Reflection and transmission in swelling porous media

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Abstract

The present investigation is concerned with the reflection and transmission of plane waves at a interface between two swelling porous elastic half-spaces in welded contact. It is found that their exist two longitudinal waves and two transverse waves. The expression for various amplitude ratios due to the incidence of longitudinal wave in solid (PS)/longitudinal wave in fluid (PF)/ transverse wave in solid (SVS)/ transverse wave in fluid (SVF) have been obtained and depicted graphically. It is found the amplitude ratios of reflected and transmitted waves are the functions of angle of incidence and are affected by the swelling porosity of the media. Surface response at the interface have also been obtained. The results so obtained have been compared with (without swelling porous) elastic medium (EL). The present investigation has immense application in structural problems.

Keywords: Amplitude ratios, longitudinal waves, transversal waves.

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1 Introduction

Dynamic analysis of theories of porous media is a subject with application in various branches of geophysics, civil and mechanical engineering. Based on the work of Von

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Terzaghi [1,2], Biot [3] proposed a general theory of three dimensional deformations of fluid saturated porous elastic solids. Subsequently, Biot [4,5,6,7] presented the models for describing the dynamic behaviour of fluid saturated porous media. He examine both high and low frequency limits and showed the existence of two longitudinal waves and one shear wave, which are dispersive and dissipative. Biot theory was based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and the basis for subsequent analysis in acoustic, geophysics and other such fields. Based on the Fillunger model [8], (which is further based on the concept of volume fractions combined with surface porosity coefficients), Bowen [9], Boer and Ehlers [10,11] and Ehlers[12] developed and used another interesting theory in which all the constituents of a porous medium are assumed of soil; solid constituents are incompressible and liquid constituents which are generally water or oils are also incompressible.

It is accepted that the swelling of soils, drying of fibres, wood, paper, plants etc. are problems concerning porous media theory. Swelling Porous medium (material) is a Porous material that swells (shrinks) upon whetting (drying). Eringen [13] pointed out the importance of theories of mixtures to the applied field of swelling porous elastic soils as a continuum theory of mixtures for porous elastic solids filled with fluid and gas. This theory provides a fundamental basis for the study of various practical problem in field of slurries, oil exploration and industry, since oil is viscous and is accompanied by gas in underground rocks. The swelling of porous materials are attributed to a wide range of forces: osmotic repulsive force, brownian motion, hydration forces, Van der Waals forces and electro-static forces [14,15]. Bofill and Quintanilla [16] discussed the problem of Anti-plane shear deformations of swelling porous elastic soils in case of fluid saturation or gas saturation.

Gales [17] investigated the spatial behavior of solutions describing harmonic vibrations of right cylinder in the isothermal linear theory of swelling porous elastic soils. Gales [18] investigated some theoretical problems concerning waves and vibra-
tions within the context of the isothermal linear theory of swelling porous elastic soils with fluid, or gas saturation. He gave two representations of the solutions of field equations. He used these results to study plane harmonic waves, Rayleigh waves and to construct fundamental solutions for steady vibrations. Tersa and Bennethum [19], Kleintelten, Park and Cushman [20] studied various problems on swelling porous elastic soils. Tomar and Gogna [21,22] studied the reflection and refraction of wave at an interface between two micropolar elastic solids in welded contact. Kumar and Singh [23,24,25] studied some problems on reflection and transmission of waves.

The exact nature beneath the earth surface is not known. For the purpose of theoretical investigation about the earth interior one has to consider various appropriate model. The problem of waves and their reflection is very useful to understand the internal structure of earth and to explore various useful material in form of rocks buried inside the earth, for example mineral and crystals etc.

In the present paper we have studied the reflection and transmission of plane waves in swelling porous elastic field at the boundary surface. The amplitude ratios of reflected waves and transmitted waves are computed and shown graphically. Surface response due to the incidence of longitudinal and transversal waves in case of solid and fluid are obtained at the interface and shown graphically. The results so obtain are compared with (without swelling porous) elastic medium (EL).

