Three-Dimensional Elasticity Solution for Thick Laminated Cylinder with Piezoelectric Layer

M. Shakeri¹, M.R. Saviz and M.H. Yas²

Elasticity solution is presented for simply-supported, orthotropic, piezoelectric cylindrical shell with finite length under pressure and electrostatic excitation. The coupled partial differential equations (pde) are reduced to ordinary differential equations (ode) with variable coefficients by means of trigonometric function expansion in longitudinal direction. The resulting ordinary differential equations are solved by Galerkin finite element method. Results for shell with one piezoelectric layer as sensor are compared with similar ones. The numerical examples are presented for [0/90/Piezo] lamination and compared with the published ones.

Keywords: piezoelectric, composite, orthotropic, cylindrical shell, sensor

1- Introduction

Due to their intrinsic characteristics, i.e., the direct and inverse piezoelectric effects, piezoelectric materials have been widely used as distributed sensors and actuators in the area of smart structures and active structural control. On the other hand, lightweight laminated shells of revolution have found extensive application in the structure of space vehicles. Because of these reasons, shell-type smart structures containing piezoelectric layers have been under investigation in recent years. Analysis of these structures demands numerical tools for calculating the displacement and stress field in these structures [1].

Very few exact solutions of the three-dimensional field equations are available for the coupled response of piezoelectric elements to electromechanical loading. These analytical solutions are needed to assess the accuracy of the various two-dimensional shell theory formulations. The equations of linear piezoelectric materials and the governing equations to the vibrations of piezoelectric shell were presented in [2]. The governing equations for the piezoelectric shell vibrations were derived, using the Hamilton’s principle and linear piezoelectricity [3]. The circular cylindrical layer of piezoelectric shell under static axisymmetric load has been solved by means of finite element method and Love-Kirchoff assumptions [4]. In another work the coupled displacement and electrical field equations were derived for a piezoelectric cylindrical shell, based on third order shear deformation theory[5]. The equations were solved by finite element method. A discrete-layer shell theory and associated finite element model were presented for

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An exact solution for static response of laminated cylindrical panel with infinite length subjected to axisymmetric loading and simply supported boundary conditions as well as for simply supported multilayered orthotropic cylindrical panel with infinite length and piezoelectric layer as actuator subjected to electrical loading have been presented in [7], [8]. Related solutions for axisymmetric cylindrical shells under pressure and electrical load has been proposed by [9], using three dimensional elasticity solution. Elastic solution of orthotropic thick laminated cylindrical panels subjected to dynamic loading was obtained in [10]. Recently, elasticity solution for infinite thick laminated panel with piezoelectric actuator layer was presented in [11] too.

In the present work, the elastic solution of axisymmetric cross-ply laminated cylindrical shell with piezoelectric layer is presented. The shell is subjected to axisymmetric loading and electric excitation. The cylindrical shell with finite length is simply supported at both ends and three dimensional elasticity approach is used. The highly coupled partial differential equations (p.d.e.) are reduced to ordinary differential equations (o.d.e.) with variable coefficients by means of trigonometric function expansion in axial direction. The resulting ordinary differential equations are solved by Galerkin finite element method. The results for one layered piezoelectric cylindrical shell under uniform pressure are presented for different thicknesses and compared with results of [9] in which the piezoelectric material (PVDF) has been considered to be isotropic with polarization along the radial direction. This comparison shows a good agreement between the results. Then a three-layered cylindrical shell with one piezoelectric layer as actuator and two orthotropic layers under electromechanical load has been solved. The results are discussed and compared with similar ones in the literatures. Finally, the piezoelectric layer in the three-layered shell has been used to sense the pressure on the inner surface of shell.

