Evaluation and ranking of suppliers with fuzzy DEA and PROMETHEE approach

R. Radfar *, F. Salahi †

Abstract

Supplier selection is a multi-Criteria problem. This study proposes a hybrid model for supporting the suppliers' selection and ranking. This research is a two-stage model designed to fully rank the suppliers where each supplier has multiple Inputs and Outputs. First, the supplier evaluation problem is formulated by Data Envelopment Analysis (DEA), since the regarded decision deals with uncertainly and ambiguity of data as well as experts and manager linguistic judgment the proposed model is equipped with Fuzzy approach, then in this research we use of Fuzzy DEA for first stage. In the second stage, efficient suppliers are ranked with Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) model. Fuzzy DEA PROMETHEE ranking does not replace the DEA classification model; rather it furthers the analysis by providing full ranking in DEA context for efficient suppliers.

Keywords: Supplier Selection; Data Envelopment Analysis; Fuzzy; Efficient; Multi-Criteria; Preference Ranking Organization Method for Enrichment Evaluation.

1 Introduction

Supplier selection is an important issue in Supply Chain Management. In recent year, determining the best supplier in supply chain has a key strategic consideration. However, these decisions usually involve several objectives or criteria, traditionally based on invoice cost, supplier’s ability to meet quality requirements and delivery schedule and it is often necessary to compromise among possibly conflicting factors. Due to the increasing acceptance of the concept of lean supply and the paradigm of lean production, as well as many organizational and managerial medications’ developed in vendor-rating systems.

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Supplier selection is a multi criteria decision making problem. Criteria and decision making techniques are two important elements in a supplier Selection problem. Thus, the Multiple Criteria Decision Making (MCDM) becomes a useful approach to solve this kind of problem, considering both tangible and intangible criteria. But, supplier selection is a difficult problem for managers because the performances of suppliers are varied based on each criterion [14]. Many of today’s buyers-supplier relationships are operating well beyond the traditional arm’s length mode, with different degree of alliance. Closer buyer-supplier relationships can be effective ways to guarantee good quality materials, diffuse new technologies rapidly, enter a new market, overcome financial constraints, bypass governmental restriction, and learn quickly from the leading firms in given field [11, 16]. Supplier-Customer relationship literature has developed descriptive and normative re-
Supplier differentiation refers to differences derived from supplier characteristics such as organizational culture, manufacturing procedure, technology capability and geographic location distribution [5]. Weber et al [21] sent a questionnaire to several companies. They identified most important criteria including price, delivery, quality, facilities, geographic location, and technology. Dickson (1966) identified over 20 supplier attributes which manager trade o when choosing a supplier. The criteria may have quantitative as well as qualitative dimension. In this paper we propose a hybrid model for supporting the suppliers’ selection and ranking. This research is a two-stage model. In the first stage we used Fuzzy DEA for suppliers evaluation then in the second stage, suppliers are ranked with PROMETHEE model.

2 Research Background

The literature review has been carried out by referring to leading journal databases. In the field of supplier selection and evaluation, a lot of articles have been published. Weber and Desai demonstrated that DEA method aids the buyer in classifying the suppliers into two categories: the efficient suppliers and the inefficient suppliers and how DEA can be used as a tool for negotiating with inefficient suppliers [20]. Ha and Krishnan [13], developed a hybrid model that including AHP, DEA and Neural Network approaches to supplier selection. Wu [22] introduced a hybrid model using data envelopment analysis, decision tree and Neural Network to assess supplier performance, the model consists of two modules: module 1 applies DEA and classifies suppliers into efficient and inefficient clusters based on resulting efficiency score. Module 2 utilized firm performance-related data to train DT, NN model and apply the train decision tree model to new supplier. Chang and Wu [6] introduced a new method for finds criteria that influence supplier selection, and construct the strategy map among these criteria using DEMATEL. The strategy map finds interdependencies among these criteria and their strengths. Businesses typically evaluate select supplier criteria according to product quality, price, services and delivery performance of the supplier. Dulmin and Mininno [10] proposed a model for supplier selection, an effort is made to highlight those aspects that are crucial to process qualitative and quantitative performance measures. In this paper, the contribution of a multi criteria decision aid method (Promethee/Gaia) to such problems is investigated, together with how to allow for a simultaneous change of the weights (importance of performance criteria).

