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Research Article

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A method for solving first order fuzzy differential equation

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Abstract

In this paper, a new approach for solving first-order fuzzy differential equation (FDE) with fuzzy initial value under strongly generalized H-differentiability is considered. The idea of the presented approach is constructed based on the extending 0-cut and 1-cut solution of original FDE. First, under H-differentiability the solutions of fuzzy differential equations in 0-cut and 1-cut cases are found and convex combination of them, is considered. Then we choose the initial interval that 0-cut of original problem is positive. The obtained convex combination on this interval is the solution of FDE.

Keywords : Fuzzy differential equation (FDE); Interval differential equation; Strongly generalized H-differentiability.

1 Introduction

The topic of fuzzy differential equation (FDE) has been rapidly growing in recent years. Kandel and Byatt [19] applied the concept of (FDE) to the analysis of fuzzy dynamical problems. The (FDE) and the initial value problem (Cauchy problem) were rigorously treated by Kaleva [17, 18], Seikkala [23], He and Yi [13], Kloeden [20] and by other researchers (see [3, 9, 10, 12, 16]). The numerical methods for solving FDE are introduced in [1, 2, 4, 5]. Bede [7] applied the concept of Strongly generalized H-differentiability to solving first order linear fuzzy differential equations and Allahviranloo et. al. [6] proposed a method to obtain analytical solutions for FDE under strongly generalized H-differentiability.

The idea of the presented approach is constructed based on the extending 0-cut and 1-cut solution of original FDE. Obviously, 0-cut of FDE is interval differential equation or ordinary differential equation. First, under H-differentiability FDE has been divided in two differential equations and solutions of each fuzzy differential equations in 0-cut and 1-cut cases are found. Then in each cases of differentiability, the initial interval that 0-cut of original problem is positive, is found. The obtained convex combination on this interval,is an solution of FDE.

The structure of this paper is organized as follows. In Section 2, some basic definitions and notations which will be used are brought. In Section 3, first order fuzzy differential equation is introduced and the proposed approach is given in detail. In Section 4, the proposed method is illustrated by solving several examples. Conclusion is drawn in Section 5.

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2 Basic Definitions and Notations

Definition 2.1 *An arbitrary fuzzy number is represented by an ordered pair of functions $(\underline{u}(r), \overline{u}(r))$ for all $r \in [0,1]$, which satisfy the following requirements [15].*

- *$\underline{u}(r)$ is a bounded left continuous nondecreasing function over $[0,1]$;*
- *$\overline{u}(r)$ is a bounded left continuous non-increasing function over $[0,1]$;*
- *$\underline{u}(r) \leq \overline{u}(r), \quad 0 \leq r \leq 1$.*

Let E be the set of all upper semi-continuous normal convex fuzzy numbers with bounded r-level intervals. This means that if $\widetilde{v} \in E$ then the r-level set

$$[v]_r = \{s | v(s) \geq r\},$$

is a closed bounded interval which is denoted by $[v]_r = [\underline{v}(r), \overline{v}(r)]$ for $r \in (0,1]$ and $[v]_0 = \bigcup_{r \in (0,1]} [v]_r$.

Two fuzzy numbers $\widetilde{u}$ and $\widetilde{v}$ are called equal $\widetilde{u} = \widetilde{v}$, if $u(s) = v(s)$ for all $s \in \mathbb{R}$ or $[u]_r = [v]_r$ for all $r \in [0,1]$.

Lemma 2.1 *[22] If $\widetilde{u}, \widetilde{v} \in E$, then for $r \in (0,1]$,*

$$[u+v]_r = [\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r)],$$
$$[u.v]_r = [\min k_r, \max k_r],$$

where

$$k_r = \{\underline{u}(r)\underline{v}(r), \underline{u}(r)\overline{v}(r), \overline{u}(r)\underline{v}(r), \overline{u}(r)\overline{v}(r)\}.$$

The Hausdorff distance between fuzzy numbers given by $D : E \times E \longrightarrow R^+ \bigcup \{0\}$,

$$D(u,v) = \sup_{r \in [0,1]} \max\{|\underline{u}(r) - \underline{v}(r)|, |\overline{u}(r) - \overline{v}(r)|\},$$

where $u = (\underline{u}(r), \overline{u}(r))$, $v = (\underline{v}(r), \overline{v}(r)) \subset E$ is utilized (see [8]). Then it is easy to see that D is a metric in E where for all $u, v, w, e \in E$ has the following properties (see [21]).