2- Problem Formulation

The linear constitutive equations of a piezoelectric medium, ignoring temperature change are given by stress-strain, electric field and electric displacement-strain, electric field matrices

\[
\begin{align*}
\{\sigma\} &= [C]\{\varepsilon\} - [e]^T\{E\} \\
\{D\} &= [d]\{\sigma\} + [\eta]^T\{E\}
\end{align*}
\]

(1)

these equations can be written in terms of stress matrix

\[
\begin{align*}
\{\varepsilon\} &= [C]^{-1}\{\sigma\} + [d]^T\{E\} \\
\{D\} &= [d]\{\sigma\} + [\eta]^T\{E\}
\end{align*}
\]

(1-a)

where:

\[
[\eta]^T = [\eta^e] + [d]^T[e]
\]

the superscript T denotes the transpose of a matrix. [C] is the matrix of elastic coefficients, [e] and [d] are piezoelectric coupling coefficients which are related with \( e = dC \) and [\eta] denotes dielectric constants of the piezoelectric material in constant strain or stress. The components of stress matrix \{\sigma\}, strain matrix \{\varepsilon\}, electric field vector \{E\} and the electric displacement vector \{D\} are given in cylindrical coordinates.
The equations of equilibrium, in the absence of body force and the equation of charge equilibrium of electrostatics, in cylindrical coordinates under axisymmetric loading are:

\[
\sigma_{rr} + \sigma_{\theta\theta} + \tau_{r\theta} = 0
\]
\[
\tau_{r\theta} + \tau_{zr} + \tau_{t\zeta} = 0
\]
\[
D_{r} + \frac{D_{\theta}}{r} + D_{z} = 0
\]

where \(\sigma_{i,j}\) and \(\tau_{i,j}\) are the normal and shear stresses, respectively. The strain-displacement relations and the electric field-potential(\(\psi\)) relations, of the piezoelectric medium in axisymmetric condition are:

\[
\varepsilon_{r} = u_{r,r} + \varepsilon_{\theta} = \frac{u_{r}}{r}, \quad \varepsilon_{\theta} = u_{x,x} + \gamma_{\theta\zeta} = 0
\]
\[
\gamma_{r\theta} = 0, \quad \gamma_{z\theta} = u_{r,z} + u_{z,r}
\]
\[
E_{r} = -\psi_{r}, \quad E_{z} = -\psi_{z}
\]

then inserting the strain components from Eqs. (5) in Eq. (1), the stress and electrical displacement components will be obtained as following:

\[
\sigma_{r} = C_{11}u_{r,r} + C_{12}u_{r} + C_{13}u_{z,z} + e_{33}\Psi_{r}
\]
\[
\sigma_{\theta} = C_{12}u_{r,r} + C_{22}u_{r} + C_{33}u_{z,z} + e_{31}\Psi_{\theta}
\]
\[
\sigma_{z} = C_{13}u_{r,r} + C_{23}u_{r} + C_{33}u_{z,z} + e_{32}\Psi_{z}
\]
\[
\tau_{r\theta} = C_{35}(u_{z,r} + u_{r,z}) - e_{24}(-\Psi_{r}) \quad \tau_{r\theta} = 0 \quad \tau_{\theta\zeta} = 0
\]
\[
D_{r} = e_{33}u_{r,r} + e_{31}u_{r} + e_{32}u_{z,z} - \eta_{r}\Psi_{r}
\]
\[
D_{z} = e_{24}(u_{z,r} + u_{r,z}) - \eta_{z}\Psi_{z}
\]

After substitution of Eqs. (6) into Eqs. (3) and (4), the governing equations of equilibrium in terms of displacements for each layer of cylindrical shell are obtained.
The simply supported boundary conditions are taken as:

\[ z = 0, L \quad \text{at} \quad u_\theta = u_r = \sigma_r = \psi = 0 \]  
(8)

for a laminate consisting of N laminae, the continuity and equilibrium conditions to be enforced at any arbitrary interface between the (k)th and (k+1)th layers are as following.

\[
\begin{align*}
\sigma_r)_k &= \sigma_r)_{k+1} \\
\tau_r)_k &= \tau_r)_{k+1} \\
\tau_r)_{k} &= \tau_r)_{k+1} \\
\psi)_k &= \psi)_k = 0
\end{align*}
\]  
(9-a, 9-b)

The boundary conditions on the inner surface are:

\[ \tau_n = \psi = 0 \quad \text{at} \quad r = R_i \]  
(10-a)

and the boundary conditions for the piezoelectric layer considered as actuator on the outer surface

\[ \tau_r = 0, \quad \psi = V_0 \quad \text{at} \quad r = R_o \]  
(10-b)

are the electrical boundary condition for the piezoelectric sensor layer on the outer surface is

\[ D_r = 0 \quad \text{at} \quad r = R_o \]  
(10-c)

and the boundary condition on the inner surface is.

