Simulation of static sinusoidal wave in deep water environment with complex boundary conditions using proposed SPH method

A. H. Javid; M. Abbaspour; S. A. Mirbagheri; *H. JanfeshanAraghi

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Abstract: The study of wave and its propagation on the water surface is among significant phenomena in designing quay, marine and water structures. Therefore, in order to design structures which are exposed to direct wave forces, it is necessary to study and simulate water surface height and the wave forces on the structures body in different boundary conditions. In this study, the propagation of static sinusoidal wave in deep water environment with complex boundary conditions are simulated by using Smoothed Particle Hydrodynamics (SPH) technique. The governing equations are programmed using VISUAL FORTRAN 6.5 and the solution results are visualized using TECPLIT. After determining the suitable number of particles for simulation, the duration of sinusoidal wave oscillation are measured by simulation and are compared with analytical solution. After ensuring the accuracy and veracity of proposed SPH method in simulation of static sinusoidal wave motion on the deep water surface, the simulation are carried out in more complex boundary conditions which there are no analytical solutions.

Key words: SPH method; Static sinusoidal wave; Complex boundary condition; Deep water

Introduction

The Navier-Stokes equation is governing equation for many phenomena in fluid mechanics science. Although this equation is in linear form, but don’t have exact solution excluding in simple problem with many assumptions. The Approximate methods which called numerical methods in computational fluid dynamics (CFD) were extended for solving the Navier-Stokes equation. Using these methods, the continuum domain of problem discretized to finite number of point or cell. Also in numerical methods, differential equations convert to algebraic terms and employ the trial-error approach for solves converted equations. Two types of numerical methods were development; First type is mesh-base methods, which in these methods, the problem domain is discretized to finite number of cell. The cell is stationary and can not move in simulation progress. Second type of numerical method is mesh-less method. In this method, continuum domain of solution is discretized to number of point or particle which doesn’t have specific location and can move in simulation. SPH method is one of the mesh-less method which can simulate free surface flow with large deformation and complex boundary condition in Lagrangian description (Violeau and Issa, 2007).

Simulation of impulsive waves generated by landslides is presented in Ataei-Ashtiyani and Shobeyri (2007), Qiu (2008) and Capone et al. (2010) research. They used I-SPH method for simulation of generated wave and considered landslide in two type; i.e. rigid and deformable landslide. The results prove the efficiency and applicability of the I-SPH method for simulation of complex free surface problems with wave in surface. Yim et al., (2008) was investigated about the water wave generation by falling rigid body. They applied two different numerical approaches, RANS and SPH, for simulation the time histories of
fluid motion, free surface deformation, and the vertical displacement of a rectangular-shape rigid body. Numerical solutions for the velocity fields, pressure distributions, and turbulence intensities in the vicinity of the falling rigid body are good arrangement with experimental data (Yim et al., 2006). In the past researches, the wave motion and its propagation was simulated by SPH method in tuned liquid damper (TLD) and sloshing behavior. The shape of wave surface, water height and force in body wall was successfully determined (Bulian et al., 2010, Colagrossi et al., 2010).

Interaction between wave and structure is importance problem in design of coastal engineering structure. For design and construction of coastal structure must be considering wave height and force in structure walls. The governing equation for simulation of wave propagation in deep water environment is Navier-Stokes equation. This problem is free surface flow with large deformation and can not solve this equation with general approach. Consequently, the SPH method was selected for simulation the wave propagation in complex boundary condition. After simulation the problem and verification the output result with analytical solution, simulation will be done in complex boundary condition and wave propagation shape will be predicted.