- $D(u \oplus w, v \oplus w) = D(u,v)$,
- $D(k \odot u, k \odot v) = |k| D(u,v), \quad \forall k \in R$,
- $D(u \oplus v, w \oplus e) \leq D(u,w) + D(v,e)$,
- (D, E) is a complete metric space.

Definition 2.2 *[14]. Let $f : R \longrightarrow E$ be a fuzzy valued function. If for arbitrary fixed $t_0 \in R$ and $\varepsilon > 0$, $\delta > 0$ such that*

$$|t - t_0| < \delta \Longrightarrow D(f(t), f(t_0)) < \varepsilon,$$

f is said to be continuous.

Definition 2.3 *Let $x, y \in E$. If there exists $z \in E$ such that $x = y + z$, then z is called the H-difference of x and y and it is denoted by $x \ominus y$.*
In this paper we consider the following definition of differentiability for fuzzy-valued functions which was introduced by Bede et.al. [8] and investigate by Chalco-Cano et.al. [11].

Definition 2.4 *Let $f : (a,b) \longrightarrow E$ and $x_0 \in (a,b)$. We say that f is strongly generalized H-differentiable at x_0. If there exists an element $f'(x_0) \in E$, such that:*
(1) for all $h > 0$ sufficiently near to 0, $\exists f(x_0 + h) \ominus f(x_0)$, $\exists f(x_0) \ominus f(x_0 - h)$ such that the following limits hold.

$$\lim_{h \longrightarrow 0^+} \frac{f(x_0+h) \ominus f(x_0)}{h} = \lim_{h \longrightarrow 0^+} \frac{f(x_0) \ominus f(x_0-h)}{h} = f'(x_0)$$

(2) for all $h < 0$ sufficiently near to 0, $\exists f(x_0) \ominus f(x_0 + h)$, $\exists f(x_0 - h) \ominus f(x_0)$ such that the following limits hold.

$$\lim_{h \longrightarrow 0^+} \frac{f(x_0) \ominus f(x_0+h)}{h} = \lim_{h \longrightarrow 0^+} \frac{f(x_0-h) \ominus f(x_0)}{h} = f'(x_0)$$

In the special case when f is a fuzzy-valued function, we have the following results.

Theorem 2.1 *[11]. Let $f : R \longrightarrow E$ be a function and denote $f(t) = (\underline{f}(t;r), \overline{f}(t;r))$, for each $r \in [0,1]$. Then*
(1) if f is differentiable in the first form (1) in definition 2.4 then $\underline{f}(t;r)$ and $\overline{f}(t;r))$ are differentiable functions and $f'(t) = (\underline{f}'(t;r), \overline{f}'(t;r))$.
(2) if f is differentiable in the second form (2) in definition 2.4 then $\underline{f}(t;r)$ and $\overline{f}(t;r))$ are differentiable functions and $f'(t) = (\overline{f}'(t;r), \underline{f}'(t;r))$.

The principal properties of the H-derivatives in the first form (1), some of which still hold for the second form (2), are well known and can be found in [17] and some properties for the second form (2) can be found in [11].
Notice that we say fuzzy-valued function f is (I)-differentiable if satisfies in the first form (1) in Definition 2.4 and we say f is (II)-differentiable if satisfies in the second form (2) in Definition 2.4.

