Optimization of Architecture and Workspace of a Planar 3-DOF Parallel Manipulator

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This paper presents a new architecture design for planar parallel manipulators, which helps greatly to increase their maneuverability and enlarging their workspace. In many cases it is very difficult to express interlace of links as constraint and it is an obstacle, which doesn’t let the computations for kinematics, or workspace is the same as in practical case. This new architectural design makes this wish comes true for computation and practical cases. To show how it works, a 3-DOF planar parallel manipulator is selected. In the mechanism, the prismatic actuators are fixed to the base which leads to a reduction of the moving links inertia, hence makes it attractive, particularly when high speeds are required and electric actuation is considered. First, the mechanism is introduced; then a new design is introduced based on the geometry of the manipulator to increase flexibility and accessibility. Finally a workspace analysis is performed. Optimization of workspace is considered with 3 different methods: Monte Carlo, Exact Method (introduced for the first time) and Global Condition Index. Then the results are compared.

Keywords: Architecture, Workspace, Optimization, 3-DOF Planar Parallel Manipulators

1 - Introduction

Parallel mechanisms are being more widely used in a diversity of advanced applications, such as flight simulators or machine tools manipulating. To a great extent, most attention on these devices centered on the 6-DOF parallel mechanism which originally proposed by Gough (Stewart) platform [1]. This platform has been shown in many works, to have very good performance, good balance between workspace, dexterity and stiffness. Alternative 6-DOF architectures have also been studied, for example by Merlet and Gosselin [2] and Nahon et al. [3], and Sima’an and Glozman [4] have demonstrated good performance. However, many applications do not require a full six degree of freedom moving platform. Reduced DOF architectures have the advantages of needing fewer actuators, thereby reducing the total cost of the device. Meanwhile some researchers tend to optimize structures in regards to application and make some changes into architecture of mechanisms to improve their abilities and performances.

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For example Lee and Shah [5] worked on 3-DOF parallel manipulator kinematics. Pouliot et al. [6] studies certain 3-DOF parallel architectures for low-cost flight simulator motion platform applications. Carretero, Podhorodeski and Nahon with Gosselin [7] and without him [8], optimized a 3-DOF parallel mechanism architecture. The physical model of solution space for 3-DOF planar parallel manipulator has been used by Liu, Wang and Gao [9] to find a optimal design for workspace volume. Tsai and Joshi [10], [11] and Kim and Tsai [12] have compared several 3-DOF parallel manipulators. Their performance was found to be comparable to that of the Gough (Stewart) platform.

It is known that parallel manipulators have desirable stiffness and accuracy but they don’t have proper workspace. With this new design parallel planar manipulators maneuverability improves and many kinds of answers of optimization parameters, which were not feasible before to be built before, can be used. Thus we try to enlarge the workspace with changes in positions and angles of the fixed prismatic actuators and lengths of manipulator's links to make it more suitable for high-performance robotic applications such as positioning and orientation device.

In this paper, architecture of planar 3-DOF parallel manipulator is based on existing design by Gosselin and his colleagues [13]. The prismatic actuators are fixed to the bases. Note that, each of the chain connecting the base of the platform is composed of fixed prismatic actuator and two moving passive revolute joints. This architecture is in fact the planar counterpart of the spatial architecture proposed by Gosselin [2].

At first the positioning and velocity equations of the planar parallel manipulator are given in sections 3 and 4.

In section 5 a new design is applied to the manipulator base, which gives more flexibility and maneuverability to the manipulator. The concept can be used for other planar parallel manipulators. To show the applicability of the concept an example of this new design is given.

Workspace of the manipulator is discussed in section 6. Three different methods are introduced to find the workspace. The results are compared.

In section 7, with some defined parameters and using workspace description, maximum workspace has been derived.

Numerical results are presented in section 8 and conclusion is given in section 9.

