CALCULATION OF FREQUENCY OF RETAINING WALL BY BACK PROPAGATION NEURAL NETWORK

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ABSTRACT

A method is used to obtain the fundamental frequency of a retaining wall quite accurately and carry out a dynamic analysis of such wall based on modal response technique. The present procedure establishes both the general and particular cases of dynamic response of retaining wall based on improved Rayleigh-Ritz method. The wall will be assumed to be a flexural member. The fundamental frequency of the retaining wall with soil mass has been computed. The results based on proposed method are then used to back propagation neural network (BPN). In the present work, the fundamental frequency of a retaining wall is calculated by BPN. A significant benefit of BPN is its ability to learn relationships between variables with repeated exposure to those variables. Therefore, instead of deriving an analytical relationship from mathematical formulations, the BPN learns the relationship through an adaptive training process. Numerical example shows the merit of the BPN.

Keywords: Frequency; retaining walls; back propagation neural network

1. INTRODUCTION

Dynamic pressure induced in the soil both due to active and passive conditions are computed using a pseudo-static force acting within the failed soil wedge and equivalent dynamic coefficients of active and passive earth pressures. Okabe [1] and Mononobe and Matsuo [2] obtained the pressures considering the wall as gravity type having an infinite stiffness with ground acceleration at its maximum value. However, a typical retaining wall is more flexible than the gravity wall and such analysis could be either too conservative or unsafe. Again, the acceleration coefficient depends on the natural period of the wall-soil system, which in turn is a function of the stiffness and mass distributions.

The most commonly used methods to design the retaining wall under seismic conditions are the force equilibrium based pseudo-static analysis [1,3], pseudo-dynamic analysis [2,4] and displacement based sliding block methods [5-6]. In the design of this retaining wall, knowledge of earth pressures under both active and passive conditions should be cleared to compute the earth forces and their point of applications. In Refs. [7-8] the seismic earth

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pressures is calculated by an artificial neural network. Then the effect of aspect ratio on earth pressures for at-rest conditions are investigated [9]. In Ref. [10-11] dynamic analysis of retaining wall is performed.

In the codes does not provide any rational method for calculation of frequency of the retaining wall, seismic analysis for such system is still based on what one can term as a pseudo static analysis. Many analyses have come up for the solution of such problems based on finite element method. The basic lacunae in most of these analyses is that the earth pressure is generated under incipient failure of the soil medium, while most of the commercially available standard finite element packages restrict themselves to elastic analysis where simulating such failure soil profile is not possible. Also, for a problem usually modeled as plane strain element connected to the wall and usually modeled as beam element, the number of modes to be considered in the analysis becomes a major stumbling block for the first 5/6 modes (may be the soil modes are still uncoupled and the wall modes are yet to take part in the vibration). Under such a situation, the mode number at which the wall gets coupled is difficult to gauge. The normal practice is to consider a large number of modes, which makes the analysis time consuming as well as expensive in terms of computational effort. Finally, for routine engineering work, unless very critical, such sophisticated analysis is usually not warranted for a retaining wall analysis.

In this present paper, a method is used by which it is possible to obtain the fundamental frequency of a retaining wall quite accurately and to carry out an alternate method for the dynamic analysis of such walls based on modal response technique. The wall will be assumed to be a flexural member. The fundamental frequency of the retaining wall with soil mass has been computed for active and passive earth pressure condition. In the active earth pressure the earthquake shock pushes the wall away from the retained soil, and in the passive case the earthquake shock pushes the wall towards the retained soil.

2. CALCULATION OF FUNDAMENTAL FREQUENCY

When the earthquake force tries to move the retaining wall away from the soil, the soil behind the retaining wall is already under incipient failure having failure profile inclined at $45 + \frac{\phi}{2}$ as shown in Figure 1. For the passive case, failure profile is inclined at $45 - \frac{\phi}{2}$. It may be argued that during an earthquake shock, the soil under the failed condition will not contribute to stiffness but will contribute its inertia to the deflection of the wall. Based on the above assumption a cantilever wall is considered for analysis, which will be subjected to the following loads under static condition:

Considering the self-weight of the wall to be negligible compared to the soil mass. Shown in Figure 2 is the mass distribution of the failed soil wedge ABD. For an elemental strip $dz$ in vertical direction mass distribution is given by $m(z) = \gamma dz / \tan \alpha$. However, if one is willing to consider the self weight of the retaining wall he may change the expression for $m(z)$ to $m(z) = \gamma r / g \tan \alpha + \gamma_c z t / g$, in which $\gamma$ is unit weight of soil, $\gamma_c$ is unit weight of concrete, $r$ is average thickness of the wall, and $g$ is gravitational acceleration.
As the first step for frequency analysis, stiffness and equivalent mass contributing to the dynamic response of the system are developed. To this end an improved Rayleigh-Ritz method has been used to obtain the stiffness and mass of the wall-soil system. For a conservative system if $E$ is the kinetic energy and $V$ the potential energy of the system, at any time $t$, the energy equations may be written in the form \[12\]

\[\int_0^H m(z) \left[ \frac{\partial y(z,t)}{\partial t} \right]^2 dz\]

where

\[y(z,t) = \sum_{i=1}^n \phi_i(z) q_i(t)\]

in which $y(z,t)$ is displacement function; $\phi_i(z)$ is admissible function; and $q_i(t)$ is generalized coordinate. Using Eq. (1), the energy equation can be written as
\[
E(t) = \frac{1}{2} \int_0^H m(z) \left[ \sum_{i=1}^{n} \phi_i(z)q_i(t) \right] \left[ \sum_{j=1}^{n} \phi_j(z)q_j(t) \right] dz = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_i(t)q_j(t) \left[ \int_0^H m(z)\phi_i(z)\phi_j(z)dz \right]
\] 

(3)

From which it can be concluded that the mass coefficient has the form

\[
m_{ij} = \int_0^H m(z)\phi_i(z)\phi_j(z)dz \quad (\text{for } i, j=1, 2, 3, \ldots, n)
\]

(4)

The potential energy, \( V \) can be written as [12]

\[
V(t) = \frac{1}{2} \int_0^H EI(z) \left[ \frac{\partial^2 y(z,t)}{\partial z^2} \right]^2 dz = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} q_i(t)q_j(t) \left[ \int_0^H EI(z) \frac{d^2 \phi_i(z)}{dz^2} \frac{d^2 \phi_j(z)}{dz^2} \right] dz
\]

(5)

And the stiffness \( k_{ij} \) has the form

\[
k_{ij} = \int_0^H EI(z) \frac{d^2 \phi_i(z)}{dz^2} \frac{d^2 \phi_j(z)}{dz^2} dz \quad (\text{for } i, j=1, 2, 3, \ldots, n)
\]

(6)

The wall is a cantilever flexural member, the displacement \( y(z,t) \) can be represented by

\[
y(z,t) = A_i \sin((2i-1)\pi z / 2H) \quad (\text{for } i=1, 2, 3, \ldots, n)
\]

(7)

where \( n \) is number of modes. For the first mode, \( \phi_1 = \sin(\pi z / 2H) \), for the second mode, \( \phi_2 = \sin(3\pi z / 2H) \), and so on.

Hence, for failure wedge of the soil, the mass contribution \( M \) for the first mode is given by

\[
M = m_{11} = \int_0^H \frac{\gamma c}{g \tan \alpha} \sin^2 \frac{\pi z}{2H} dz = \frac{\gamma H^2}{g \tan \alpha} \left[ \frac{\pi^2 + 4}{4\pi^2} \right]
\]

(8)

If \( I \) is the average moment of inertia of the wall, the stiffness \( K \) can be written as

\[
K = k_{11} = EI \int_0^H \frac{\pi^4}{16H^4} \sin^2 \frac{\pi z}{2H} dz = \frac{\pi^4 EI}{32H^3}
\]

(9)

in which \( E \) is Young’s modulus of concrete and \( H \) is height of the wall. Considering the fundamental frequency as \( \omega = \sqrt{K/M} \) [12] substituting the above values of \( K \) and \( M \) the fundamental frequency is
The above frequency would be sufficiently accurate. This is because we have arrived at the above frequency assuming a shape function of \( \phi_1 = \sin(\pi H / 2) \). How accurate the frequency will be, depends on how realistic has been the assumed shape function and there could be an error in the estimation in frequency depending on this choice. However, in Ref. [10] has shown that if higher order shape functions are chosen, and eigenvalues evaluated with higher order matrix.

The fundamental frequency based on the proposed formula and the matrix solution, is insignificant for all practical purposes and does not warrant any higher order matrix solution to arrive at a more accurate fundamental frequency for earthquake response of retaining walls.

3. FUNDAMENTAL FREQUENCY IN THE GENERAL CASE

Having established the fundamental approach to the problem for a simplified version with the top surface horizontal, the theory is extended to more general case where soil behind the wall is inclined at an angle \( \beta \) to the horizontal.