3 First Order Fuzzy Differential Equations

In this section, we are going to investigate solution of FDE. Consider the following first order fuzzy differential equation:

$$\begin{cases} y^{'}(t) = f(t, y(t)) \\ \widetilde{y}(t_0) = \widetilde{y}_0, \end{cases} \tag{3.1}$$

where $f : [a, b] \times E \longrightarrow E$ is fuzzy-valued function, $\widetilde{y}_0 \in E$ and strongly generalized H-differentiability is also considered which is defined in Definition 2.4

Now, we describe our propose approach for solving FDE (3.1). First, we shall solve FDE (3.1) in sense of 1-cut and 0-cut as a follows:

$$\begin{cases} (y^{'})^{[1]}(t) = f^{[1]}(t, y(t)), \\ y^{[1]}(t_0) = \widetilde{y}_0^{[1]}, \qquad t_0 \in [0, T]. \end{cases} \tag{3.2}$$

$$\begin{cases} (y^{'})^{[0]}(t) = f^{[0]}(t, y(t)), \\ y^{[0]}(t_0) = \widetilde{y}_0^{[0]}, \qquad t_0 \in [0, T]. \end{cases} \tag{3.3}$$

If Eq. (3.2) and Eq. (3.3) be a crisp differential equation we can solve it as usual, otherwise, if Eq. (3.2) and Eq. (3.3) be an interval differential equation we will solve it by stefanini et. al.'s method is proposed and discussed in [24]. Notice that the solutions of differential equation (3.2) and Eq. (3.3) are presented with notation $y^{[1]}(t)$ and $y^{[0]}(t)$ respectively. Then unknown $\widetilde{y}(t)$ must be determined, this approach leads to obtain

$$y(t) = [\underline{y}(t; r), \overline{y}(t; r)] = \tag{3.4}$$
$$[(1 - r)\underline{y^{[0]}}(t) + r\underline{y^{[1]}}(t), (1 - r)\overline{y^{[0]}}(t) + r\overline{y^{[1]}}(t)]$$

Pleased notice that, we assumed the 0-cut and 1-cut solutions are differentiable. Consequently, based on type of differentiability we have two following cases:

Case I. Suppose that $\widetilde{y}(t)$ in Eq. (3.3) is (I)-differentiable, then we get:

$$y^{'}(t) = [\underline{y}^{'}(t; r), \overline{y}^{'}(t; r)] \tag{3.5}$$

Consider Eq. (3.5) and original FDE (3.1), then we have the following for all $r \in [0, 1]$:

$$\begin{cases} \underline{y}^{'}(t; r) = \underline{f}(t; r), \quad t_0 \leq t \leq T, \\ \overline{y}^{'}(t; r) = \overline{f}(t; r), \quad t_0 \leq t \leq T. \end{cases} \tag{3.6}$$

Therefore

$$\begin{cases} (1 - r)(\underline{y^{[0]}})'(t) + r(\underline{y^{[1]}})'(t) = \\ (1 - r)(\underline{f^{[0]}})(t) + r(\underline{f^{[1]}})(t), \\ (1 - r)(\overline{y^{[0]}})'(t) + r(\overline{y^{[1]}})'(t) = \\ (1 - r)(\overline{f^{[0]}})(t) + r(\overline{f^{[1]}})(t), \\ \underline{y}(t_0; r) = (1 - r)\underline{y^{[0]}}(t_0) + r\underline{y^{[1]}}(t_0), \\ \overline{y}(t_0; r) = (1 - r)\overline{y^{[0]}}(t_0) + r\overline{y^{[1]}}(t_0). \end{cases} \tag{3.7}$$

Then we have

$$\begin{cases} \underline{y^{[0]}}^{'}(t) = \underline{f^{[0]}}(t), \\ \overline{y^{[0]}}^{'}(t) = \overline{f^{[0]}}(t), \\ \underline{y^{[0]}}(t_0) = \underline{y_0}^{[0]}, \\ \overline{y^{[0]}}(t_0) = \overline{y_0}^{[0]}, \end{cases} \tag{3.8}$$

and

$$\begin{cases} \underline{y^{[1]}}^{'}(t) = \underline{f^{[1]}}(t), \\ \overline{y^{[1]}}^{'}(t) = \overline{f^{[1]}}(t), \\ \underline{y^{[1]}}(t_0) = \underline{y_0}^{[1]}, \\ \overline{y^{[1]}}(t_0) = \overline{y_0}^{[1]}. \end{cases} \tag{3.9}$$