2 Basic equations

Following Eringen [13], the field equations in linear theory of swelling porous elastic soils are

\[
\mu \mu s_{ij,jj} + (\lambda + \mu) \mu s_{j,ji} - \sigma f \mu f_{j,ji} + \xi f f (\dot{u}^f_{i} - \dot{u}^s_{i}) + \xi^f_{i} f = \rho^s_0 \ddot{u}^s_{i}, \tag{1}
\]

\[
\mu f \dot{u}^f_{ij,ji} + (\lambda f + \mu f) \dot{u}^f_{j,ji} - \sigma f u^s_{j,ji} - \sigma f f u^f_{j,ji} - \xi f f (\dot{u}^f_{i} - \dot{u}^s_{i}) + \xi^f_{i} f = \rho^f_0 \ddot{u}^f_{i}, \tag{2}
\]

\[
t^s_{ij} = (-\sigma f u^f_{r,r} + \lambda f u^s_{r,r}) \delta_{ij} + \mu (u^s_{i,j} + u^s_{j,i}), \tag{3}
\]

\[
t^f_{ij} = (-\sigma f u^f_{r,r} - \sigma f f u^f_{r,r} + \lambda f \dot{u}^f_{r,r}) \delta_{ij} + \mu f (\dot{u}^f_{i,j} + \dot{u}^f_{j,i}), \tag{4}
\]
for $i,j=1,2,3$, where, the superscripts $s$ and $f$ denote respectively, the elastic solid and the fluid; $u_i^s$ and $u_i^f$ are the displacement components of solid and fluid respectively. The functions $(f_i^s, f_i^f)$ are the body forces, $\rho_0^s, \rho_0^f$ are the densities of each constituent and $\lambda, \mu, \lambda, \mu, \sigma^s, \sigma^f, \sigma^s, \xi^f, \xi^f$ are constitutive constants. Subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate, and a superposed dot denotes time differentiation, $t_{ij}^s, t_{ij}^f$ are the partial stress tensors.

3 Formulation of Problem

We consider two homogenous, isotropic, swelling porous elastic half space medium $M_1$ ($0 < x_3 < \infty$) and medium $M$ ($-\infty < x_3 < 0$). The media are in contact with each other at the plane surface, which we designate as the plane $x_3 = 0$ of the rectangular Cartesian coordinate system $\alpha x_1 x_2 x_3$ with an origin on the surface $x_3 = 0$ and $x_3$ axis pointing vertically into the medium. The complete geometry of the problem is shown in Fig.1. We write all the variables without a hat in the region $x_3 > 0$ (medium $M_1$) and attach a hat to denote the variables in the region $x_3 < 0$ (medium $M$). For two dimensional problem, we assume the displacement vector

\[
\overrightarrow{u}^i = (u_1^i, 0, u_3^i) \quad i = s, f \tag{5}
\]

We define the non-dimensional quantities as

\[
x_1' = \frac{\omega^*}{c_2^*} x_1, \quad x_3' = \frac{\omega^*}{c_2^*} x_3, \quad u_1'^i = \frac{\omega^*}{c_2^*} u_1^i, \quad u_3'^i = \frac{\omega^*}{c_2^*} u_3^i, \quad t_{ij}^s = \frac{t_{ij}^s}{\mu}, \quad t_{ij}^f = \frac{t_{ij}^f}{\mu}, \quad \omega^* = \frac{\xi^f}{\mu},
\]

\[
c_1^* = \frac{\lambda + 2\mu}{\rho_0^s}, \quad c_2^* = \frac{\mu}{\rho_0^s}, \quad t' = \omega^* t, \quad c_2^* = \frac{\mu}{\rho_0^s} \tag{6}
\]