MATERIALS AND METHODS

1. Numerical Modeling

1.1 Governing Equations

Lagrangian governing equations for a two dimensional incompressible flow are expressed as

\[
\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0
\]

Continuity equation (1)

\[
\frac{D\mathbf{v}^\alpha}{Dt} = \frac{1}{\rho} \frac{\partial}{\partial x^\beta} \left( -\rho \delta^{\alpha\beta} + \tau^{\alpha\beta} \right) + \mathbf{g}
\]

Momentum equation (2)

where \( D/Dt \) is the material derivative, \( \rho \) is the density of fluid, \( \mathbf{v} \) is flow velocity, \( \mathbf{g} \) is the acceleration of gravity, \( \alpha \) and \( \beta \) denote the coordinate directions (x or y), \( p \) is pressure stress, \( r \) is shear stress, and \( \delta^{\alpha\beta} \) is the Dirac Delta function (Ancye and Cochard, 2009). Eqs. (1) and (2) are applicable for Newtonian and non-Newtonian fluids. It is known that the relations between shear stress and the rate of shear strain for Newtonian fluid given as

\[
\dot{\gamma} = \mu \dot{\epsilon}
\]

where \( \mu \) is dynamic viscosity and \( \dot{\epsilon} \) is the rate of shear strain (Dufour and Cabot, 2005).

1.2 Equation of State

In the SPH method, the pressure \( p \) is expressed by the equation of state as \( p = p(\rho, C) \), where \( \rho \) is particle density and \( C \) is the speed of sound. The state equation employed in this study is the Tait’s state equation and is expressed as

\[
p = \frac{C_0^2 \mathbf{n}}{\mathbf{a}} \left( \frac{\mathbf{n}}{\rho_0} \right)^{\frac{3}{\gamma}} - 1
\]

(4)

where \( \rho_0 \) is the reference density and the value of \( \gamma = 7 \). Generally, the oscillation of density in incompressible fluid in numerical methods is allowed up to 1%. In SPH method, it is recommended that the speed of sound be considered 10 times more than the maximum speed of the fluid for an incompressible flow (Lopez et al., 2010, Gomez-Gesteria et al., 2010).

1.3 Sph Formulation

SPH method divides the domain of solution into a number of discrete particles (N). These particles have a spatial distance (h) over which their properties are smoothed by a kernel function. Method of interpolation in SPH is an integral descriptive method and the spatial variable is approximated using the available information in the local area around the desired point. In SPH method, discretization form of variable \( f \) and its derivative will be as follows

\[
< f(x_j) > = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W_{ij} ;
\]

\[ W_{ij} = W(x_j, h) \]
\[
\n< \nabla . f(x_i) > = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_i) \nabla_i W_{ij} 
\]
here \( f \) is a function of the three-dimensional position vector \( x \), \( i \) is the desired particle which the unknown variable is calculated, \( j \) are particles which are placed inside the influence domain of particle \( i \) and \( h \) is the smoothing length which defining the influence domain of kernel function, \( m_j \) is the mass of the particle, and \( W(x_i, h) \) is called the kernel function in SPH method (Bui et al., 2007). The value of \( f(x) \) in the \( i \)th point is approximated through the values of function in particles within influence domain of the \( i \)th particle.

The kernel function is normalized over its influence domain and outside this domain, the kernel function gets zero value and within that domain, it gets a positive value (Liu et al., 2003). In this study, a 3rd order and two dimensional kernel function is used as

\[
W(R) = \frac{1}{6} \left( 2 - R \right)^3 f \quad 0 \leq R < 1
\]

\[
= \frac{1}{2} R^2 + \frac{1}{2} R^3 f \quad 1 \leq R < 2
\]

\[
= 0 \quad R \geq 2
\]

\[
R = \left| x_i - x_j \right| / h
\]

The values of \( a \) for one, two and three dimensional simulations are \( 1/h \), \( 15/7\pi h^2 \) and \( 3/2\pi h^3 \) respectively (Fang et al., 2009).