\[ \sigma_r = P_0(\theta,t) \quad \text{at} \quad r = R_o \]  
(10-d)

Following non dimensional radius, thickness, displacements and stresses recommended in[10]

are applied to equations of equilibrium and boundary conditions;

\[
(\overline{u}_r, \overline{u}_\theta, \overline{u}_z) = \frac{100E_f}{HS^3} (u_r, u_\theta, u_z) , \quad \overline{r} = \frac{r - R_m}{H} , \quad S = \frac{R_m}{H}
\]
where $R_m$ is the radius of the midplane and $H$ is the thickness of shell.

3- Solution of Governing Equations

The following series solution satisfies the simply supported boundary conditions (8)

$$u_r = \sum_{n=1}^{\infty} \phi_n(r) \sin(b_n z) \quad u_z = \sum_{n=1}^{\infty} \phi_n(r) \cos(b_n z) \quad \Psi = \sum_{n=1}^{\infty} \psi_n(r) \sin(b_n z)$$

where

$$b_n = \frac{n\pi}{L}.$$

After substituting Eqs.(11) into Eqs.(7), the partial differential equations reduce to ordinary differential equations. So the above equations can be factorized and written as follow:

$$
\begin{align*}
\begin{bmatrix}
\frac{c_{11}}{r} \frac{d^2}{dr^2} + \frac{c_{11}}{r} \frac{d}{dr} \left( \frac{c_{22}}{r^2} + c_{33} \frac{d^2}{dr^2} \right) \phi_n + \frac{d}{dr} \left[ b_n \left( \alpha_{33} + c_{55} \right) \frac{d}{dr} + b_n \frac{c_{23} - c_{13}}{r} \right] \phi_n \\
+ \left[ e_{33} \frac{d^2}{dr^2} + e_{33} - e_{31} \frac{d}{dr} + \frac{e_{33}}{r} \right] \phi_n = 0
\end{bmatrix}
\end{align*}
$$

$$
\begin{align*}
\begin{bmatrix}
\frac{c_{23} + c_{13}}{r} \frac{d}{dr} + \frac{c_{23} + c_{55}}{r} b_n \phi_n + \left[ e_{33} \frac{d^2}{dr^2} + e_{33} \frac{d}{dr} - \left( e_{33} - e_{31} \right) \phi_n \right] \phi_n + \left[ e_{32} + e_{34} \right] b_n \frac{d}{dr} + e_{32} b_n \psi_n \phi_n \\
- \left[ e_{33} \frac{d^2}{dr^2} + e_{33} \frac{d}{dr} - \left( e_{33} - e_{31} \right) \phi_n \right] \phi_n + \left[ e_{32} + e_{34} \right] b_n \frac{d}{dr} + e_{32} b_n \psi_n \phi_n \\
\end{bmatrix}
\end{align*}
$$

This system of equations are solved by considering linear shape functions $N_i$ and $N_j$ for $\phi_r, \phi_\theta, \phi_z, \psi$ and then applying the formal Galerkin finite element method to the governing ordinary differential equations, the result is written in the following finite element equilibrium equation for each non boundary elements.

$$[K]_e \{X\}_e = \{F\}_e$$

where $[K]_{8x8}$ and $\{F\}_{8x1}$ are the stiffness and force matrices. Degree of freedom for each element is:

$$\{X\}_e^T = \{\phi_{i,j}, \psi_i, \phi_{i,j}, \psi_j\}^T$$
Deriving equilibrium condition (9) in term of displacement by using eqs. (1), (5) and (6), the displacement components on the inner boundaries are obtained in term of the values at neighboring nodes as following.