Making use of equations (5)-(6) in equations (1)-(4), in absence of body forces and after supressing primes for convenience, we obtain

\[
u_{i,jj}^s + a_1 u_{j,ji}^s - a_2 u_{j,jj}^f - a_3 (\dot{u}_i^f - \dot{u}_i^s) = \ddot{u}_i^s, \tag{7}
\]

\[
\dot{u}_{i,jj}^f + h_1 \dot{u}_{j,ji}^f - h_2 u_{j,jj}^s - h_3 u_{j,jj}^f - h_4 (\dot{u}_i^f - \dot{u}_i^s) = h_5 \ddot{u}_i^f, \tag{8}
\]


\[ t_{ij}^s = \left( -\frac{\sigma_f}{\mu} u_{rr}^s + \frac{\lambda}{\mu} u_{rr}^s \right) \delta_{ij} + (u_{i,j}^s + u_{j,i}^s), \]

\[ t_{ij}^f = \left( -\frac{\sigma_f}{\mu} u_{rr}^f + \frac{\lambda_{ff}}{\mu} u_{rr}^f \right) \delta_{ij} + \frac{\mu_{ff}}{\mu} (u_{i,j}^f + u_{j,i}^f), \]

(9)

(10)

where,

\[ a_1 = \frac{\lambda + \mu}{\mu}, \quad a_2 = \frac{\sigma_f}{\mu}, \quad a_3 = \frac{\sigma_{ff} c_2^2}{\omega^2 \mu}, \quad h_1 = \frac{\lambda_0 + \mu_0}{\mu_0}, \quad h_2 = \frac{\sigma_f}{\mu_0 \omega^2}, \quad h_3 = \frac{\sigma_{ff}}{\mu_0 \omega^2}. \]

The displacements components \( u_1^s, u_3^s, u_1^f, u_3^f \) are related by the potential functions

\[ u_1^i = \frac{\partial \phi^i}{\partial x_1} - \frac{\partial \psi^i}{\partial x_3}, \quad u_3^i = \frac{\partial \phi^i}{\partial x_3} + \frac{\partial \psi^i}{\partial x_1} \]

(11)

We assume the solutions of the system of equations (7)-(8) in the form

\[ \phi^s, \phi^f, \psi^s, \psi^f = [\phi_1^s, \phi_1^f, \psi_1^s, \psi_1^f] e^{i[k(x_1 \sin \theta - x_3 \cos \theta) - \omega t]} \]

(12)

where \( \kappa \) is the wave number and \( \omega \) is the complex circular frequency. Making use of equation (12) in (7)-(8) we obtain two quadratic equations in \( V^2 \) given by

\[ AV^2 + BV^2 + C = 0 \]

(13)

\[ A_1 V^4 + B_1 V^2 + C_1 = 0 \]

(14)

Where \( V = \omega / \kappa \) is the velocity of the waves: \( V_1, V_2 \) are the velocities of the reflected longitudinal PS and PF waves respectively, given by equation (13), and \( V_3, V_4 \) are the velocities of transverse SVS and SVF waves respectively given by equation (14) where,

\[ A = \frac{i(\text{h}_5 a_3 + h_4)}{\omega} + h_5, \quad C = (-i \omega (1 + h_1) - h_3) (1 + a_1) - h_2 a_2, \quad B_1 = i (\omega - \frac{h_4}{\omega}) - h_5 - a_3, \]

\[ C_1 = -i \omega, \quad B = (1 + h_1)(i \omega - a_3) - (1 + a_1) \gamma_1 + h_3 \gamma_0 + \frac{\omega}{\omega} (h_4 a_2 + h_2 a_3), \quad \gamma_1 = (h_5 + \frac{\omega}{\omega}) \]

\[ A_1 = \frac{a_3}{\omega} (h_5 - h_4) + \frac{\omega}{\omega} (a_3 h_5 + h_4) + h_5, \quad \gamma_0 = (1 + \frac{\omega}{\omega}) \]

4 Reflection and transmission

We consider a longitudinal wave in solid (PS)/longitudinal wave in fluid (PF) /transverse wave in solid (SVS)/ transverse wave in fluid(SVF) propagating through the
medium \( M_1 \) which is designated as the region \( x_3 > 0 \) and incident at the plane \( x_3 = 0 \), with its direction of propagating with angle \( \theta_0 \) normal to the surface. Corresponding to each incident wave, we get reflected PS,PF,SVS,SVF waves and transmitted PS,PF,SVS,SVF waves in medium \( M \) as shown in Fig. 1.

