \frac{\ln(bw_{\text{max}} - bw_{\text{min}})}{i_{\text{max}}}$$

(33)

where $bw_i$ is the bandwidth for each iteration, $bw_{\text{min}}$ and $bw_{\text{max}}$ are the minimum and maximum bandwidth, respectively. In this paper $PAR_{\text{min}}$ and $PAR_{\text{max}}$ are set to 0.3 and 0.99, respectively, [13].

7. NUMERICAL EXAMPLES

In this study, two single layer barrel vaults are selected from [1] in which Kaveh et al. used some CSS-based methods to optimize barrel vaults. They used the standard CSS, MCSS, ICSS and IMCSS algorithms and compare the results with each other. Since the proposed algorithm is based on the CSS, we select their examples to compare the results with. In all examples, the material density is 0.2836 lb/in$^3$ (7850 kg/m$^3$) and the modulus of elasticity is 30450 ksi (2.1E6 kg/m$^2$). The yield stress $F_y$ of steel is taken as 34135.96 psi (2400 kg/m$^2$) for both problems. Also, member sections are pipe shape and taken from the AISC-LRFD code [8].

7.1. A 173-member single barrel vault frame

The geometry of this example containing 3D and plan view are shown in Fig. 3. The member groups and support conditions are presented in the figure. Member sections are categorized in 15 groups as shown in Fig. 3b, [1]. The span, length and height of single barrel vaults are 30, 30 and 8 meter, respectively. This example has 173 members and 108 joints. In this study, similar to Ref. [1], the loading cases contain 3-types of static loads; dead load, snow load and wind load. The uniform dead load equal to 100 kg/m$^3$ applied to the roof. The snow and wind loads are shown in Fig. 4.
Table 1 compares the result of the new hybrid algorithm with some other CSS-based methods. The CSS-PSO algorithm finds the best solutions with 15000 number of analyses. In comparing with the CSS, MCSS, ICSS and IMCSS algorithms, the hybrid algorithm has the best solution and the best weights of present method is 45297.82 lb, while it is 50295.90 lb, 50247.66 lb, 49411.27 lb and 48985.05 lb for the CSS, MCSS, ICSS and IMCSS algorithms, respectively. It is clear that the CSS-PSO algorithm gives an economical design compared to the other algorithms. Also, the maximum strength ratio for the new method is 90.06%. Fig. 5 shows the strength ratios for all elements of the 173-member single layer barrel vault frame for optimal results of the new algorithm. As shown in Fig. 5, the strength ratios of elements are lower than 1, and the stress distribution is good and all of the constraints are satisfied.
Figure 4. The 173-bar single layer barrel vault frame subjected to: (a) Snow loading, (b) Wind loading

Figure 5. Strength ratios for the elements of the 173-bar single layer barrel vault frame
Table 1: Optimal design comparison for the 173-bar single layer barrel vault frame

<table>
<thead>
<tr>
<th>Element Group</th>
<th>Optimal sections and cross-section Area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Section Name</td>
</tr>
<tr>
<td>1</td>
<td>P1</td>
</tr>
<tr>
<td>2</td>
<td>P1</td>
</tr>
<tr>
<td>3</td>
<td>XP1.5</td>
</tr>
<tr>
<td>4</td>
<td>P0.75</td>
</tr>
<tr>
<td>5</td>
<td>XP0.5</td>
</tr>
<tr>
<td>6</td>
<td>XP1.25</td>
</tr>
<tr>
<td>7</td>
<td>P1</td>
</tr>
<tr>
<td>8</td>
<td>P10</td>
</tr>
<tr>
<td>9</td>
<td>P10</td>
</tr>
<tr>
<td>10</td>
<td>P10</td>
</tr>
<tr>
<td>11</td>
<td>P10</td>
</tr>
<tr>
<td>12</td>
<td>P10</td>
</tr>
<tr>
<td>13</td>
<td>P6</td>
</tr>
<tr>
<td>14</td>
<td>P6</td>
</tr>
<tr>
<td>15</td>
<td>P10</td>
</tr>
</tbody>
</table>

Weight (lb) 50295.90 50247.66 49411.27 48985.05 45297.82
No. of analysis 20000 20000 20000 19800 14000

7.2. A 292-member single barrel vault frame

The geometry of a 292-member single barrel vault frame is presented in Fig. 6. The member groups and support conditions are presented as well. Member sections are categorized into 30 groups as shown in Fig. 6b. The span, length and height of single barrel vaults are 36, 20 and 10 meter, respectively. This example has 292 members and 117 joints.