The workspace analysis, optimized architecture and the approach that is presented in this paper can be a great help in the design, trajectory planning and control of such devices. Moreover, it can also be used in other applications such as vehicle motion simulators, camera positioning devices and other devices, which require high-precision or high-bandwidth control.

2 - Architecture of the Mechanism

The architecture of the proposed planar 3-DOF parallel manipulator is illustrated schematically in Figure 1. A reference frame $O_{xy}$ is fixed to the base and a moving reference frame $C_{x'y'}$ is attached to the platform. The three prismatic actuators are fixed to the base ($A_i$). Each actuator is defined by angle $\alpha_i$ (fixed angle). Moreover, vector $e_i$ is defined as a fixed unit vector in the direction of the axis of the $i$th prismatic actuator. The moving part of the $i$th actuator is then attached to a second moving link of length $l_i$ by an punctuated revolute joint located at point $B_i$. The joint coordinate associated with the $i$th actuator is defined as $\rho_i$, the distance between points $A_i$ and $B_i$. Finally, the moving link of length $l_i$ is connected to
the platform by a revolute joint located at point $C_i$. The Cartesian coordinate vector of the manipulator is given by the position and orientation of the platform and can be written as

$$\mathbf{P} = [x_c \ y_c \ \phi]^T$$

Where $(x_c, y_c)$ are the position coordinates of point $C$ expressed in the fixed frame, and $\phi$ is the angle between the $x$ axis of the fixed frame and axis $x'$ of the moving frame, as indicated on the Figure 1. The actuated joint coordinate vector is defined as

$$\mathbf{\rho}=[\rho_1 \ \rho_2 \ \rho_3]^T$$

Where each of the components are defined above. Finally angle $\beta_i$ is defined as the angle between the $x$ axis of the fixed frame and the intermediate link of length $l_i$.

By controlling the three actuators, it is possible to control the Cartesian coordinate (position and orientation) of the platform, as expected. Moreover, as explained in the introduction, the architecture of each of kinematic chain connecting the base to the platform is of the $PRR$ type, where $P$ and $R$ respectively denote prismatic and revolute joints.

However, if electrical actuators are used (for instance with a ball screw system), transversal forces are considerably less critical and the design proposed here is of great interest since it reduces the inertia of the moving parts.

3 - Kinematic Analysis

Kinematic issues to be addressed, when considering new manipulators, are the solutions of the inverse and direct kinematic problems. Following the notation defined above, the geometry of the kinematic chains connecting the base to the platform allows one to write the coordinates of point $C_i$, i.e., $(x_{c_i}, y_{c_i})$, as

$$x_{c_i} = x_{ai} + \rho_i \cos \alpha_i + l_i \cos \beta_i \quad i = 1,2,3 \quad (3)$$

$$y_{c_i} = y_{ai} + \rho_i \sin \alpha_i + l_i \sin \beta_i \quad i = 1,2,3 \quad (4)$$

Moreover, these coordinates can also be written as functions of the Cartesian coordinates of the platform, i.e.,

$$x_{c_i} = x_c + x'c_i \cos \phi - y'c_i \sin \phi \quad i = 1,2,3 \quad (5)$$

$$y_{c_i} = y_c + y'c_i \cos \phi + x'c_i \sin \phi \quad i = 1,2,3 \quad (6)$$

Where $x'c_i$ and $y'c_i$ are constant parameters describing the geometry of the platform, i.e., they are the coordinates of point $C_i$ in the coordinate frame attached to the platform.