![Fig. 1 Geometry of the problem](image)

5 Boundary Conditions:

The boundary conditions are:

\[
\begin{align*}
(i) t_{33}^s &= t_{33}^s \\
(ii) t_{33}^f &= t_{33}^f \\
(iii) t_{31}^s &= t_{31}^s \\
(iv) t_{31}^f &= t_{31}^f \\
(v) \dot{u}_3^s - \dot{u}_3^s &= 0 \\
(vi) \dot{u}_3^f - \dot{u}_3^f &= 0 \\
(vii) \dot{u}_1^s - \dot{u}_1^s &= 0 \\
(viii) \dot{u}_1^f - \dot{u}_1^f &= 0
\end{align*}
\]

In view of (12), we assume the values of \( \phi^s, \phi^f, \psi^s, \psi^f \) for medium \( M_1 \) and \( \overline{\phi}^s, \overline{\phi}^f, \overline{\psi}^s, \overline{\psi}^f \).
for medium \( \mathcal{M} \) satisfying the boundary conditions as

\[
\begin{align*}
\{\phi^s, \phi^f\} &= \sum_{i=1}^{2} \{1, \eta_i\} [A_{0i} e^{i(\kappa_i x \sin \theta_0 - x_3 \cos \theta_0) - \omega_i t} + P_i] \\ 
\{\psi^s, \psi^f\} &= \sum_{j=3}^{4} \{1, \eta_j\} [B_{0j} e^{i(\kappa_j x \sin \theta_0 - x_3 \cos \theta_0) - \omega_j t} + P_j] \\
\{\bar{\phi}^s, \bar{\phi}^f\} &= \sum_{i=1}^{2} \{1, \eta_i\} [\bar{A}_i e^{i(\kappa_i x \sin \theta - x_3 \cos \theta) - \omega t} + \bar{P}_i] \\
\{\bar{\psi}^s, \bar{\psi}^f\} &= \sum_{j=3}^{4} \{1, \eta_j\} [\bar{B}_j e^{i(\kappa_j x \sin \theta - x_3 \cos \theta) - \omega t} + \bar{P}_j]
\end{align*}
\]

where,

\[
P_i = A_i e^{i(\kappa_i x \sin \theta_0 - x_3 \cos \theta_0) - \omega_i t}, \quad P_j = B_j e^{i(\kappa_j x \sin \theta_0 - x_3 \cos \theta_0) - \omega_j t}, \quad \eta_i = \frac{\omega (1 + a_1) - a_2 V_0 - \omega V_i^2}{a_2 \omega - a_3 V_i^2},
\]

\[
\eta_j = \frac{-\omega + (a_3 + \omega)V_j^2}{a_3 V_j^2}, \quad \bar{\eta}_i = \frac{\omega (1 + \bar{\kappa}_i) - a_2 \bar{V}_0 - \omega \bar{V}_i^2}{a_2 \omega - a_3 \bar{V}_i^2}, \quad \bar{\eta}_j = \frac{-\omega + (a_3 + \omega)\bar{V}_j^2}{a_3 \bar{V}_j^2}, \quad (i = 1, 2; j = 3, 4)
\]

\( A_{0i} \) are the amplitudes of the incident PS wave, PF wave and \( B_{0j} \) are the amplitudes of the incident SVS wave, SVF wave respectively. \( A_i \) are the amplitudes of the reflected PS wave, PF wave and \( B_j \) are the amplitudes of the reflected SVS wave and SVF wave, \( \bar{A}_i \) are the amplitudes of the transmitted PS wave, \( \bar{A}_j \) are the amplitudes of transmitted SVS wave respectively.