Similar to the previous example, the geometry and load conditions of this example are taken from Ref. [1]. The uniform dead load equal to 100 kg/m³ applied to roof. The snow and wind loads are shown in Fig. 7.
Figure 6. The 292-member single layer barrel vault frame: (a) 3D view, (b) Member groups in top view

Table 2 indicates a comparison between the results of the CSS, MCSS, ICSS, IMCSS and the CSS-PSO algorithms for this example. Table 2 shows that the best weights of present method is 60584.24 lb, while it is 68324.57 lb, 65892.33, 63694.69 lb and 62968.19 lb for the CSS, MCSS, ICSS and IMCSS algorithms. In comparing with the CSS, MCSS, ICSS and IMCSS algorithms, the CSS-PSO algorithm has a good solution. The maximum strength ratio for the design obtained by the new method is 92.50%. This algorithm needs 16000 analyses to find the optimum design as shown in Fig. 8.
HYBRID CHARGED SYSTEM SEARCH - PARTICLE SWARM OPTIMIZATION...

Figure 7. The 292-member single layer barrel vault frame subjected to: (a) Snow loading, (b) Wind loading

Figure 8. Convergence history for the 292-bar single layer barrel vault frame

Table 2: Optimal design comparison for the 292-bar single layer barrel vault frame

<table>
<thead>
<tr>
<th>Element Group</th>
<th>Optimal sections and cross-section Area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CSS</td>
</tr>
<tr>
<td></td>
<td>Area (in²)</td>
</tr>
<tr>
<td>1</td>
<td>P12 14.6</td>
</tr>
<tr>
<td>2</td>
<td>P10 11.9</td>
</tr>
<tr>
<td>3</td>
<td>XP8 12.8</td>
</tr>
<tr>
<td>4</td>
<td>P6  5.58</td>
</tr>
<tr>
<td>5</td>
<td>XP8 12.8</td>
</tr>
<tr>
<td>6</td>
<td>P10 11.9</td>
</tr>
<tr>
<td>7</td>
<td>P10 11.9</td>
</tr>
<tr>
<td>8</td>
<td>P12 14.6</td>
</tr>
<tr>
<td>9</td>
<td>P10 11.9</td>
</tr>
<tr>
<td>10</td>
<td>XP12 19.2</td>
</tr>
</tbody>
</table>
## 8. DISCUSSION

As it can be seen from the obtained results and the related figures for two above-mentioned structures, we just considered one direction for the wind load. For example in the first structure, the section for group 10 is P8 (with area equals 8.4 in\(^2\)) and it is P10 (11.9 in\(^2\)) for elements in group 13 while they are the same locations compared to the peak point of the barrel vault. This means that some elements (that are similar considering the geometry), take completely different optimum sections. Therefore, 15 and 30 different design groups are defined in Ref. [1]. However, according to design codes, both positive and negative direction of the wind load should be taken in account as combination of loads. If one defines such loading combinations, the above structures will be symmetric and the number of group becomes smaller (8 for the first example and 18 for the other one as shown in Fig. 9. Optimization point of view, a small number of design variables creates a small search space and needs a small computational costs. We know that this may cause heavier structures, however structural point of view, the obtained design can be utilized directly because all loading conditions defined by design codes were considered. For the examples prepared in the previous section by using the new design groups and defining all load combinations, the CSS-PSO algorithm is applied and the results are presented in Table 3. As it can be seen, for the first and second examples the obtained weights are 14% and 2.5% more than the designs with the previous loading conditions however the required number of analyses are reduced to 9000 and 12000, respectively.