Equations (5) and (6) can then be substituted into eqs. (3) and (4), respectively, which leads to two equations from which angle $\beta_i$ is easily eliminated. This leads to a quadratic equation in $\rho_i$ which can be solved to lead to:
\[ \rho_i = M_i \pm N_i, \quad i = 1, 2, 3 \]  
(7)

Where

\[ M_i = (x_{c_i} - x_d) \cos \alpha_i + (y_{c_i} - y_d) \sin \alpha_i \]  
(8)

\[ N_i = \sqrt{l_i^2 - S_i^2} \]  
(9)

\[ S_i = (x_{c_i} - x_d) \sin \alpha_i - (y_{c_i} - y_d) \cos \alpha_i \]  
(10)

And \( x_{c_i} \) and \( y_{c_i} \) are defined in Eqs. (5) and (6). Eq. (7) provides a closed-form solution to the inverse kinematics problem. Indeed, for a given position and orientation of the platform, the joint coordinates can be computed using this equation. Since two solutions are found for each of the joint coordinates, it is clear that the inverse kinematics problem of this manipulator has 8 different solutions.

The second important problem to address is the direct kinematics problem, which is finding the position and orientation of the platform for given values of the joint coordinates. This problem has been solved for the existing planar 3-DOF parallel manipulators [14-16]. It is shown to lead to a polynomial of degree 6 which can have up to 6 real solutions.

4 - Velocity equation

Eq. (7) can be differentiated with respect to time in order to obtain the velocity equations. This leads to an equation of the form

\[ A\dot{\rho} + B\dot{\rho} = 0 \]  
(11)

Where \( \dot{\rho} \) is the vector of joint velocities defined as

\[ \dot{\rho} = [\dot{x}_c, \dot{y}_c, \dot{\phi}]^T \]  
(12)

And \( \dot{\rho} \) is the vector of joint velocities defined as

\[ \dot{\rho} = [\dot{\rho}_1, \dot{\rho}_2, \dot{\rho}_3]^T \]  
(13)

Matrices \( A \) and \( B \) are the \( 3 \times 3 \) Jacobian matrices of the manipulator and can be expressed as

\[ A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \]  
(14)

\[ B = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \]  
(15)

Where one has, for \( i = 1, 2, 3 \)
\[ a_i = -\rho_i \cos \alpha_i x_{Ai} + xc + x'c_i \cos \phi - y'c_i \sin \phi \]  
(16)

\[ b_i = -y_{Ai} + yc + y'c_i \cos \phi + x'c_i \sin \phi - \rho_i \sin \alpha_i \]  
(17)

\[ c_i = -x'c_i y_{Ai} \cos \phi + x'c_i yc \cos \phi + \rho_i y'c_i \cos (\phi - \alpha_i) + x_{Ai} y'c_i \cos \phi - xc y'c_i \cos \phi + x_{Ai} x'c_i \sin \phi - xc x'c_i \sin \phi + y_{Ai} y'c_i \sin \phi \]  
(18)

\[ d_i = \rho_i + x_{Ai} \cos \alpha_i - xc \cos \alpha_i - x'c_i \cos (\phi + \alpha_i) + y'c_i \sin (\phi - \alpha_i) + y_{Ai} \sin \alpha_i - yc \sin \alpha_i \]  
(19)

Eq. (11) relates the Cartesian velocities of the mechanism to its joint velocities. This relationship is also very important for the description of the different types of singularities of the manipulator. The conditioning of the Jacobian matrices can also be used to characterize the dexterity of the mechanism.

5 - A New Design Approach

Numerical computations have shown that most of the time maximum workspace can be obtained when \( \theta_i \)s are very close to each other. So practically, it is not feasible to build such an arrangement, because there is not enough angular separation to hold actuators and links. Another important matter is that, computations specify the workspace but cannot give certainty that links never cross each other when all links are in one plane. An approach is proposed to solve these drawbacks.

To overcome these drawbacks, we change the mechanism to omit the angular separation and to assure that end-effector can go to all points found by computation routine. In this design we placed every fixed prismatic actuator and its links in one plane as shown in Figure 2, thus now without any problem the end-effector can go to every point that is predicted as a workspace by computation. It should be noted that this design may cause an increase in mechanism weight and volume but is a practical way for all kind of planar parallel manipulators.