Some researchers have investigated application of Fuzzy approach in supplier selection. Bottani and Rizzi [4] applied Fuzzy TOPSIS for selecting the best suppliers. Liu and Chuang [15] introduced a new method for solving Fuzzy BCC (Fuzzy DEA), by transformed a pair of two-level mathematical problems. Lee [1] proposed to select suppliers under a Fuzzy approach. A Fuzzy analytic hierarchy process (Fuzzy AHP) model which incorporates the benefits, opportunities, cost and risk (BOCR) concept, is constructed to evaluate to various aspect of suppliers. Sanaye, Mousavi and Yazdankhah [17] Developed new model for supplier selection, in this paper, linguistic values are used to assess the ratings and weights for quantitative and qualitative factors. These linguistic ratings can be expressed in trapezoidal or triangular Fuzzy numbers. Then, a hierarchy MCDM model based on Fuzzy sets theory and VIKOR method is proposed to deal with the supplier selection problems in the supply chain system.

3 Methodology

The model can function as a classification model; it generally consists of two modules. Module 1 applies DEA to calculate the DEA score given to each supplier. Regarded to decision makers always the time deals with uncertainly and ambiguity of data, we use of Fuzzy DEA to calculate the DEA score, typically classified as efficient and inefficient clusters the calculated DEA scores are used to derive the class for each supplier. Module 2 utilizes PROMETHEE method for ranking the suppliers.

3.1 Fuzzy DEA

Data Envelopment Analysis (DEA) is a non-parametric technique for measuring the relative
efficiency of the decision making units (DMUs) that have homogenous inputs and outputs. DEA applies linear programming techniques to the observed inputs/outputs of DMUs by constructing an efficient production frontier based on the best practices. Supplier efficiency is defined as the ratio of the weighted sum of its outputs (i.e. the performance of the supplier) to the weighted sum of its inputs (i.e. the costs of using the supplier). Each DMU’s efficiency is then measured relative to its distance to this frontier.

Assume that there are \( n \) suppliers indexed by \( j (j = 1, 2, \ldots, n) \) to be evaluated. The \( j^{th} \) supplier, \( E_j \) has \( m \) different inputs \( x_{ij} \) and \( s \) different outputs \( y_{rj} \). The relative efficiency of \( E_j \) is calculated as:

\[
E_j = \frac{\sum_{r=1}^{s} U_r \cdot Y_{rj}}{\sum_{i=1}^{m} V_i \cdot X_{ij}} \quad (3.1)
\]

Where \( V_i, i = 1, 2, \ldots, m \) and \( U_r, r = 1, 2, \ldots, s \) are input and output weight vectors, respectively.

The standard form of CCR model for assessing \( DMU_p \) is written as:

\[
Max \quad \sum_{r=1}^{s} U_r \cdot Y_{rp} \quad (3.2)
\]

\[
st. \quad \sum_{i=1}^{m} V_i \cdot X_{ip} = 1
\]

\[
\sum_{r=1}^{s} U_r \cdot Y_{rj} - \sum_{i=1}^{m} V_i \cdot X_{ij} \leq 0
\]

\[
U_r, V_i \geq \varepsilon
\]

The above model can only be used for cases where the data are precisely measured. Fuzzy DEA is a powerful tool for evaluating the performance of DMUs with imprecise data (or interval data). Fuzzy input-output variables can be introduced to DEA in the following Fuzzy Linear Programming model.

\[
Max \quad \sum_{r=1}^{s} U_r \cdot \tilde{Y}_{rp} \quad (3.3)
\]

\[
st. \quad \sum_{i=1}^{m} V_i \cdot \tilde{X}_{ip} = 1
\]

\[
\sum_{r=1}^{s} U_r \cdot \tilde{Y}_{rj} - \sum_{i=1}^{m} V_i \cdot \tilde{X}_{ij} \leq 0
\]

\[
U_r, V_i \geq \varepsilon
\]

Where \( \tilde{Y}_{rj} \) and \( \tilde{X}_{ij} \) are Fuzzy inputs and Fuzzy outputs, respectively. \( \varepsilon \) is a non-Archimedean small positive number. Saati Mohtadi et al [18] suggested a different CCR model for assessment of Fuzzy data by transferring the standard CCR model to a possibility programming problem. Their basic idea is using \( \alpha - cut \) approach to transform the Fuzzy CCR model into a crisp linear programming problem such as the standard DEA model. Their proposed approach assumes that the solution lies in the interval and the result for each DMU is an interval efficiency score rather than a crisp efficiency score. The main drawback in this approach is that their model cannot retain the uncertainty information completely since it is based on simple \( \alpha - cut \) approach. In other words, the Fuzzy numbers are simply converted to intervals using the same membership numbers in the entire of interval.