As shown in Figure 3, other than the load from the failed wedge ABD, the load of triangle ABC also contributes to the earthquake response. Since the triangle ABC has an oblique shape, as the first step, the triangle is redefined to a shape where the integration basis remains same as ABD. Hence,

\[
AC = \frac{H \cos \alpha}{\sin(\alpha - \beta)} \tag{11}
\]

The area of the triangle ABC is

\[
\text{Area} = 0.5AC \times OB = \frac{0.5H^2 \cos^2 \alpha \sin \beta}{\sin \alpha \sin(\alpha - \beta)} \tag{12}
\]

![Figure 3. Retaining wall with soil sloped at an angle behind the wall](https://www.SID.ir)
For the equivalent right angled triangle, ABE, where \( BE = h \), with the area same as that of ABC and one can write

\[
h = \frac{\cos \alpha \sin \beta}{\sin(\alpha - \beta)} H = \eta H
\]  

(13)

where

\[
\eta = \frac{\cos \alpha \sin \beta}{\sin(\alpha - \beta)}
\]

(14)

Substituting the limits 0 to \( H \) and 0 to \( h \) and also considering \( h = \eta H \), the above expression reduces to

\[
M = \frac{\gamma H^2}{g \tan \alpha} \left[ \frac{3 + 0.5\pi^2(1 + \eta^2) - (\cos \eta \pi + \eta \pi \sin \eta \pi)}{2\pi^2} \right]
\]

(15)

It can be observed that for \( \beta = 0 \), \( \eta = 0 \), Eq. (15) converges to Eq. (8). Albeit this idealization will not change the results for the total mass content for \( \Delta ABC \) and \( \Delta ABE \) remain unaltered. As such, the total lumped mass contribution remains constant. The stiffness term being independent of the soil mass remains the same as Eq. (9) for the earlier case. In this case the fundamental frequency is

\[
\omega = \frac{\pi^6 E I g \tan \alpha}{\sqrt{16}\gamma H^3 [3 + 0.5\pi^2(1 + \eta^2) - (\cos \eta \pi + \eta \pi \sin \eta \pi)]}
\]

(16)

4. BACK PROPAGATION NEURAL NETWORK

Back propagation neural network have been applied successfully in various fields. BPN can be used to learn from the training dataset the non-linear relationships between multiple inputs and outputs without requiring specific information on the fundamental mechanisms relating them [13-14]. The learning mimics the human learning process by correcting the errors continuously. BPN is composed of interconnected computational processing elements called neurons that process input information and give outputs.

The neurons are divided into layers. A typical BPN consists of an input layer representing the input variables, an output layer corresponding to the output variables and a hidden layer [13-15]. Neurons between two adjacent layers are fully connected by branches. Attached to each branch, there is a weight reflecting the strength of the connections. The training of the network involves finding the connection weights that minimize the sum of squares of the differences between the network outputs and the target values. The BPN learning process involves a forward propagation pass calculating the outputs using the inputs, weights and neuron transfer functions, as well as a back propagation pass correcting the weights using the error between the predicted and target values. The major advantage of BPN model is its ability to learn from examples without requiring principal knowledge of
domain problems. In addition, it is very effective in dealing with large amounts of data. The structure of BPN model can easily be constructed according to the domain problem and the availability of data attributes.

The node receives weighted activation from other nodes through its coming connections. First, these are added (summation function). The result is then passed through an activation function, the outcome being the activation of the neuron. For each of the outgoing connections, this activation value is multiplied with the specific weight and transferred to the next neuron [14,17].

During training of the network, data is processed through the input layer to hidden layer, until it reaches the output layer. In this layer, the output is compared to the measured values. The difference or error between both is processed back through the network updating the individual weights of the connections and the biases of the individual neurons. The input and output data are mostly represented as vectors called training pairs. The process as mentioned above is repeated for all the training pairs in the dataset, until the network error converged to a threshold minimum defined by a corresponding cost function; usually the root mean squared error (RMS) or summed squared error (SSE). In Figure 4 the \( j \)th neuron is connected with a number of inputs.