6 - Manipulator Workspace

To find the manipulator workspace, the entire possible workspace is introduced by an square that surrounds the circle that passes through three points which are the base points \( (A_i) \) of prismatic links as shown in Figure 2.

There are three numerical solutions methods, Monte Carlo, Exact Method and Global Condition Index.

In the following, each method is outlined.

6 - 1 - Monte Carlo Method

Step1: Consider the entire possible workspace of the manipulator (Figure 3).
Step2: A large number of points, \( n_{total} \), are randomly selected with in the unit square.
Step3: Each point is tested to determine if it falls within manipulator workspace. This is accomplished by solving the inverse kinematics problem for each leg as described by equations (3) to (10). If all the \( \rho_i \) is real and is in wanted range of \( \rho_i \), then the point is within the workspace.
Step 4: The number of points that fall within the workspace, \( n_w \).

Step 5: The workspace area is estimated by the ratio of the points that fall in the workspace to the total number of points selected (entire feasible workspace is a square with unit area).

\[
W = \frac{n_w}{n_{total}} \quad (20)
\]

### 6 - 2 - Exact Method

Step 1: Consider the entire possible workspace manipulator (Figure 2).

Step 2: Entire possible workspace (unit circle) must be included by specified points that are distributed in square regularly.

Step 3-5: The same as used for Monte Carlo Method.

### 6 - 3 - Manipulator Global Condition Index

A Global Condition Index \( \eta \), that considers the condition number of the Jacobian over the entire workspace is defined for the manipulator as [17]:

\[
\eta = \int_\lambda \frac{1}{\lambda} \, dw \quad (21)
\]

Where \( \lambda \) is the condition number of the Jacobian at a given position in the workspace and \( w \) is the manipulator workspace.

The condition number of the Jacobian matrix, \( J \), is defined as:

\[
\lambda = \| J \| \cdot \| J^{-1} \| \quad (22)
\]

Where \( \| \| \) is the 2 norm of the matrix.

For parallel manipulator, the Jacobian matrix, maps the velocity of the moving platform in Cartesian space to the actuated joint velocities.

In this case, the Jacobian matrix is calculated from velocity Eq. (11).

\[
J = -A^*B^{-1} \quad (23)
\]

The complexity of generating a closed-form solution to equation (21) compels the use of a numerical solution technique. Here, Exact Method is employed and is outlined as follows:

Step 1-3: Same as steps 1-3 for Exact Method.

Step 4: The condition index sum \( S \), which is the sum of the reciprocal of the condition number of each point that falls within the workspace.

Step 5: The global condition \( \eta \), is determined by the condition index sum, dividing by the total number of selected points.

\[
\eta = \frac{S}{n_{total}} \quad (24)
\]

### 7 - Workspace Optimization

The optimization of parallel manipulator workspace volume is dependent upon determination of the workspace of a manipulator for a given set of variables. To have symmetrical solution, let \( l_1 = l_2 = l_3 = l \). The objective of total workspace optimization is to determine the values of the manipulator design variables that result in the largest total manipulator workspace. The design variable considered, are:
• The link lengths, \( l_i, \rho_i \);
• The relative angular position of the prismatic bases \( \alpha_i \);
• Prismatic actuator base locations on circle \( \theta_i \);
• In order to bound the solution and to ensure a practical realization, the objective function is subject to the following constraints:
  • All links length must be positive;
  • Length \( \rho_i \) is limited \( \rho_{\text{min}} < \rho_i < \rho_{\text{max}} \)

7 - 1 - Monte Carlo and Exact Method Optimization

Given this problem formulation, the optimization is computed using the Matlab optimization toolbox.

7 - 2 - Global Condition Index Optimization

The objective of the well-conditioned workspace optimization is to determine the values of the manipulator design variables that result in the best Global Condition Index. The same set of design variables that were used during the total workspace optimization is also used for the well-conditioned workspace optimization. The objective function is also subject to the same constraints as were used during the total workspace optimization. The well-conditioned workspace optimization is computed by using Matlab optimization toolbox.