Dum’s have flexibility in select the weights. In other word, few number DMUs and lot number inputs and outputs cause increase feasible region so, by considering of weights flexibility, the more number of DMUS be efficient. There are some different methods for weighting restriction. One of these methods that we have used in research is Common Set of Weights (CSW).
3.2 Evaluation CSW

First Stage (determine bounds):
Outputs weight upper bounds

\[
\begin{align*}
\text{Max } & \quad U_p \\
st. & \quad \sum_{i=1}^{m} V_i \cdot X_{ip} = 1 \quad j = 1, 2, \ldots, n \\
& \quad \sum_{r=1}^{s} U_r \cdot Y_{rj} - \sum_{i=1}^{m} V_i \cdot X_{ij} \leq 0 \\
& \quad U_r, V_i \geq \varepsilon
\end{align*}
\]

Inputs weight upper bounds:

\[
\begin{align*}
\text{Max } & \quad V_p \\
st. & \quad \sum_{i=1}^{m} V_i \cdot X_{ip} = 1 \\
& \quad \sum_{r=1}^{s} U_r \cdot Y_{rj} - \sum_{i=1}^{m} V_i \cdot X_{ij} \leq 0 \\
& \quad U_r, V_i \geq \varepsilon
\end{align*}
\]

By solving \( m+s \) linear programming problem, determine inputs and outputs weight upper bounds.

Second Stage (determine CSW):

\[
\begin{align*}
\text{Max } & \quad \phi \\
st. & \quad \sum_{r=1}^{s} Y_{rj} - \sum_{i=1}^{m} V_i \cdot X_{ij} \leq 0 \quad j = 1, 2, \ldots, n \\
& \quad \phi \cdot U_r \leq U_r \leq (1 - \phi) \cdot U_r \quad r = 1, 2, \ldots, s \\
& \quad \phi \cdot V_i \leq V_i \leq (1 - \phi) \cdot V_i \quad i = 1, 2, \ldots, m \\
& \quad U_r, V_i \geq \varepsilon
\end{align*}
\]

Then, calculate DMU efficient:

\[
E_j = \frac{\sum_{r=1}^{s} U_r \cdot Y_{rj}}{\sum_{i=1}^{m} V_i \cdot X_{ij}} \tag{3.7}
\]

Fuzzy upper bounds: (for max \( U_p \) and max \( V_i \))

\[
\begin{align*}
\text{Max } & \quad U_p \\
st. & \quad \sum_{i=1}^{m} V_i (X_{ij}^m \cdot X_{ij}^\alpha \cdot X_{ij}^\beta) \leq \tilde{1} \quad j = 1, \ldots, n \\
& \quad \sum_{r=1}^{s} U_r (Y_{rj}^m \cdot Y_{rj}^\alpha \cdot Y_{rj}^\beta) - \sum_{i=1}^{m} V_i (X_{ij}^m \cdot X_{ij}^\alpha \cdot X_{ij}^\beta) \leq 0 \\
& \quad U_r, V_i \geq 0
\end{align*}
\]

\[
\begin{align*}
\text{Max } & \quad V_i \\
st. & \quad \sum_{i=1}^{m} V_i (X_{ij}^m \cdot X_{ij}^\alpha \cdot X_{ij}^\beta) \leq \tilde{1} \quad j = 1, \ldots, n \\
& \quad \sum_{r=1}^{s} U_r (Y_{rj}^m \cdot Y_{rj}^\alpha \cdot Y_{rj}^\beta) - \sum_{i=1}^{m} V_i (X_{ij}^m \cdot X_{ij}^\alpha \cdot X_{ij}^\beta) \leq 0 \\
& \quad U_r, V_i \geq 0
\end{align*}
\]

Calculate CSW by distance numbers:

\[
\begin{align*}
\text{Max } & \quad \phi \\
st. & \quad \sum_{r=1}^{s} \tilde{Y}_{rj} - \sum_{i=1}^{m} \tilde{X}_{ij} \leq 0 \quad j = 1, \ldots, n \\
& \quad \phi \cdot U_r \leq U_r \leq (1 - \phi) \cdot U_r \quad r = 1, 2, \ldots, s \\
& \quad \phi \cdot V_i \leq V_i \leq (1 - \phi) \cdot V_i \quad i = 1, 2, \ldots, m \\
& \quad U_r, V_i \geq \varepsilon
\end{align*}
\]