\[
\begin{align*}
\phi^k_{(k+1)} &= A\phi^k_{(k+1)} + B\phi^k_{(k+1)} + C\phi^k_{(k+1)} + D\phi^k_{(k+1)} + F\phi^k_{(k+1)} + G\phi^k_{(k+1)} + H\phi^k_{(k+1)} \\
\phi^k_{(k+1)} &= A'\phi^k_{(k+1)} + B'\phi^k_{(k+1)} + C'\phi^k_{(k+1)} + D'\phi^k_{(k+1)} + F'\phi^k_{(k+1)} + G'\phi^k_{(k+1)} + H'\phi^k_{(k+1)} \\
\psi^k_{(k+1)} &= A''\phi^k_{(k+1)} + B''\phi^k_{(k+1)} + C''\phi^k_{(k+1)} + D''\phi^k_{(k+1)} + F''\phi^k_{(k+1)} + G''\phi^k_{(k+1)} + H''\phi^k_{(k+1)}
\end{align*}
\]  

(14)

Where \(A, B, \ldots, H''\) are constants which are given in [11]. Substituting Eqs. (14) into , the finite element equilibrium equations for two neighboring elements at interior \((k)\)th and \((k+1)\)th interfaces are obtained.

\[
[K]_{k+1} \{X\}_{k+1} = \{0\}, \quad [K] \{X\}_k = \{0\}
\]  

(15)

Applying the boundary conditions (10) for the first and last nodes in the inner and outer surfaces and using Eqs. (13), the displacement values for these nodes are as following:

\[
\begin{align*}
\phi_1 &= C_{10} \psi_2 + E_{10} \phi_2 + F_{10} \psi_2 \\
\phi_2 &= C_{10} \phi_2 + E_{10} \psi_2 + F_{10} \psi_2 \\
\psi_1 &= C''_{10} \phi_2 + E''_{10} \phi_2 + F''_{10} \psi_2
\end{align*}
\]  

(16-a)

\[
\begin{align*}
\phi_{(11)} &= G_{10} \phi_{(11)} + I_{10} \phi_{(11)} + J_{10} \psi_{(11)} + \left( W_{10} P_{0} + Y_{10} V_{0} \right) \\
\phi_{(11)} &= G''_{10} \phi_{(11)} + I''_{10} \phi_{(11)} + J''_{10} \psi_{(11)} + \left( W''_{10} P_{0} + Y''_{10} V_{0} \right) \\
\psi_{(11)} &= G_{10} \phi_{(11)} + I_{10} \phi_{(11)} + J_{10} \psi_{(11)} + \left( W_{10} P_{0} + Y_{10} V_{0} \right)
\end{align*}
\]  

(16-b)

where \(C_{10}, E_{10}, \ldots, Y''_{10}\) are constants which are given in [11]. By substituting Eqs. (16-a) and (16-b) into (13), the finite element equilibrium equations for the first and last elements become:

\[
[K]_{1} \{X\}_1 = \{F\}_1, \quad [K]_{k+1} \{X\}_{k+1} = \{0\}
\]  

(17)

Assembling Eqs. (13), (15) and (17), the general finite element equilibrium equation is obtained as follows:

\[
[K] \{X\} = \{F\}
\]  

(18)

Once the finite element equilibrium is established, the Gauss-Sidel numerical method has been used to solve the equations.

4- Numerical Results and Discussion
The results for a one layer piezoelectric cylindrical shell under uniform pressure on the outer surface are obtained across the thickness and compared with the exact results of [9] which has considered the PVDF to be isotropic with polarization along the radial direction. The material properties of piezoelectric layer are

\[
E_r = E_\theta = E_z = 2.0 \times 10^9 \text{ Pa} \quad \nu_{z\nu} = \nu_{\nu\theta} = \nu_{\theta\theta} = 0.33
\]

\[
\eta = \begin{bmatrix}
0.1062 & 0 & 0 \\
0 & 0.1062 & 0 \\
0 & 0 & 0.1062
\end{bmatrix} \times 10^{-9} \text{ F.m}^{-1}
\]

\[
d_1 = -30 \times 10^{-12} C.N^{-1} \quad d_2 = 23 \times 10^{-12} C.N^{-1} \quad d_3 = 3 \times 10^{-12} C.N^{-1} \quad d_5 = d_6 = 0
\]