In order to satisfy the boundary conditions, the extension of the Snell’s law will be

\[
\frac{\sin \theta_0}{V_0} = \frac{\sin \theta_1}{V_1} = \frac{\sin \theta_2}{V_2} = \frac{\sin \theta_3}{V_3} = \frac{\sin \bar{\theta}_1}{\bar{V}_1} = \frac{\sin \bar{\theta}_2}{\bar{V}_2} = \frac{\sin \bar{\theta}_3}{\bar{V}_3} = \frac{\sin \bar{\theta}_4}{\bar{V}_4}
\]

where

\[
V_r = \frac{\omega}{k_r}, \quad \bar{V}_r = \frac{\omega}{\bar{k}_r}, \quad (r = 1, 2, 3, 4)
\]

\[
\kappa_1 V_1 = \kappa_2 V_2 = \kappa_3 V_3 = \kappa_4 V_4 = \pi_1 V_1 = \pi_2 V_2 = \pi_3 V_3 = \pi_4 V_4 = \omega \quad \text{at} \ x_3 = 0
\]

Making use of potentials given by Eqs. (15)-(18) in boundary conditions, we obtain a system of eight non-homogeneous equations which can be written as

\[
\sum_{i,j=1}^{8} a_{ij} Z_j = Y_i
\]
where,
\[
\begin{align*}
a_{1i} &= \left( \frac{\sigma^2_i}{\mu} \eta_i - \frac{1}{\mu} \right) \frac{2(\frac{V_i}{V_0})^2}{\pi} \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \kappa_i^2, \\
a_{1j} &= -2\kappa_j^2 \sin \theta_j \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}, \\
a_{1k} &= -2\pi \kappa_j^2 \sin \theta_j \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}, \\
a_{1l} &= -2\pi \kappa_j^2 \sin \theta_j \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}, \\
a_{2i} &= \left( \frac{\sigma^2_i}{\mu} \eta_i + \frac{\sigma^2_i}{\mu} \eta_i \right) \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \kappa_i^2, \\
a_{2j} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{2k} &= -\left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{2l} &= -\left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{3i} &= -2\kappa_i^2 \sin \theta_i \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}, \\
a_{3j} &= -2\kappa_j^2 \sin \theta_j \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}, \\
a_{3k} &= -2\kappa_j^2 \sin \theta_j \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}, \\
a_{3l} &= -2\kappa_j^2 \sin \theta_j \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}, \\
a_{4i} &= \left( \sin \theta_i \theta_j \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_i^2, \\
a_{4j} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{4l} &= \left( \sin \theta_i \theta_j \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_i^2, \\
a_{5i} &= \left( \sin \theta_i \theta_j \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_i^2, \\
a_{5j} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{5k} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{5l} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{6i} &= \left( \sin \theta_i \theta_j \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_i^2, \\
a_{6j} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{6k} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{6l} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{7i} &= \left( \sin \theta_i \theta_j \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_i^2, \\
a_{7j} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{7k} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{7l} &= \left( \sin \theta_j \theta_i \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_j^2, \\
a_{8i} &= \left( \sin \theta_i \theta_j \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \kappa_i^2, \\
Z_1 &= \frac{A_1}{\mu}, \\
Z_2 &= \frac{A_2}{\mu}, \\
Z_3 &= \frac{A_3}{\mu}, \\
Z_4 &= \frac{A_4}{\mu}, \\
Z_5 &= \frac{A_5}{\mu}, \\
Z_6 &= \frac{A_6}{\mu}, \\
Z_7 &= \frac{A_7}{\mu}, \\
Z_8 &= \frac{A_8}{\mu}.
\end{align*}
\]

Considering the phase of the reflected waves for the incident PS,PF,SVS,SVF waves can be written by using equations (19) and (20) as
\[
\begin{align*}
\cos \frac{\theta_i}{V_i} &= \frac{2}{V_0} \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}, \\
\cos \frac{\theta_j}{V_j} &= \frac{1}{V_0} \left( \frac{V_j}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}.
\end{align*}
\]

Following [26] if we write
\[
\cos \frac{\theta_i}{V_i} = \cos \frac{\theta_i}{V_i} + t \cos \frac{\theta_i}{V_i} \pi
\]

then
\[
\begin{align*}
\cos \frac{\theta_i}{V_i} &= \frac{1}{V_0} \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2}, \\
cr &= 2\pi Im \left( \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right)
\end{align*}
\]

where \( V_r' \), the real phase speed and \( \theta_r' \), the angle of reflection are given by
\[
\begin{align*}
\frac{V_r'}{V_0} &= \frac{\sin \theta_r'}{\sin \theta_0} + \left[ Re \left( \left( \frac{V_i}{V_0} \right)^2 (\sin^2 \theta_0 - \sin^2 \theta_0) \frac{1}{2} \right) \right] \frac{1}{2},
\end{align*}
\]

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and \( c_r \), the attenuation in a depth is equal to the wavelength of incident wave i.e. \( \frac{2\pi v_0}{\omega} \).