\( \gamma \in [0, 1] \) is a parameter. Now, we can measure DMU’S efficiency.
\[ e_j = \frac{\sum_{r=1}^{s} U_r^* \cdot Y_{rj}}{\sum_{i=1}^{m} V_i^* \cdot X_{ij}} \]  
\[ e_j^m = \frac{\sum_{r=1}^{s} U_r^* \cdot Y_{rj}^m}{\sum_{i=1}^{m} V_i^* \cdot X_{ij}^m} \]
\[ e_j^\beta = \frac{(\sum_{r=1}^{s} U_r^* \cdot Y_{rj}^\beta \cdot \sum_{i=1}^{s} V_{i, j}^\beta \cdot X_{ij}^\beta)}{(\sum_{i=1}^{m} V_i^* \cdot X_{ij}^m)^2} \]
\[ e_j^{\alpha} = \frac{(\sum_{r=1}^{s} U_r^* \cdot Y_{rj}^{\alpha} \cdot \sum_{i=1}^{s} V_{i, j}^{\alpha} \cdot X_{ij}^{\alpha})}{(\sum_{i=1}^{m} V_i^* \cdot X_{ij}^m)^2} \]

3.3 PROMETHEE Method

The PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) method is a Multi Criteria Decision Making Technique developed by Brans and Vincke [3]. It is well adapted to problems where finite a number of alternative actions are to be ranked considering several, sometimes conflicting criteria [12]. Among numerous methods of MCDM, the PROMETHEE is significantly suitable for ranking applications [2].

Let A be a finite set of alternatives for MCDM problems, and suppose a preference function \( f_j \) has been defined for each \( g_j \), for each couple of alternatives \( a, b \in A \), when \( a > b \) in \( j \) criterion, \( f_j(a, b) = f_j(dabj) \) indicates that the degree of alternative a prefers to alternative b (a over b) with different distance of performance value \( dabj = g_j(a) - g_j(b) \) in j criterion; and \( \pi(a, b) \) is a preference index over all the criteria defined by: \( \pi(a, b) = \sum w_j f_j(a, b) \).

Leaving flow:
\[ \phi^+(a) = \sum \pi(a, x) \quad x = (b, c, d, \ldots) \]

Entering flow:
\[ \phi^-(a) = \sum \pi(x, a) \quad x = (b, c, d, \ldots) \]

Net flow:
\[ \phi_{net}(a) = \phi^+(a) - \phi^-(a) \]  
\[ \phi_{net}(a) = \phi(a) \quad x = (b, c, d, \ldots) \]

The leaving flow \( \phi^+(a) \) is the measure of how a dominates all the other alternatives of A, where we assume that each alternatives is belonging to the set of A of alternatives. Symmetrically, the entering flow \( \phi^-(a) \) gives that how is dominated by all the other alternatives of A. \( \phi_{net}(a) \) represents a value function, whereby a higher value reflects a higher attractiveness of alternative and is called net flow. In PROMETHEE methods, the higher the leaving flow and the lower the entering flow, the better the alternative. The leaving and entering flow induce, respectively, the following preorders on alternatives on A:

\[ A = \begin{cases} 
  a P^+ b & \text{iff } \phi^+(a) > \phi^+(b) \\
  a I^+ b & \text{iff } \phi^+(a) = \phi^+(b) 
\end{cases} \]

\[ A = \begin{cases} 
  a P^- b & \text{iff } \phi^-(a) < \phi^-(b) \\
  a I^- b & \text{iff } \phi^-(a) = \phi^-(b) 
\end{cases} \]

Where \( P \) and \( I \) represent preference and indifference, respectively.