### 1.4 Discretization

The derivation of the basic SPH form of Eq. (1) is

\[
\sum_{j=1}^{N} m_j v_{ij} \frac{\partial W_{ij}}{\partial x_i} \frac{\partial}{\partial t}
\]

where \( v_{ij} \) is the difference of the velocities of particles \( i \) and \( j \) in all directions and \( \partial W_{ij} / \partial x_i \) is the derivation of the kernel function in the considered direction. Eq. (2) is discretized as

\[
\frac{Dv_i^0}{Dt} = - \sum_{j=1}^{N} \left( \frac{p_i + p_j}{\rho_i \rho_j} \frac{\partial W_{ij}}{\partial x_i} + \frac{\mu_{ij} \epsilon_{ij} \alpha \beta}{\rho_i \rho_j} \frac{\partial W_{ij}}{\partial x_i} + \Pi_{ij} \right)
\]

where \( \Pi_{ij} \) is the artificial viscosity. The right hand side terms of Eq. (9) are the SPH approximation of pressure and the existence of viscous forces, respectively the term \( \epsilon_{ij} \) is written, based on its discrete definition as

\[
\epsilon_{ij}^{\alpha \beta} = \sum_{j=1}^{N} \frac{m_j v_{ij} \partial W_{ij}}{\rho_j \partial x_i} + \frac{\mu_{ij} \epsilon_{ij}^{\alpha \beta}}{\rho_j \partial x_i} - \frac{2}{3} \sum_{j=1}^{N} \frac{m_j v_{ij} \partial W_{ij}}{\rho_j \partial x_i}
\]

Using the Eq. (10) and by calculating strain rate and substituting it in the Eq. (9), the acceleration value of each particle is obtained and thus the speed and location of particle in each time step is found (Liu and Liu 2003).

### 1.5 Artificial Viscosity

For preventing the unphysical oscillations in the numerical results and to simulate shock waves, the artificial viscosity in SPH method is used. Many forms of artificial viscosity have been proposed. The Monaghan’s type definition for artificial viscosity is as follows

\[
D_t = \begin{cases} 
-\bar{a}_i \bar{c}_i \bar{\alpha}_i \bar{\beta}_i + \bar{a}_i \bar{\alpha}_i^2 & v_j x_j < 0 \\
0 & v_j x_j > 0 
\end{cases}
\]

where

\[
\bar{\varphi}_j = \frac{h v_j x_j}{|x_j|^2 + 0.01 h^2} \quad \bar{c}_j = \frac{c_i + c_j}{2} \\
\bar{\alpha}_j = \frac{\bar{\alpha}_i + \bar{\alpha}_j}{2} \quad \bar{v}_j = v_j - v_j \quad \bar{x}_j = x_i - x_j
\]

In Eq. (12) \( a \) and \( \beta \) are constants that all typically set approximately equal to 1. In Eq. (11), the first term associated with \( a \) produces a bulk viscosity, while the second term associated with \( \beta \) use at high Much number which is for keeping the particle interpenetration. The 0.01 \( h^2 \) is inserted to prevent numerical divergence when two particles are approaching each other and \( c \) and \( v \) are the speed of sound and particle velocity, respectively (Mehra and Chaturvedi, 2006).
RESULTS AND DISCUSSION

Phenomenon of wave propagation and its effect on structures are most important issues in the design of structures such as jetties, oil platforms and fluid reservoirs. Therefore, to become more information with this phenomenon and predict its effect on structures, it is necessary to simulate the wave propagation in the water environment. To do so, we consider a two-dimensional reservoir with \( L = 1 \) m which half filled with water \( (H = 0.5 \) m) (Fig. 1). In addition, to simulate the wave propagation and its motion in the reservoir, it is supposed that the sinusoidal wave is on the surface of water at the first moment. The wave amplitude is \( A = 0.05 \) m as follows

\[
h(x) = A \sin\left(2\pi \frac{x}{L} - \frac{\pi}{2}\right) + 0.5
\]

(13)

Where, \( h \) is the height of water and \( A \) is the wave amplitude.

To simulate the sinusoidal wave in the deep water, it is necessary to determine the appropriate number of particles first. In order to determine the number of appropriate particles to solve the governing equations using SPH method, it is necessary for the simulation to be carried out with different number of particles. Fig. 2 shows water surface profile at \( t = 0.85 \) sec and simulation with different number of particles. First, the simulation is carried out with 1250 particles. Therefore, solution domain discretion to 1250 particles and the distance between particles would be 20 mm. In the next step, simulation was carried out with 1800, 2450, 3600 and 4050 particles. As shown in Fig. 2, the simulation results of water surface profile for simulating with 3200 particles are approximately the same with more number of particles. It indicates that the output results for simulations with 3200 particles and more than 3200 particles have negligible changes. As the computational cost of simulation with 3200 particles is less than of 4050 particles, simulation with 3200 particles is chosen as the optimum number of particles.