<table>
<thead>
<tr>
<th></th>
<th>XP2.5</th>
<th>2.25</th>
<th>P0.5</th>
<th>0.25</th>
<th>P1.25</th>
<th>0.669</th>
<th>XP2</th>
<th>1.48</th>
<th>P2</th>
<th>1.07</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>P1.25</td>
<td>0.669</td>
<td>XP0.75</td>
<td>0.433</td>
<td>XP2</td>
<td>1.48</td>
<td>P1.5</td>
<td>0.799</td>
<td>P1.5</td>
<td>0.799</td>
</tr>
<tr>
<td>12</td>
<td>P2.5</td>
<td>6.11</td>
<td>P2.5</td>
<td>1.7</td>
<td>P1</td>
<td>0.494</td>
<td>P1.25</td>
<td>0.669</td>
<td>P1.25</td>
<td>0.669</td>
</tr>
<tr>
<td>13</td>
<td>XP3.5</td>
<td>3.68</td>
<td>XP1</td>
<td>0.639</td>
<td>XP2</td>
<td>1.48</td>
<td>P1.5</td>
<td>1.07</td>
<td>P1</td>
<td>0.494</td>
</tr>
<tr>
<td>14</td>
<td>P4</td>
<td>3.17</td>
<td>XP1.5</td>
<td>1.07</td>
<td>XP1.5</td>
<td>1.07</td>
<td>P1.5</td>
<td>0.799</td>
<td>P1.5</td>
<td>0.799</td>
</tr>
<tr>
<td>15</td>
<td>XP2</td>
<td>1.48</td>
<td>P3.5</td>
<td>2.68</td>
<td>P1.5</td>
<td>0.799</td>
<td>XP2.5</td>
<td>2.25</td>
<td>P3</td>
<td>2.23</td>
</tr>
<tr>
<td>16</td>
<td>XP2</td>
<td>1.48</td>
<td>XP1.5</td>
<td>1.07</td>
<td>XP2</td>
<td>1.48</td>
<td>P1.25</td>
<td>0.669</td>
<td>P1.5</td>
<td>0.779</td>
</tr>
<tr>
<td>17</td>
<td>XXP3</td>
<td>5.47</td>
<td>XP1</td>
<td>0.639</td>
<td>XP1</td>
<td>0.639</td>
<td>XP1</td>
<td>0.639</td>
<td>P2</td>
<td>1.07</td>
</tr>
<tr>
<td>18</td>
<td>XP2</td>
<td>2.66</td>
<td>XP2</td>
<td>1.48</td>
<td>XP3</td>
<td>3.02</td>
<td>P1.5</td>
<td>0.799</td>
<td>P1.5</td>
<td>0.799</td>
</tr>
<tr>
<td>19</td>
<td>P5</td>
<td>4.3</td>
<td>P3</td>
<td>2.23</td>
<td>XP1.5</td>
<td>0.881</td>
<td>XP2</td>
<td>1.48</td>
<td>P2</td>
<td>1.07</td>
</tr>
<tr>
<td>20</td>
<td>XXP2</td>
<td>2.25</td>
<td>XP1.5</td>
<td>1.07</td>
<td>XP2</td>
<td>1.48</td>
<td>XP1.5</td>
<td>1.07</td>
<td>P1.5</td>
<td>0.779</td>
</tr>
<tr>
<td>21</td>
<td>XP1</td>
<td>0.639</td>
<td>P3</td>
<td>2.23</td>
<td>XP0.75</td>
<td>0.433</td>
<td>P1.5</td>
<td>0.799</td>
<td>P1.5</td>
<td>0.779</td>
</tr>
<tr>
<td>22</td>
<td>XP2</td>
<td>1.48</td>
<td>P2.5</td>
<td>1.7</td>
<td>XP0.75</td>
<td>0.433</td>
<td>P1.5</td>
<td>0.799</td>
<td>1.07</td>
<td>P2</td>
</tr>
<tr>
<td>23</td>
<td>XP2</td>
<td>1.48</td>
<td>P2.5</td>
<td>1.7</td>
<td>XP0.75</td>
<td>0.433</td>
<td>P1.5</td>
<td>0.799</td>
<td>P1.5</td>
<td>0.779</td>
</tr>
<tr>
<td>24</td>
<td>P1.5</td>
<td>0.799</td>
<td>P1.25</td>
<td>0.669</td>
<td>XP1.25</td>
<td>0.881</td>
<td>P1.25</td>
<td>0.669</td>
<td>P1.25</td>
<td>0.669</td>
</tr>
<tr>
<td>25</td>
<td>XP1.5</td>
<td>1.07</td>
<td>P1.25</td>
<td>0.669</td>
<td>XP1.25</td>
<td>0.881</td>
<td>XP1.25</td>
<td>0.881</td>
<td>P1.5</td>
<td>0.779</td>
</tr>
<tr>
<td>26</td>
<td>P1.5</td>
<td>0.799</td>
<td>P2.5</td>
<td>1.7</td>
<td>P1.5</td>
<td>0.799</td>
<td>P1.5</td>
<td>0.779</td>
<td>P1.5</td>
<td>0.779</td>
</tr>
<tr>
<td>27</td>
<td>XP2</td>
<td>1.48</td>
<td>P3</td>
<td>2.23</td>
<td>XP1.25</td>
<td>0.881</td>
<td>P1.5</td>
<td>0.799</td>
<td>P2.5</td>
<td>1.7</td>
</tr>
<tr>
<td>28</td>
<td>XP1.5</td>
<td>1.07</td>
<td>P1.5</td>
<td>0.799</td>
<td>XP2</td>
<td>1.48</td>
<td>XP2</td>
<td>1.48</td>
<td>P2.5</td>
<td>1.7</td>
</tr>
<tr>
<td>29</td>
<td>XP1.5</td>
<td>1.07</td>
<td>P1.5</td>
<td>0.799</td>
<td>P4</td>
<td>3.17</td>
<td>XP8</td>
<td>12.8</td>
<td>XP2.5</td>
<td>2.25</td>
</tr>
</tbody>
</table>

