The b value estimation by the moment method for Earthquakes of Hormozgan province, Southern Iran *

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Abstract
The earthquake size distribution is generally considered to obey the Gutenberg-Richter law. In this study we have introduced the concept of the b value spectrum based on the moment method to investigate the deviation of the actual magnitude distribution of earthquakes in Hormozgan province, Southern Iran from this law. This enables us to describe characteristic features of the magnitude frequency distribution of earthquakes. We found also a simple relation between the η value and the b value spectrum. Analysis using this scheme showed that the actual size distributions of earthquakes have large variations from case to case and sometimes deviate considerably from the widely assumed the Gutenberg-Richter formula.

Keywords: Catalogue, Body waves, b value spectrum, η value spectrum, Moment method, Hormozgan province

1- Introduction
The earthquake size distribution is described by Gutenberg-Richter law

\[ \log N(m) = a - bm \]  

(1)

Where, \( N \) is the frequency of earthquakes that have magnitude equal or exceeding \( m \). This formula shows a power law with respect to the energy (E) distribution, since an empirical relation \( \approx \log E \) holds; \( a \) and \( b \) are parameters and this \( b \) represents an important feature of the earthquake populations. Recent studies have shown, however, that actual magnitude frequency distributions do not always obey this law. Sometimes they deviate significantly from this law and some modified formulae have been postulated (e.g., Tsuboi 1952; Utsu 1978; Bender 1983; Seino et al. 1989; Beauval et al. 2003, Edoardo et al. 2003, Lombardi 2003, Wyss et al. 2004, Wyss et al. 2006). However, complicated models are not statistically desirable, although they may fit better to certain data sets. Also, the deviation of actual magnitude frequency distributions from the Gutenberg-Richter law has been investigated (e.g., Utsu 1978; Smith 1981; Okuda et al. 1992). Utsu (1978) presented the measure η that describes the situation of departure of magnitude frequency distributions from the...
Gutenberg-Richter formula. However, physical meaning of this measure is not well understood yet. In this paper, we introduce the concept of the b value spectrum by the moment methods (Utsu 1965) to describe the characteristic feature of magnitude frequency distributions of earthquakes. This enables us detailed and quantitative analysis of magnitude frequency distributions of earthquakes. Also the relation between this b value spectrum and the $\eta$ value is discussed.

2. Method

The b value estimation by the moment method (Utsu 1965) is as follows. Let the magnitudes of observed earthquakes be $M_1, M_2, \ldots, M_N$ in decreasing order, where $N$ denotes the number of events. If the magnitude frequency distributions obey the Gutenberg-Richter law, we get the following approximate relation for the moment of order $\gamma$:

$$\sum_{i=1}^{N} (M_i - M_0)^\gamma \approx N \Gamma(\gamma + 1) / (b \ln 10)^\gamma$$

(2)

Here, the integral is replaced by a summation. Utsu (1965) treated only the case for $\gamma = 1$, and he derived

$$b = \ln e / \frac{1}{N} \sum_{i=1}^{N} (M_i - M_0)$$

(3)

This estimation is equivalent to the maximum-likelihood method (Aki, 1965). For general $\gamma$, we obtain

$$b_\gamma = \Gamma(\gamma + 1) \frac{1}{\ln 10} J_\gamma$$

(4)

$$J_\gamma = \frac{1}{N} \sum_{i=1}^{N} (M_i - M_0)^\gamma$$

(5)

For $\gamma$, we can consider fractional values as well as integer values. If data are complete and magnitude frequency distributions of earthquakes obey exactly the Gutenberg-Richter law, then we can get the same estimates of b values which are independent of $\gamma$. However, these requirements are not always fulfilled in actual situations. Besides, we need not restrict $\gamma$ be 1, although that is the conventional case. Conversely, we can investigate the dependency of estimated b values on $\gamma$, which will give us valuable information on the feature of magnitude frequency distributions of earthquakes. Utsu (1978) has postulated the following measure

$$\eta = \frac{< X^2 >}{< X >^2}$$

(6)

Where $X = M - M_0$ and $< >$ denotes the average operation. This measure represents the degree of deviation of magnitude frequency distributions from the Gutenberg-Richter law. In general, $1 < \eta < \infty$. When the magnitude frequency distributions follows exactly the Gutenberg-Richter formula (1), then $\eta = 2$, since $< X^2 > = 2 < X^2 >^2$. When the magnitude frequency distributions in logn vs. M plot shows upwardly convex structure, this parameter becomes $\eta < 2$, but when the magnitude frequency distributions shows opposite shape, then $\eta > 2$. Utsu (1988) found that $\eta < 2$ for most cases of natural earthquakes and foreshocks have, in general, smaller $\eta$ values than those of aftershocks. By Equations (4) and (5)

$$b_1 = \Gamma(2) / \ln 10 J_1$$

(7)

$$b_2 = \Gamma(3) \frac{1}{2} / \ln 10 J_\frac{3}{2}$$

(8)

$$J_2 / J_1^2 = 2 (b_1 / b_2)^2$$

(9)