where C is colon. Diagrams of negative non-dimensional radial and axial components of stress displacement, electric potential and transverse shear stress for different shell thicknesses (s=4,6,20) are shown in figures (1-6). The distribution of non-dimensional radial stress \( \sigma_r \) at \( Z = 0.5L \) is given in Fig. 1, \( \sigma_r \) is maximum(=1) on the loaded surface for all values of S. In Fig.2, the distribution of longitudinal stress is shown, for thin shell(S=20), \( \sigma_z \) is almost linear, resulting in a membrane state. It is observed from Fig. 3 that at midspan(\( Z = L/2 \)), the radial displacement, \( \bar{u}_r \) is linear for S>6 and uniform for S=20. However \( \bar{u}_r \) becomes nonuniform as the shell becomes thicker. \( \bar{u}_r \) of the outer and inner surfaces differ by more than 5% for S=4. In Fig. 4 the non-dimensional axial displacement, \( \bar{u}_z \) at \( Z = 0 \) is presented, it has uniform distribution for thin shells and varies nonlinearly for S=4. Fig. 5 shows the negative electric potential for piezo-shell, it is almost zero for thick shell and parabolic shape for thinner ones. The transverse shear stress is presented in the Fig. 6, it has almost a parabolic distribution, the maximum value of \( \tau_{zr} \) shifts from the midsurface for \( s \leq 6 \). The results in the middle of thickness(\( \bar{r} = 0 \) ) are shown in table(1) and compared with analytical solutions of [9] in which the one-layer piezoelectric material(PVDF) is isotropic with polarization along the radial direction. This comparison shows a good agreement between the results.

A three-layered cylindrical shell with one piezoelectric layer as actuator and two orthotropic layers has been also solved. The three-layered cross-ply cylindrical shell with sequence lay-up (0/90/P) composed of graphite-epoxy and piezoelectric material, is considered completely bonded at layers. The piezoelectric material considered here is elastically orthotropic. The material properties of piezoelectric lamina are

\[
C = \begin{bmatrix}
11.3 & 7.43 & 7.43 & 0 \\
7.43 & 13.9 & 7.78 & 0 \\
7.43 & 7.78 & 13.9 & 0 \\
0 & 0 & 0 & 2.56
\end{bmatrix} \times 10^{10} \text{ Pa}
\]

\[
\eta = \begin{bmatrix}
5.62 & 0 & 0 \\
0 & 6.46 & 0 \\
0 & 0 & 6.46
\end{bmatrix} \times 10^{-9} \text{ F.m}^{-1}
\]

The material properties of the graphite-epoxy composite are.

\[
E_z = 76.8 \times 10^9 \text{ Pa} \quad E_r = E_\theta = 5.5 \times 10^9 \text{ Pa} \quad G_{z\theta} = G_{zr} = 2.07 \times 10^9 \text{ Pa} \quad G_{\theta r} = 1.4 \times 10^9 \text{ Pa}
\]

\[
\nu_{z\nu} = \nu_{\nu\theta} = 0.34 \quad \nu_{\theta\theta} = 0.37
\]
The forcing function is chosen as:

\[ P_0(\theta, z) = P_0 = \text{const}. \]

In all the calculations, \( R_m \) is chosen to be 0.4m and \( L/R_m = 1 \). The constant pressure on the outer surface is \( P_0 = -1 \text{Pa} \) and \( S \) is 10 corresponding to thick shells. Only the inverse effect of piezoelectric is considered (the piezoelectric layer is served as an actuator). In Fourier series, a few terms can give satisfactory results for the case studied (\( S=10 \)). Therefore, 20 terms are used throughout the following analyses for axial direction. The results are discussed and compared with similar ones.

Fig. 7 shows the electric potential distribution corresponding to different applied voltages. The distribution of the electric potential depends on the applied voltage. When the applied voltage is much stronger than the induced electric potential by strain, the electric potential in the actuator tends to change linearly, which is different from the studies of [8] and [9], and similar to [10] which only considered the piezoelectricity analysis for a circular cylindrical shell. Fig. 8 illustrate the radial stress distributions across the thickness with different voltages. The boundary and inter laminar conditions are satisfied in this figure. The stresses depend strongly on the radial coordinate and its absolute value increases with voltage. In Fig. 9, the distributions of the radial displacement due to the applied voltage are presented. The figure shows that radial displacement is continuous and vary nonlinearly across the thickness. As it is expected in Figs. 10 and 11 the value of \( \sigma_\theta \) is zero across the thickness in the \( 0^\circ \) layer and \( \sigma_z \) is zero in the \( 90^\circ \) layer. \( \sigma_\theta \) and \( \sigma_z \) both tend to become nonlinear by applying voltage. Fig.12 shows the through-thickness distribution of transverse shear stress \( \tau_{rz} \). In each layer, the shear stress distribution is very close to a parabola form. According to these figures the electrical effect becomes much stronger than the mechanical loading with increasing the applied voltage.