8 - Numerical Results

Computation results have shown in Table 1 and 2.

9 - Conclusion

A new approach is proposed to overcome link interlace possibility and makes the computations for kinematics or workspace be the same as what is in practical case and eliminate need for angular separation where is applicable for a vast variety of planar parallel manipulators.

The design of 3-DOF transnational parallel manipulator is optimized for both total workspace and Global Conditioning Index. As shown in Table 1 and Table 2 workspaces (\( w \)) that are obtained from three methods are closed together but the best answers are achieved from Exact Method and Monte Carlo. It must be mentioned that in defined function for computation of workspace, dexterity has been considered because every single point in our entire workspace is tested for several positions.

For this reason Monte Carlo and Exact Method have acceptable results and comparable with Global Conditioning Index and even better than it, and workspaces that obtained from first two methods are not ill conditioned. Although Monte Carlo is a well known method as a base function of optimization but it should be mentioned that Exact Method is also a proper way and comparable with Monte Carlo, that is introduced by authors and applied in this paper.

Also results have shown that if this kind of manipulator is considered to be designed it is efficient to choose \( l_i \)(link length) approximately equal to:

\[
l_i \approx \rho_{\text{max}} - \rho_{\text{min}}
\]  

(25)
References


Table 1: Optimal design values for $\alpha_i=120^0$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Monte Carlo</th>
<th>Exact Method</th>
<th>Global Condition index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>260.4°</td>
<td>253.87°</td>
<td>228.83°</td>
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<tr>
<td>$\alpha_2$</td>
<td>3.92°</td>
<td>352.06°</td>
<td>347.25°</td>
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<tr>
<td>$\alpha_3$</td>
<td>98.83°</td>
<td>105.66°</td>
<td>131.68°</td>
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<tr>
<td>$l_i$</td>
<td>1.1139m</td>
<td>1.1197m</td>
<td>1.1109m</td>
</tr>
<tr>
<td>$\rho_{min}$</td>
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<td>0.1m</td>
<td>0.1m</td>
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<td>$\rho_{max}$</td>
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<td>1.1m</td>
<td>1.1m</td>
</tr>
<tr>
<td>$W$</td>
<td>0.8087</td>
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<td>$n_{total}$</td>
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Table 2: Optimal design values when $\alpha_i$, $\theta_i$ and $l_i$ are variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Monte Carlo</th>
<th>Exact Method</th>
<th>Global Condition index</th>
</tr>
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<td>$\theta_1$</td>
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<td>159.47°</td>
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<td>$l_i$</td>
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<td>0.1m</td>
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<td>$n_{total}$</td>
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Figure 1 Architecture of planar 3-DOF parallel manipulator

Figure 2 An approach to give certainty of movement.
Figure 3 Entire possible workspace
چکیده:

بهینه‌سازی سازه ربات‌ها و انتخاب مناسب‌ترین ابعاد برای عضوها در ربات‌ها و به ویژه ربات‌های موازی از اهمیت بسیار زیادی برخوردار است. ربات‌های موازی در کنار سرعت، قدرت و دقت بالا خود، فضای کاری که از گسترده‌ای را پوشش نمی‌دهند و به همین دلیل بهینه‌سازی آنها از اهمیت زیادی برخوردار است. در این تحقیق ابتدا طرح جدیدی به صورت عمومی برای ربات‌های موازی صفحه‌ای ارائه می‌شود. که این طرح از تفاوت عضوها با یکدیگر جلوگیری کرده و باعث همگونی محاسبات تحلیلی و جواب‌های تجزیه‌ای می‌شود. سپس بر روی یک نوع از این نوع ربات‌ها اعمال می‌شود و پس از بهینه‌سازی نوع سازه، با هدف بهینه‌سازی فضای کاری ربات به کمک سه روش مختلف بهینه‌سازه و نتایج با هم مقایسه می‌شوند.