Indeed, we will find $\underline{y^{[0]}}(t)$, $\overline{y^{[0]}}(t)$, $\underline{y^{[1]}}(t)$, $\overline{y^{[1]}}(t)$ by solving $ODEs$ (3.8), (3.9). Hence, solution of original FDE (3.1) is derived based on 1-cut solution and 0-cut solution as follows:

$$\widetilde{y}(t) = [\underline{y}(t; r), \overline{y}(t; r)] = [(1 - r)\underline{y^{[0]}}(t) + r\underline{y^{[1]}}(t), (1 - r)\overline{y^{[0]}}(t) + r\overline{y^{[1]}}(t)],$$

where for all $0 \leq r \leq 1$ and $t \in [0, T]$ such that:

$$\underline{y}(t; r) = (1 - r)\underline{y^{[0]}}(t) + r\underline{y^{[1]}}(t),$$
$$\overline{y}(t; r) = (1 - r)\overline{y^{[0]}}(t) + r\overline{y^{[1]}}(t).$$

Case II. Suppose that $\widetilde{y}(t)$ in Eq. (3.3) is (II)-differentiable, then we get:

$$y^{'}(t) = [\overline{y}^{'}(t; r), \underline{y}^{'}(t; r)] \tag{3.10}$$

Similarly, *ODEs* (3.8) and (3.9) can be rewritten in sense of (II)-differentiability as following:

$$\begin{cases} \underline{y^{[0]}}'(t) = \overline{f^{[0]}}(t), \\ \overline{y^{[0]}}'(t) = \underline{f^{[0]}}(t), \\ \underline{y^{[0]}}(t_0) = \overline{y_0}^{[0]}, \\ \overline{y^{[0]}}(t_0) = \underline{y_0}^{[0]}, \end{cases} \tag{3.11}$$

and

$$\begin{cases} \underline{y^{[1]}}'(t) = \overline{f^{[1]}}(t), \\ \overline{y^{[1]}}'(t) = \underline{f^{[1]}}(t), \\ \underline{y^{[1]}}(t_0) = \overline{y_0}^{[1]}, \\ \overline{y^{[1]}}(t_0) = \underline{y_0}^{[1]}. \end{cases} \tag{3.12}$$

Finally, by solving above *ODEs* (3.11) and (3.12) $\underline{y^{[0]}}(t)$, $\overline{y^{[0]}}(t)$, $\underline{y^{[1]}}(t)$, $\overline{y^{[1]}}(t)$ are determined and follows we can drive solution of original *FDE* (3.1) in sense of (II)-differentiability by using

$$\underline{y}(t;r) = (1-r)\underline{y^{[0]}}(t) + r\underline{y^{[1]}}(t),$$
$$\overline{y}(t;r) = (1-r)\overline{y^{[0]}}(t) + r\overline{y^{[1]}}(t),$$

for all $0 \le r \le 1$ and $t \in [0, T]$.

4 Examples

In this section, some examples are given to illustrate our method and we show that our approach is coincide with the exact solutions.

Example 4.1 *Let consider the following FDE:*

$$\begin{cases} y'(t) = y(t) + \widetilde{a}, \\ y(0;r) = [r, 2-r], \quad \widetilde{a} = [r-1, 1-r] \quad 0 \le r \le 1. \end{cases} \tag{4.13}$$

Case I. Suppose that $\widetilde{y}(t)$ is (I)-differentiable. The exact solution of above system is:

$$\widetilde{y}(t) = [(2r-1)e^t - r + 1, -(2r-3)e^t r - 1].$$

Based on *ODEs* (3.8) and (3.9), we have:

$$\begin{cases} \underline{y^{[0]}}'(t) = \underline{y^{[0]}}(t) - 1, \\ \overline{y^{[0]}}'(t) = \overline{y^{[0]}}(t) + 1, \\ \underline{y^{[0]}}(0) = 0, \\ \overline{y^{[0]}}(0) = 2, \end{cases} \tag{4.14}$$

$$\begin{cases} \underline{y^{[1]}}'(t) = \underline{y^{[1]}}(t), \\ \overline{y^{[1]}}'(t) = \overline{y^{[1]}}(t), \\ \underline{y^{[1]}}(0) = 1, \\ \overline{y^{[1]}}(0) = 1. \end{cases} \tag{4.15}$$