And by Equation (6)

$$< X > = < M - M_0 > = < M > - M_0 =$$

$$\frac{1}{N} \sum_{i=1}^{N} M_i - \frac{1}{N} \sum_{i=1}^{N} M_0$$

(10)

And

$$< X^2 > = < (M - M_0)^2 > = < M^2 > - M_0^2 - 2 M_0 < M >$$
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\[
\begin{align*}
\frac{1}{N} \sum_{i=1}^{N} M_i^2 + \frac{1}{N} \sum_{i=1}^{N} M_0^2 - 2 \\
M_0 \frac{1}{N} \sum_{i=1}^{N} M_i \\
= \frac{1}{N} \sum_{i=1}^{N} (M_i - M_0)^2 = J_2
\end{align*}
\]

From Equations (6), (9), (10) and (11) \( \eta \) value is related with the b value spectrum as

\[
\eta = \frac{J_2}{J_1^2} = 2 \left( \frac{b_1}{b_2} \right)^2
\]

This relation indicates that the \( \eta \) value is a simple and representative index of the b value spectrum.

3. Analysis of magnitude frequency distributions of earthquakes

Hormozgan province is located in the south of Iran (Figure 1). In this region, over 1100 earthquakes have been recorded between April 1930 and July 2008. Figure 2 shows the frequency of events in some regions in this province. Within Figure 2, there are three seismic zones with high frequency. These regions are: Bandarabbas Zone (BZ), Hajiabad Zone (HZ) and Minab Zone (MZ). Figure 3 also shows the magnitude frequency distributions of earthquakes in Bandarabbas zone, Hajiabad zone, Minab zone and Hormozgan province in (a), (b), (c) and (d) respectively. In this study by magnitude frequency distributions of earthquakes we have applied moment method to see the actual situation of the b value spectra. We have calculated the b value spectra for the magnitude frequency distributions of earthquakes for the four zones (Figure 4).
Figure 2. Earthquake epicenters (1930-2008) with magnitude 2.8-7.0 were distributed within the seismic zones: Bandarabbas zone (BZ), Hajiabad zone (HZ) and Minab zone (MZ) by red points denote the location of earthquake epicenters and blue points denote the location of cities (after Kavei et al. 2008).

Figure 3. The magnitude frequency distributions of earthquakes in: (a) Bandarabbas zone, (b) Hajiabad zone, (c) Minab zone and (d) Hormozgan.
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Figure 4. The b value spectra for the magnitude frequency distributions of earthquakes shown in Figure 3.

Figure 5. The magnitude frequency distributions of earthquakes in all Japan and region of northeast Japan (a) and (b) respectively (Ouchi 1997).

Figure 6. The b-value spectra for the magnitude frequency distributions of earthquakes have shown in Figures 5(a) and 5(b) respectively (Ouchi 1997).
4. Discussion
Magnitude frequency distributions of earthquakes represent certainly characteristic feature of seismic activities and thus have valuable information about the physical state of the crust. It is natural to expect that they show variations both in time and space, since geological conditions certainly differ from place to place and time to time. Also, they might reveal some precursory anomalies prior to large earthquake. Thus it is important to subtract such information from the analysis of magnitude frequency distributions of earthquakes. Indeed, numerous studies have been done about this subject. However, these results were not always reliable because previous analyses were insufficient. If the data are complete both in quality and quantity and magnitude frequency distributions do obey exactly the Gutenberg-Richter law, the ordinal analysis will yield important information contained in magnitude frequency distributions of earthquakes. In reality, this is not the case and these requirements are not always fulfilled. This problem also gives rise to dependency of the estimation of b values on the methods. Conventionally, b values are estimated by the maximum-likelihood (moment) method or least square method and results sometimes differ significantly. Thus some new approach has been needed to analyze magnitude frequency distributions of earthquakes more efficiently. This is the reason why we have introduced the notion of the b value spectrum. Through this b value spectrum, we can represent the characteristic feature of magnitude frequency distributions of earthquakes and deviation from the Gutenberg-Richter law. This enables us detailed and quantitative analysis of magnitude frequency distributions of earthquakes, namely, how and to what extent actual distributions deviate from the Gutenberg-Richter law etc... Also, the meaning of the η value becomes clear, since a simple relation holds between them.

5. Conclusions
We have introduced the concept of the b value spectrum based on the moment method. By this concept, we can analyze magnitude frequency distributions of earthquakes more effectively and investigate the deviation of the magnitude frequency distributions from the Gutenberg-Richter law or fractal structure. A simple relation is found between the b value spectrum and the η value. Analysis by his b value spectrum shows that actual magnitude frequency distributions of earthquakes have large variations and in some cases deviate significantly from the conventionally assumed Gutenberg-Richter law or fractal structure. This indicates that a simple fractal law like the Gutenberg-Richter law is not sufficient to characterize actual magnitude frequency distributions of earthquake populations. This idea will be used for more general analysis of the fractal structure that appears in various natural phenomena.

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Reference
– Bender B, 1983. Maximum likelihood estimation of b values for
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