5.1 For incident PS-wave:

\[
A^* = A_{01}, A_{02} = B_{03} = B_{04} = 0
\]

\[
Y_1 = -a_{11}, Y_2 = -a_{21}, Y_3 = a_{31}, Y_4 = a_{41}, Y_5 = -a_{51}, Y_6 = -a_{61}, Y_7 = a_{71}, Y_8 = a_{81},
\]

5.2 For incident PF-wave:

\[
A^* = A_{02}, A_{01} = B_{03} = B_{04} = 0
\]

\[
Y_1 = -a_{12}, Y_2 = -a_{22}, Y_3 = a_{32}, Y_4 = a_{42}, Y_5 = -a_{52}, Y_6 = -a_{62}, Y_7 = a_{72}, Y_8 = a_{82},
\]

5.3 For incident SVS-wave:

\[
A^* = B_{03}, A_{01} = A_{02} = B_{04} = 0
\]

\[
Y_1 = a_{13}, Y_2 = a_{23}, Y_3 = -a_{33}, Y_4 = -a_{43}, Y_5 = a_{53}, Y_6 = a_{63}, Y_7 = -a_{73}, Y_8 = -a_{83},
\]

5.4 For incident SVF-wave:

\[
A^* = B_{04}, A_{01} = A_{02} = B_{03} = 0
\]

\[
Y_1 = a_{14}, Y_2 = a_{24}, Y_3 = -a_{34}, Y_4 = -a_{44}, Y_5 = a_{54}, Y_6 = a_{64}, Y_7 = -a_{74}, Y_8 = -a_{84},
\]

6 Surface response

The following responses of the solid and liquid constituents at the interface \( x_3 = 0 \) of two medium is calculated.
6.1 For incident PS-wave:

6.1.1 Solid constituent:

The expressions for the displacements are given by

1) \( u_3^s = \overline{w_3} \) at \( x_3 = 0 \) i.e.

\[
(\kappa_1 (1 + Z_1) \sin \theta_1 + \kappa_2 \sin \theta_2 Z_2 - \kappa_3 \cos \theta_3 Z_3 - \kappa_4 \cos \theta_4 Z_4 - \kappa_1 \sin \theta_1 Z_1 - \kappa_2 \sin \theta_2 Z_2 - \kappa_3 \cos \theta_3 Z_3 - \kappa_4 \sin \theta_4 Z_4 ) i A_0 \exp \{ i \kappa_0 x_1 \}
\]

2) \( u_4^s = \overline{w_4} \) at \( x_3 = 0 \) i.e.

\[
(\kappa_1 (1 + Z_1) \cos \theta_1 + \kappa_2 \cos \theta_2 Z_2 + \kappa_3 \sin \theta_3 Z_3 + \kappa_4 \sin \theta_4 Z_4 + \kappa_1 \cos \theta_1 Z_1 + \kappa_2 \cos \theta_2 Z_2 - \kappa_3 \sin \theta_3 Z_3 - \kappa_4 \sin \theta_4 Z_4 ) i A_0 \exp \{ i \kappa_0 x_1 \}
\]

6.1.2 Liquid constituent:

The expression for the displacement of the liquid constituent are given by

1) \( u_3^l = \overline{w_3} \) at \( x_3 = 0 \) i.e.

\[
(\kappa_1 (1 + Z_1) \eta_1 \cos \theta_1 + \kappa_2 \eta_2 \cos \theta_2 Z_2 + \kappa_3 \eta_3 \sin \theta_3 Z_3 + \kappa_4 \eta_4 \sin \theta_4 Z_4 + \kappa_1 \eta_1 \cos \theta_1 Z_1 + \kappa_2 \eta_2 \cos \theta_2 Z_2 - \kappa_3 \eta_3 \sin \theta_3 Z_3 - \kappa_4 \eta_4 \sin \theta_4 Z_4 ) i A_0 \exp \{ i \kappa_0 x_1 \}
\]