Finally in the three-layered cylindrical shell, the piezoelectric material is considered as sensor. Figs.13 and 14 illustrate the non dimensional normal and shear stresses distributions across the thickness with \( S \). The boundary and inter laminar conditions are satisfied. The stresses depend strongly on the radial coordinate. It can be inferred from Fig.13 that the radial stress is unit at the inner surface. Fig.14 shows the through-thickness distribution of transverse shear stress \( \tau_{rz} \), which in each layer is very close to a parabola form.

In Fig.15, the distribution of the electric potential in the piezoelectric layer due to different \( S \) are presented. When \( S \) is 10 for almost thin shell, the electric potential in the sensor changes almost linearly. The electric potential in the outer layer grows with thickness. Fig.16 illustrates the variations of sensed voltage of outer surface versus the internal pressure for different \( S \) ratios. The displacement in the radial direction due to different \( S \) are presented in Fig. (17), This Figure shows that radial displacement does not vary linearly across the thickness specially for thick shell(S=4). Fig. (18) shows the circumferential normal stress, as it is expected \( \sigma_\theta \) is zero in the \( 0^\circ \) layer and maximum in outer surface. The inter-laminar continuity of circumferential stress is not needed.

From the above studies, we can conclude that the linear variation assumption of the electric potential in the piezoelectric layer must be treated with caution in approximate solutions and the three-dimensional elasticity methods for the piezoelectric response analysis are the preferred ones.

6- CONCLUSION
A three-dimensional elasticity solution for an orthotropic cylindrical shell with piezoelectric layer has been studied. The shell is simply supported at both ends and has finite length. The present study has used the Fourier series expansion method for the mechanical displacement and the electric potential distribution.

The direct piezoelectric effect has been used for one layered piezoelectric(PVDF) shell and the effect of radius-to-thickness ratio results are studied. The solution is being extended to hybrid laminated shells with surface bounded piezoelectric layer.

The inverse and direct piezoelectric effect of the piezoelectric layer in (0/90/ PVDF) composition subjected to outer applied voltage and inner pressure are investigated in detail, respectively. It has been shown that the distributions of the mechanical displacements and electric potential of piezoelectric response are very complicated and cannot be treated as pure elastic structures or piezoelectric structures. Therefore, three-dimensional analysis of piezoelectric behavior of structures is recommended even for thin laminated structures. Since a comprehensive and exact study of active piezoelectric structures is still unavailable, the present work provides a three-dimensional insight to the mechanical and electric behaviors of this type of smart structure. The results, presented in this paper are useful for assessing the approximate analysis of piezoelectric structures.

Table 1 The results of one piezolayer in the middle of thickness( $r^* = 0$ )

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_r$</th>
<th>$\sigma_\theta$</th>
<th>$\sigma_z$</th>
<th>$u_r$</th>
<th>$u_\theta$</th>
<th>$u_z$</th>
<th>$\tau_{r\theta}$</th>
<th>$\Psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref[9]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S=4</td>
<td>0.602</td>
<td>1.122</td>
<td>-0.034</td>
<td>1.085</td>
<td>0.857</td>
<td>0.798</td>
<td>-0.059</td>
<td></td>
</tr>
<tr>
<td>S=6</td>
<td>0.578</td>
<td>1.095</td>
<td>-0.018</td>
<td>1.064</td>
<td>0.861</td>
<td>1.111</td>
<td>-0.243</td>
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</tr>
<tr>
<td>S=20</td>
<td>0.515</td>
<td>1.048</td>
<td>-0.004</td>
<td>1.035</td>
<td>0.801</td>
<td>2.02</td>
<td>-0.714</td>
<td></td>
</tr>
<tr>
<td>Present</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S=4</td>
<td>0.609</td>
<td>1.129</td>
<td>-0.035</td>
<td>1.087</td>
<td>0.900</td>
<td>0.788</td>
<td>-0.051</td>
<td></td>
</tr>
<tr>
<td>S=6</td>
<td>0.581</td>
<td>1.101</td>
<td>-0.020</td>
<td>1.061</td>
<td>0.880</td>
<td>1.109</td>
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<tr>
<td>S=20</td>
<td>0.519</td>
<td>1.050</td>
<td>-0.008</td>
<td>1.040</td>
<td>0.819</td>
<td>2.01</td>
<td>-0.710</td>
<td></td>
</tr>
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Figure 1 Distribution of $-\sigma_r$ across $\tilde{r}$, $Z = 0.5L$, $Z = 0$.