By solving *ODEs* (4.16) and (4.17), we get:

$$\underline{y^{[0]}}(t) = -e^t + 1, \quad \overline{y^{[0]}}(t) = 3e^t - 1,$$
$$\underline{y^{[1]}}(t) = e^t, \quad \overline{y^{[1]}}(t) = e^t.$$

Finally, with substituting above solution in (3.4) we have:

$$\underline{y}(t) = (1-r)(-e^t + 1) + re^t = e^t(2r-1) - r + 1,$$

$$\overline{y}(t) = (1-r)(3e^t - 1) + re^t = r - e^t(2r-3) - 1,$$

and

$$\widetilde{y}(t) = [e^t(2r-1) - r + 1, r - e^t(2r-3) - 1],$$

where $y(t)$ has valid level sets for $t > 0$ and $y(t)$ is (I)-differentiable on $t > 0$, then $y(t)$ is a solution for original problem on $t > 0$.

Case II. Suppose that $\widetilde{y}(t)$ is (II)-differentiable. The exact solution of above system is:

$$\widetilde{y}(t) = [e^t - r + (2r-2)/e^t + 1, r + e^t - (2r-2)/e^t - 1].$$

Based on *ODEs* (3.8) and (3.9), we have:

$$\begin{cases} \underline{y^{[0]}}'(t) = \overline{y^{[0]}}(t) + 1, \\ \overline{y^{[0]}}'(t) = \underline{y^{[0]}}(t) - 1, \\ \underline{y^{[0]}}(0) = 0, \\ \overline{y^{[0]}}(0) = 2, \end{cases} \tag{4.16}$$

$$\begin{cases} \underline{y^{[1]}}'(t) = \overline{y^{[1]}}(t), \\ \overline{y^{[1]}}'(t) = \underline{y^{[1]}}(t), \\ \underline{y^{[1]}}(0) = 1, \\ \overline{y^{[1]}}(0) = 1. \end{cases} \tag{4.17}$$

By solving $ODEs$ (4.16) and (4.17), we get:

$$\underline{y^{[0]}}(t) = e^t - r + (2r-2)/e^t + 1,$$
$$\overline{y^{[0]}}(t) = r + e^t - (2r-2)/e^t - 1,$$
$$\underline{y^{[1]}}(t) = e^t, \quad \overline{y^{[1]}}(t) = e^t.$$

Finally, with substituting above solution in (3.4) we have:

$$\underline{y}(t) = (1-r)(e^t - r + (2r-2)/e^t + 1) + re^t - (1/2)e^{-t}) = e^t - r + (2r-2)/e^t + 1,$$

$$\overline{y}(t) = (1-r)(r + e^t - (2r-2)/e^t - 1) + re^t = r + e^t - (2r-2)/e^t - 1$$

and

$$\widetilde{y}(t) = [e^t - r + (2r-2)/e^t + 1, r + e^t - (2r-2)/e^t - 1].$$

where $y(t)$ has valid level sets for $[0, log(2)]$ and $y(t)$ is (II)-differentiable on $t > 0$, then $y(t)$ is a solution for the original problem on $[0, log(2)]$.

Example 4.2 *Let consider the following FDE:*

$$\begin{cases} y'(t) = -y(t), \\ y(0; r) = [1+r, 5-r], \quad 0 \le r \le 1. \end{cases} \tag{4.18}$$

Case I. Suppose that $\widetilde{y}(t)$ is (I)-differentiable. The exact solution of above system is:

$$\widetilde{y}(t) = [3e^{-t} + (r-2)e^t, 3e^{-t} - (r-2)e^t].$$

Based on $ODEs$ (3.11) and (3.12), we get:

$$\begin{cases} \underline{y^{[0]}}'(t) = -\overline{y^{[0]}}(t), \\ \overline{y^{[0]}}'(t) = -\underline{y^{[0]}}(t), \\ \underline{y^{[0]}}(0) = 1, \\ \overline{y^{[0]}}(0) = 5, \end{cases} \tag{4.19}$$

$$\begin{cases} \underline{y^{[1]}}'(t) = -\overline{y^{[1]}}(t), \\ \overline{y^{[1]}}'(t) = -\underline{y^{[1]}}(t), \\ \underline{y^{[1]}}(0) = 2, \\ \overline{y^{[1]}}(0) = 4. \end{cases} \tag{4.20}$$