| Weight (lb) | 68324.57 | 65892.33 | 65694.69 | 62968.19 | 60584.24 |
| No. of analysis | 20000 | 20000 | 20000 | 17500 | 16000 |
Figure 9. The Member groups in top view for a) the first example, b) the second example.


Table 3: Optimal design obtained for the single layer barrel vault frame considering new loading conditions

<table>
<thead>
<tr>
<th>Element Group</th>
<th>Optimal sections and cross-section areas</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Example</td>
<td>Second Example</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Section Name</td>
<td>Area ( (\text{in}^2) )</td>
<td>Section Name</td>
</tr>
<tr>
<td>1</td>
<td>P0.5</td>
<td>0.25</td>
<td>P12</td>
</tr>
<tr>
<td>2</td>
<td>P0.5</td>
<td>0.25</td>
<td>P10</td>
</tr>
<tr>
<td>3</td>
<td>P0.5</td>
<td>0.25</td>
<td>P10</td>
</tr>
<tr>
<td>4</td>
<td>P0.5</td>
<td>0.25</td>
<td>P10</td>
</tr>
<tr>
<td>5</td>
<td>P10</td>
<td>11.9</td>
<td>P10</td>
</tr>
<tr>
<td>6</td>
<td>P10</td>
<td>11.9</td>
<td>P10</td>
</tr>
<tr>
<td>7</td>
<td>P8</td>
<td>8.40</td>
<td>P2</td>
</tr>
<tr>
<td>8</td>
<td>P10</td>
<td>11.9</td>
<td>P2</td>
</tr>
<tr>
<td>9</td>
<td>P1.5</td>
<td>0.799</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>P3</td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>P1</td>
<td>0.494</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>P1.5</td>
<td>0.799</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>XP2.5</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>P2.5</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>P2.5</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>P2</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>P1.25</td>
<td>0.669</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>P1.5</td>
<td>0.779</td>
<td></td>
</tr>
</tbody>
</table>

Weight. (lb): 52784.74 62113.79  
No. of analysis: 9000 12000

9. CONCLUSIONS

Recently, a hybrid charge system search and particle swarm optimization algorithm was developed based on the charge system search (CSS) and positive properties of particle swarm optimization (PSO) are added to it. Here, some parameters of this algorithm are set in a way that the performance of the algorithm is improved. Then, two single-layer barrel vaults are selected as numerical examples. The geometry and load conditions are taken from Ref. [1]. This examples are optimized with the CSS-PSO algorithm and then compared with other CSS-based algorithms. The results show that the CSS-PSO method can find the good and economical designs of single-layer barrel vaults. Also, the loading conditions are modified according to the recommendations of design codes. This change makes the structures symmetric and therefore, the number of design groups are reduced. Optimization point of view, this change may increase optimum weight more or less, however the computational costs will be reduced.
REFERENCES