Figure 2 Distribution of $-\sigma_z$ across $\tilde{r}$.
Figure 3 Distribution of $-\mathbf{u}_r$ across $\bar{r}$, $Z = 0.5L$

Figure 4 Distribution of $-\mathbf{u}_z$ across $\bar{r}$, $Z = 0$.

Figure 5 Distribution of Electric Potential across $\bar{r}$, $Z = 0.5L$

Figure 6 Distribution of $-\mathbf{v}_r$ across $\bar{r}$, $Z = 0$.

Figure 7 Distribution of $\psi$ across the $\bar{r}$ with $V_0(v)$ ($0/90/P S=10, Z=L/2$)

Figure 8 Distribution of $\sigma_r$ across the $\bar{r}$ with $V_0(v)$ ($0/90/P S=10, Z=L/2$)
Figure 9 Distribution of $\sigma_z$ across the $r$ with $V_0(v)$
(0/90/P S=10 , Z=L/2)

Figure 10 Distribution of $\sigma_z$ across the $r$ with $V_0(v)$
(0/90/P S=10 , Z=L/2)

Figure 11 Distribution of $\sigma_z$ across the $r$ with $V_0(v)$
(0/90/P S=10 , Z=0)

Figure 12 Distribution of $\sigma_z$ across the $r$ with $V_0(v)$
(0/90/P S=10 , Z=0)

Figure 13 Distribution of $\sigma_z$ across the $r$ with $S$
(0/90/sensor, Z=L/2)

Figure 14 Distribution of $\sigma_z$ across the $r$ with $S$
(0/90/sensor, Z=0)
Figure 15 Distribution of $\psi$ across $\bar{r}$
(0/90/sensor, Z=L/2) (0/90/P)

Figure 16 The variations of voltage with pressure
(0/90/PS=10, Z=L/2)

Figure 17 Distribution of $\mu$ across the $\bar{r}$ with $S$
(0/90/ sensor, Z=L/2)

Figure 18 Distribution of $\sigma_\theta$ across the $\bar{r}$ with $S$
(0/90/ sensor, Z=L/2)

References


چکیده:

در این مقاله حل الاستیسیته پوسته استوانه ای چند لایه با یک لاشه پیزوالکترونیک به عنوان عملکرشکر مورد توجه قرار گرفته است. پوسته تحت بار استاتیکی و ولتاژ الکتریکی پیدا و حل الاستیسیته استفاده شده است. پوسته با طول محدود در دو انتهای پیوسته گاه‌های ساده می‌باشد. معادلات کویره مشتق‌های جزئی، با پیلی توابع عمود بر هن مثلثی (سری فوریه) به معادلات با مشتق‌های عمومی تبدیل شده اند. این معادلات با مشتق‌های عمومی و ضرایب متغیر با روش الگوریتم محدود خطي «گلرکین» در جهت شما حل شده اند. در اثره نتایج، آندازه پوسته استوانه ای چند لایه پیزوالکترونیک تحت فشار یک‌چاپ‌کن‌های خارجی مورد بررسی قرار گرفته است و جواب‌های بدست آمده با نتایج روش تحلیلی موجود مقایسه شده اند. پس از حصول اطمینان از عملکرد برنامه کامپیوتری، پوسته سه لایه با یک لاشه خارجی پیزوالکترونیک بعنوان عملگر تحت بار الکتریکی حل شده است. در انتها لاشه پیزوالکترونیک بعنوان حسگر در نظر گرفته شده و پوسته تحت فشار داخلی قرار گرفته است و نتایج تا حد امکان با مقالات جاب شده مقایسه شده است.