Figure 4. Architecture of the BPN used

The net input values in the hidden layer will be:

\[
Net_j = \sum_{i=1}^{n} x_i w_{ij} + b_j
\]  

(17)

where \( x_i \) is the input units, \( w_{ij} \) the weight on the connection of \( i \)th input and \( j \)th neuron, \( b_j \) the bias neuron, and \( n \) the number of input units. So, the net output from hidden layer is calculated using tangent sigmoid function as:
The total input to the $k$th unit is

$$Net_k = \sum_{j=1}^{a} w_{jk} O_j + b_k$$

(19)

where $b_j$ is the bias neuron, $w_{jk}$ the weight between $j$th neuron and $k$th output. So, the total output from $k$th unit will be

$$O_k = f(Net_k)$$

(20)

In the learning process, the network is presented with a pair of patterns, an input pattern and a corresponding desired output pattern. The network computes its own output pattern using its (mostly incorrect) weights and thresholds. Now, the actual output is compared with the desired output. Hence, the error at any output in layer $k$ is

$$e_l = t_k - O_k$$

(21)

where $t_k$ is the desired output, and $O_k$ the actual output. The total error function is given by

$$E = 0.5 \sum_{k=1}^{n} (t_k - O_k)^2$$

(22)

Training of the network is basically a process of arriving at an optimum weight space of the network. The descent down error surface is made using the following rule:

$$\nabla W_{jk} = -\xi (\partial E / \partial W_{jk})$$

(23)

where $\xi$ is the learning rate parameter, and $E$ the error function. The update of weights for the $(n+1)$th pattern is given as

$$W_{jk}(n + 1) = W_{jk}(n) + \nabla W_{jk}(n)$$

(24)

Similar logic applies to the connections between the hidden and output layers. This procedure is repeated with each pattern pair of training exemplar assigned for training the network. Each pass through all the training patterns is called a cycle or epoch. The process is then repeated as many epochs as needed until the error within the user specified goal is reached successfully. This quantity is the measure of how the network has learned.
5. THE RESULTS

In this study, the back propagation learning algorithm is used in a feed forward, single hidden layer network. The configuration 6-10-2 (Figure 4) (6 inputs, 10 hidden and 2 output neurons) appeared to be the most optimal topology for this application. The inputs of network are $\phi$, $\gamma$, $E$, $\beta$, $H$ and $t$. The outputs of network are fundamental frequency of retaining wall in active ($\omega_a$) and passive ($\omega_p$) case. The range of values of different input parameters is used from Ref. [18] (Table 1).

A variable transfer function is used as the activation function for both the hidden and output layers. In the hidden and output layers, the activation functions a tangent sigmoid function and linear is used, respectively. Several back propagation training algorithm was repeatedly applied until satisfactory training was achieved. The best training algorithm is Levenberg-Marquardt method [19].

The values of the training and test data were normalized to a range from -1 to +1. The data set available for the input and output is included 60 data patterns that are obtained from Eqs. (10) and (16). From these, 50 data patterns were used for training the network, and the remaining 10 patterns were randomly selected are used as the test data set.

Table 1: Input parameters for network and their range

<table>
<thead>
<tr>
<th>No</th>
<th>Input parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\phi$; Soil friction angle (degree)</td>
<td>28-42</td>
</tr>
<tr>
<td>2</td>
<td>$\gamma$; Unit weight of soil ($kN/m^3$)</td>
<td>18-23</td>
</tr>
<tr>
<td>3</td>
<td>$E$; Young’s modulus of concrete ($kN/mm^2$)</td>
<td>20-30</td>
</tr>
<tr>
<td>4</td>
<td>$\beta$; Ground inclination with respect to horizontal (degree)</td>
<td>0-30</td>
</tr>
<tr>
<td>5</td>
<td>$H$; Height of the wall (m)</td>
<td>2-10</td>
</tr>
<tr>
<td>6</td>
<td>$t$; Average thickness of the wall (m)</td>
<td>0.2-1</td>
</tr>
</tbody>
</table>

The network was trained with 67 training epochs. The training performance and error elimination by sum squared error (SSE) method for datasets. In each case, the network is trained until the mean squared error is less than 1e-4. Calculated and predicted values of $\omega_a$ and $\omega_p$ have been given in Figures 5 and 6, respectively. The correlation coefficients for the predicted and calculated values are as high as 0.9981 and 0.9972 for $\omega_a$ and $\omega_p$, respectively.
6. CONCLUSION

The present analysis proposes a solution both for the general and particular cases of dynamic response of retaining wall based on improved Rayleigh-Ritz method and the BPN. The advantage with the proposed method is that it gives a rational basis for estimation of the fundamental frequency. Results of 50 out of 60 have been used to train the BPN and the
other 10 are used to test the BPN. Average relative errors of the test of the BPN were found to be 1.14% for fundamental frequency of retaining wall in active case \( (\omega_a) \) and 2.48% for fundamental frequency of retaining wall in passive case \( (\omega_p) \). The correlation coefficients values found by the BPN were 0.9981 for \( \omega_a \), and 0.9972 for \( \omega_p \). These results showed that the BPN gave a good estimation of results.

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