![Fig. 1: Sinusoidal wave arrangement in a reservoir](image1)

![Fig. 2: Water surface profile, 0.86 seconds after the start of simulation with different number of particles](image2)
To validate the simulation, the output results are compared with the analytical solution of half and full oscillation time. With respect to the ratio $H/L \geq 0.5$, consequently deep water relation use for determining the oscillation time. In deep water environment, the boundaries has low effect on the wave propagation (Dean and Dalrymple, 2000). Full oscillation time of a sinusoidal wave is obtained from the following

$$T = \frac{L}{C} = \frac{L}{\sqrt{g \cdot L / 2\pi}}$$  \hspace{2cm} (14)

Where, $T$ is the oscillation time of the wave and $C$ is the wave speed (Dean and Dalrymple, 2000). This relation is used for small amplitude wave in incompressible and inviscid fluid with assumptions irrotational flow and no ambient velocity. With respect to the reservoir dimensions ($H = 0.5$ m and $L = 1$ m), the time required for half-oscillation and full-oscillation are 0.4 and 0.8 seconds, respectively. During simulation, the obtained half-oscillation and full-oscillation time of the sinusoidal wave were 0.435 and 0.86 seconds, respectively (Fig. 3). It shows that, there are 8.75% and 7.5% discrepancy in the simulation results, respectively. This amount of discrepancy is due to considering the viscous effect in the modeling whereas this effect was ignored in the analytical solution of the wave propagation.

As compared earlier, SPH method has the appropriate precision in modeling the sinusoidal wave motion. Using this method, one is able to gain more information about water motion. Gaining such information in a laboratory would be difficult and costly. For example, by tracking the reactions occur on a specific particle in simulation, useful information about the particle, e.g. direction, speed and pressure, can be obtained. In Fig. 4, direction of particles which are at $x = 0.6$ m and $y = 0.54$ m, $x = 0.7$ m and $y = 0.5$ m, $x = 0.8$ m and $y = 0.47$ m are shown during 3 seconds. Fig. 5 shows the simulation results of the sinusoidal wave motion at different times.

![Fig. 3: Water surface profile at t=0.435 sec and t=0.86 sec after the start of simulation](image_url)

![Fig. 4: Motion of different particles within 3 seconds from the start of simulation](image_url)
With respect to the validation carried out and confidence in the precision of SPH method in simulating static sinusoidal wave propagation, this method can be used in simulating wave propagation at different boundary conditions. Boundary conditions on shores are different from the ones in reservoir and the wave motion in such environments is more complex, so the mesh-less methods are needed for simulating the wave motion. Then, the wave motion on a surface with constant and variable bed slopes is simulated. Fig. 6, Fig. 7 and Fig. 8 shows the static sinusoidal wave propagation with amplitude of $A = 0.2$ m (Eq. 13) which is in an environment with $L = 1$ m and variable bed slope. Also, Fig. 9 and Fig. 10 shows the water surface profile in simulation of large amplitude wave propagation (i.e., $A = 0.2$ m and $A = 0.3$ m) in environment with horizontal bed. The results indicate that SPH method is appropriately able to model the phenomenon of wave propagation. Surface diffusion of water and a large number of displacements on it can be simulated well.
CONCLUSION

The paper is concerned with numerical simulation of static sinusoidal wave in deep water environment with complex boundary conditions. The SPH approach, based on a lagrangian description of the fluid motion, has been followed. In this context, a numerical meshless model has been developed which allows the approximate description of the propagation of static sinusoidal wave. To validate the simulation, we compared the output results of modeling with the analytical solution for wave motion in deep water environment. This proposed model shows a good agreement with analytical solutions. With respect to the validation carried out and confidence in the precision of proposed SPH method, the method are used to model the wave propagation in the complex boundary conditions.

REFERENCES


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