**PROMETHEE I**

PROMETHEE I determines the partial preorder \( (P^I, I^I, R) \) on the alternatives of A that satisfied the following principle:

\[ a P^I b, \quad \text{if} \quad \begin{cases} 
  a P^+ b & \text{and} \quad a P^- b \\
  a I^+ b & \text{and} \quad a I^- b \\
  a R b, & \text{otherwise} 
\end{cases} \]

**PROMETHEE II**

PROMETHEE I ensure creation of indifferent and incomparable alternatives. In some ranking problems, PROMETHEE I can give a complete ranking depending on the evaluation matrix values and, this ranking cannot be different from the one achieved with PROMETHEE II [3]. PROMETHEE II gives a complete preorder \( (P^{II}, I^{II}) \) induced by the net flow and defined by:

\[ A = \begin{cases} 
  a P^{II} b & \text{if } \phi(a) > \phi(b) \\
  a I^{II} b & \text{if } \phi(a) = \phi(b) 
\end{cases} \]

\[ A = \begin{cases} 
  a P^- b & \text{iff } \phi(a) < \phi(b) \\
  a I^- b & \text{iff } \phi(a) = \phi(b) 
\end{cases} \]
Table 1: Input and Output variables case 1.

<table>
<thead>
<tr>
<th>Output variable</th>
<th>Input variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>quality</td>
<td>Time cycle</td>
</tr>
<tr>
<td>on time delivery percentage</td>
<td>cost</td>
</tr>
<tr>
<td>Market share</td>
<td>capacity internal production</td>
</tr>
<tr>
<td>Return capital</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Input and Output numbers.

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Inputs</th>
<th>DMUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>U4</td>
<td>U3</td>
<td>U2</td>
</tr>
<tr>
<td>5</td>
<td>(65, 5, 5)</td>
<td>(80, 5, 5)</td>
</tr>
<tr>
<td>10</td>
<td>(62.5, 2.5, 2.5)</td>
<td>(65, 5, 5)</td>
</tr>
<tr>
<td>15</td>
<td>(72.5, 2.5, 2.5)</td>
<td>(80, 5, 5)</td>
</tr>
<tr>
<td>5</td>
<td>(85, 5, 5)</td>
<td>(75, 5, 5)</td>
</tr>
<tr>
<td>15</td>
<td>(75, 5, 5)</td>
<td>(80, 5, 5)</td>
</tr>
<tr>
<td>10</td>
<td>(70, 5, 5)</td>
<td>(80, 5, 5)</td>
</tr>
<tr>
<td>5</td>
<td>(70, 5, 5)</td>
<td>(75, 5, 5)</td>
</tr>
<tr>
<td>5</td>
<td>(72.5, 2.5, 2.5)</td>
<td>(80, 5, 5)</td>
</tr>
<tr>
<td>10</td>
<td>(70, 5, 5)</td>
<td>(85, 5, 5)</td>
</tr>
</tbody>
</table>

Table 3: Upper bounds for criteria weights.

<table>
<thead>
<tr>
<th>U4</th>
<th>U3</th>
<th>U2</th>
<th>U1</th>
<th>V3</th>
<th>V2</th>
<th>V1</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0580</td>
<td>0.0116</td>
<td>0.0122</td>
<td>0.0121</td>
<td>0.0005</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>0.0580</td>
<td>0.0114</td>
<td>0.0121</td>
<td>0.0119</td>
<td>0.0005</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>0.0580</td>
<td>0.0113</td>
<td>0.0119</td>
<td>0.0118</td>
<td>0.0005</td>
<td>0.1</td>
<td>0.1</td>
<td>0.7</td>
</tr>
<tr>
<td>0.0580</td>
<td>0.0111</td>
<td>0.0117</td>
<td>0.0116</td>
<td>0.0005</td>
<td>0.1</td>
<td>0.1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4: Common Set of Weights.

<table>
<thead>
<tr>
<th>U4</th>
<th>U3</th>
<th>U2</th>
<th>U1</th>
<th>V3</th>
<th>V2</th>
<th>V1</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0206</td>
<td>0.0041</td>
<td>0.0043</td>
<td>0.0043</td>
<td>0.0003</td>
<td>0.0644</td>
<td>0.0644</td>
<td>0.3</td>
</tr>
<tr>
<td>0.0206</td>
<td>0.0041</td>
<td>0.0043</td>
<td>0.0042</td>
<td>0.0003</td>
<td>0.0644</td>
<td>0.0644</td>
<td>0.5</td>
</tr>
<tr>
<td>0.0206</td>
<td>0.0040</td>
<td>0.0042</td>
<td>0.0042</td>
<td>0.0003</td>
<td>0.0644</td>
<td>0.0644</td>
<td>0.7</td>
</tr>
<tr>
<td>0.0206</td>
<td>0.0039</td>
<td>0.0042</td>
<td>0.0041</td>
<td>0.0003</td>
<td>0.0644</td>
<td>0.0645</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5: Efficiency suppliers.