By solving $ODEs$ (4.21) and (4.22), we get:

$$\underline{y^{[0]}}(t) = 3e^{-t} - 2e^t, \quad \overline{y^{[0]}}(t) = 3e^{-t} + 2e^t,$$
$$\underline{y^{[1]}}(t) = 3e^{-t} - e^t, \quad \overline{y^{[1]}}(t) = 3e^{-t} + e^t.$$

Finally, with substituting above solution in (3.4) we have:

$$\underline{y}(t) = (1-r)(3e^{-t} - 2e^t) + r(3e^{-t} - e^t) = 3e^{-t} + (r-2)e^t,$$

$$\overline{y}(t) = (1-r)(3e^{-t} + 2e^t) + r(3e^{-t} + e^t) = 3e^{-t} - (r-2)e^t$$

and

$$\widetilde{y}(t) = [3e^{-t} + (r-2)e^t, 3e^{-t} - (r-2)e^t],$$

where $y(t)$ has valid level sets for $t > 0$ and $y(t)$ is (I)-differentiable on $t > 0$, then $y(t)$ is a solution for the original problem on $t > 0$.

Case II. Suppose that $\widetilde{y}(t)$ is (II)-differentiable. The exact solution of above system is:

$$\widetilde{y}(t) = [(1+r)e^{-t}, (5-r)e^{-t}].$$

Based on $ODEs$ (3.11) and (3.12), we get:

$$\begin{cases} \underline{y^{[0]}}'(t) = -\underline{y^{[0]}}(t), \\ \overline{y^{[0]}}'(t) = -\overline{y^{[0]}}(t), \\ \underline{y^{[0]}}(0) = 1, \\ \overline{y^{[0]}}(0) = 5, \end{cases} \tag{4.21}$$

$$\begin{cases} \underline{y^{[1]}}'(t) = -\underline{y^{[1]}}(t), \\ \overline{y^{[1]}}'(t) = -\overline{y^{[1]}}(t), \\ \underline{y^{[1]}}(0) = 2, \\ \overline{y^{[1]}}(0) = 4. \end{cases} \tag{4.22}$$

By solving $ODEs$ (4.21) and (4.22), we get:

$$\underline{y^{[0]}}(t) = e^{-t} \ , \ \overline{y^{[0]}}(t) = 5e^{-t} \ ,$$
$$\underline{y^{[1]}}(t) = 2e^{-t} \ , \ \overline{y^{[1]}}(t) = 4e^{-t}.$$

Finally, with substituting above solution in (3.4) we have:

$$\underline{y}(t) = (1-r)e^{-t} + 2re^{-t} = (1+r)e^{-t},$$

$$\overline{y}(t) = 5(1-r)e^{-t} + 4re^{-t} = (5-r)e^{-t}$$

and

$$\widetilde{y}(t) = [(1+r)e^{-t}, (5-r)e^{-t}],$$

where $y(t)$ has valid level sets for $t > 0$ and $y(t)$ is (II)-differentiable on $t > 0$, then $y(t)$ is a solution for original problem on $t > 0$.

5 Conclusion

A new method for solving first order fuzzy differential equations FDE with fuzzy initial value under strongly generalized H-differentiability was considered. The idea of the presented approach was constructed based on the extending 0-cut and 1-cut solution of original FDE. First, under H-differentiability the solutions of fuzzy differential equations in 0-cut and 1-cut cases were found then convex combination of them was considered. Then we found out the initial interval that 0-cut of original problem was positive. The obtained convex combination on this interval was a solution of FDE. Using 0-cut and 1-cut solutions we show that the discussed method can be applied to solve the fuzzy differential equation.

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