<table>
<thead>
<tr>
<th>1</th>
<th>0.7</th>
<th>0.5</th>
<th>0.3</th>
<th>DMUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.694, 0.738, 0.782)</td>
<td>(0.704, 0.748, 0.792)</td>
<td>(0.714, 0.759, 0.804)</td>
<td>(0.720, 0.765, 0.810)</td>
<td>D1</td>
</tr>
<tr>
<td>(0.538, 0.566, 0.594)</td>
<td>(0.577, 0.549, 0.605)</td>
<td>(0.559, 0.588, 0.617)</td>
<td>(0.564, 0.593, 0.622)</td>
<td>D2</td>
</tr>
<tr>
<td>(1.026, 1.065, 1.104)</td>
<td>(1.037, 1.076, 1.105)</td>
<td>(1.047, 1.088, 1.129)</td>
<td>(1.054, 1.094, 1.134)</td>
<td>D3</td>
</tr>
<tr>
<td>(0.790, 0.732, 0.874)</td>
<td>(0.805, 0.844, 0.883)</td>
<td>(0.816, 0.857, 0.898)</td>
<td>(0.820, 0.826, 0.904)</td>
<td>D4</td>
</tr>
<tr>
<td>(0.944, 0.9903, 1.036)</td>
<td>(0.954, 1.0003, 1.046)</td>
<td>(1.056, 1.103, 1.15)</td>
<td>(1.062, 1.109, 1.156)</td>
<td>D5</td>
</tr>
<tr>
<td>(0.740, 0.781, 0.822)</td>
<td>(0.753, 0.794, 0.835)</td>
<td>(0.765, 0.805, 0.845)</td>
<td>(0.769, 0.810, 0.851)</td>
<td>D6</td>
</tr>
<tr>
<td>(0.735, 0.799, 0.863)</td>
<td>(0.746, 0.811, 0.876)</td>
<td>(0.762, 0.825, 0.888)</td>
<td>(0.766, 0.830, 0.894)</td>
<td>D7</td>
</tr>
<tr>
<td>(0.671, 0.699, 0.727)</td>
<td>(0.683, 0.712, 0.741)</td>
<td>(0.697, 0.724, 0.751)</td>
<td>(0.701, 0.730, 0.759)</td>
<td>D8</td>
</tr>
<tr>
<td>(0.958, 1.006, 1.054)</td>
<td>(0.970, 1.019, 1.068)</td>
<td>(0.981, 1.030, 1.079)</td>
<td>(0.986, 1.035, 1.084)</td>
<td>D9</td>
</tr>
<tr>
<td>(0.976, 1.031, 1.086)</td>
<td>(0.990, 1.045, 1.110)</td>
<td>(1.007, 1.060, 1.113)</td>
<td>(1.012, 1.066, 1.120)</td>
<td>D10</td>
</tr>
</tbody>
</table>
3.4 Case Study (Numerical Example)

In this section we are going to propose a numerical example to illustrate an application of the proposed method in the previous section. Case study illustrates the stages of supplier selection and evaluation for a manufacture of gearbox and axle in Iran. Management wants to improve the efficiency of the purchasing process. After several meetings they identified possible four suppliers. Tables 1 and 2 present data of supplier selection parameters of the firm.

Crisp variable includes; Time cycle (day) (V1), Total cost (dollar) (V2), Capacity internal production (percent) (V3), and Return capital (ton/year) (U4).

Fuzzy variable include; Quality (percent) (U1), On time delivery percentage (percent) (U2), market share (percent) (U3)

By use of input and output weight upper bound function we can calculate upper bounds for criteria weights which shown in Table 3.

Then we obtain Common Set of Weights whit Fuzzy CSW model.

By considering the weights obtained from model, Two issue can be considered; the 3rd input (capacity internal production) related to 1st and 2nd inputs (time cycle, cost) is less important. Which means the priority of capacity internal production is lower than time cycle and cost. Also the weight of outputs shows the 4th output (return capital) is the first priority. So by identifying the most important and prioritized factors in the model more efficiency can be obtained.

Also the change of α from 0.3 to 1 can decrease the optimal weight of outputs. This means the outputs getting more close to the original values.

With set $V^*$ and $U^*$ in Fuzzy efficiency model, we can obtain distance efficiency ($e^m, e^a, e^b$). Therefore efficiency suppliers with considering upper and lower bound of average is like Table 5. Result shows that D3, D5, D9, D10 are efficient suppliers. Reviews show that, Reason inecient 1, 2, 4, 6, 7, 8 suppliers can result from optimal nonuse of resource or weakly in outputs. Now for ranking the suppliers we use PROMETHEE method, we use of calculated weights by CSW model for criteria weights in PROMETHEE.

The first we should define $p(a, b)$ or $f(a, b)$ for each criteria. (Determine by DM).

$$D_j = g_j(a) - g_j(b)$$

1) criterion (V1):

$$F(a, b) = \begin{cases} 
1 & \text{if } d_j < 0 \\
0 & \text{if } d_j \geq 0 
\end{cases}$$

2) criterion (V2):

$$F(a, b) = \begin{cases} 
1 & \text{if } d_j < 0 \\
0 & \text{if } d_j \geq 0 
\end{cases}$$

3) criterion (V3):

$$F(a, b) = \begin{cases} 
1 & \text{if } d_j \geq 100 \\
0 & \text{if } d_j < 100 
\end{cases}$$

4) criterion (U1):

Table 6: The criteria weights for $\alpha = 0.3$.

<table>
<thead>
<tr>
<th>W</th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>U1</th>
<th>U2</th>
<th>U3</th>
<th>U4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.3$</td>
<td>0.0644</td>
<td>0.0644</td>
<td>0.0003</td>
<td>0.0043</td>
<td>0.0043</td>
<td>0.0041</td>
<td>0.0206</td>
</tr>
</tbody>
</table>

Table 7: Results of PROMETHEE model.

<table>
<thead>
<tr>
<th>D3</th>
<th>D5</th>
<th>D9</th>
<th>D10</th>
<th>$\phi^+(a)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^-(a)$</td>
<td>0.2057</td>
<td>0.2016</td>
<td>0.1823</td>
<td>0.0673</td>
</tr>
<tr>
<td>$\phi(a)$</td>
<td>-0.0874</td>
<td>-0.148</td>
<td>-0.036</td>
<td>0.1382</td>
</tr>
</tbody>
</table>
\[ F(a, b) = \begin{cases} 1 & \text{if } d_j \geq 5 \\ 0 & \text{if } d_j < 5 \end{cases} \]

5) criterion (U2):
\[ F(a, b) = \begin{cases} 1 & \text{if } d_j \geq 5 \\ 0 & \text{if } d_j < 5 \end{cases} \]

6) criterion (U3):
\[ F(a, b) = \begin{cases} 1 & \text{if } d_j \geq 2.5 \\ 0 & \text{if } d_j < 2.5 \end{cases} \]

7) criterion (U4):
\[ F(a, b) = \begin{cases} 1 & \text{if } d_j > 0 \\ 0 & \text{if } d_j \leq 0 \end{cases} \]

Now, we obtain overall preference indexes for each alternative pair by \( \pi(a, b) = \sum w_j \cdot f_j(a, b) \). Results of PROMETHEE method show in Table 7.

By considering of result, it’s obvious that D10 has top priority in ranking suppliers, and D5, D9, D3 respectively have second, third and fourth ranks. \( D10 > D5 > D9 > D3 \)

4 Conclusion
Supplier selection, which is one of the most crucial components of production and logistics management, has a significant impact on various functional areas of business from procurement to production and delivery of the products to the end customer. The supplier selection problem is often influenced by uncertainty in practice, and in such situation Fuzzy approach is an appropriate tool to deal with this kind of problems. This paper has developed a hybrid supplier evaluation model, using Fuzzy DEA and PROMETHEE. The work described herein presents a proposal for applying a decision model to the final vendar rating phase of a process of supplier selection; the model enables us to deal with the complexity and multiple criteria including intangible criteria embedded in the supplier selection problem. Using this model, decision maker is able to choose most efficient supplier by solving Fuzzy DEA model. Our approach this model uses an MCDM technique (PROMETHEE II) for ranking suppliers.

PRPMETHEE is an outranking method, which can result in a partial (PROMETHEE I) or complete (PROMETHEE II) pre-ordering of the alternatives. The proposed method is very flexible. Using this method enables us to assess and determine the outranking order of suppliers and rate the suppliers. These rating can be used in combination with mathematical programming and other methods to deal with supplier selection in multiple sourcing environments.

References


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