2) \( u_4^l = \overline{w_4} \) at \( x_3 = 0 \) i.e.

\[
(\eta_1 \kappa_1 (1 + Z_1) \sin \theta_1 + \eta_2 \kappa_2 \sin \theta_2 Z_2 - \eta_3 \kappa_3 \cos \theta_3 Z_3 - \eta_4 \kappa_4 \cos \theta_4 Z_4 - \eta_1 \kappa_1 \sin \theta_1 Z_1 - \eta_2 \kappa_2 \sin \theta_2 Z_2 - \eta_3 \kappa_3 \cos \theta_3 Z_3 - \eta_4 \kappa_4 \cos \theta_4 Z_4 ) i A_0 \exp \{ i \kappa_0 x_1 \}
\]

where,

\[
\kappa_1 \sin \theta_{01} = \kappa_1 \sin \theta_1 = \kappa_2 \sin \theta_2 = \kappa_3 \sin \theta_3 = \kappa_4 \sin \theta_4 = \kappa_1 \sin \theta_1 = \kappa_2 \sin \theta_2 = \kappa_3 \sin \theta_3 = \kappa_4 \sin \theta_4 = \kappa_0
\]

6.2 For incident PF-wave:

6.2.1 Solid constituent:

The expressions for the displacements are given by

1) \( u_3^p = \overline{w_3} \) at \( x_3 = 0 \) i.e.

\[
(\kappa_1 \cos \theta_1 Z_1 + \kappa_2 \cos \theta_2 (1 + Z_2) + \kappa_3 \sin \theta_3 Z_3 + \kappa_4 \sin \theta_4 Z_4 + \kappa_1 \cos \theta_1 Z_1 + \kappa_2 \cos \theta_2 Z_2 -
\]

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The expressions for the displacements are given by

$$\kappa_3 \sin \theta_3 Z_3 - \kappa_4 \sin \theta_4 Z_4) \nu A_{02} \exp \{ \nu \kappa_0 x_1 \}$$

2) \( u_1^0 = \overline{w}_1 \) at \( x_3 = 0 \) i.e.

\((\kappa_1 Z_1 \sin \theta_1 + \kappa_2 \sin \theta_2 (1 + Z_2) - \kappa_3 \cos \theta_3 Z_3 - \kappa_4 \cos \theta_4 Z_4 - \kappa_1 \sin \theta_1 Z_1 - \kappa_2 \sin \theta_2 Z_2 - \kappa_3 \cos \theta_3 Z_3 - \kappa_4 \cos \theta_4 Z_4) \nu A_{02} \exp \{ \nu \kappa_0 x_1 \} \)

### 6.2.2 Liquid constituent:

The expression for the displacement of the liquid constituent are given by

1) \( u_4^0 = \overline{w}_4 \) at \( x_3 = 0 \) i.e.

\((\kappa_1 \eta_1 \cos \theta_1 Z_1 + \kappa_2 \eta_2 \cos \theta_2 (-1 + Z_2) + \kappa_3 \eta_3 \sin \theta_3 Z_3 + \kappa_4 \eta_4 \sin \theta_4 Z_4 + \kappa_1 \eta_1 \cos \theta_1 Z_1 + \kappa_2 \eta_2 \cos \theta_2 Z_2 - \kappa_3 \eta_3 \sin \theta_3 Z_3 - \kappa_4 \eta_4 \sin \theta_4 Z_4) \nu A_{02} \exp \{ \nu \kappa_0 x_1 \} \)

2) \( u_1^0 = \overline{w}_1 \) at \( x_3 = 0 \) i.e.

\((\eta_1 \kappa_1 \sin \theta_1 Z_1 + \eta_2 \kappa_2 \sin \theta_2 (1 + Z_2) - \eta_3 \kappa_3 \cos \theta_3 Z_3 - \eta_4 \kappa_4 \cos \theta_4 Z_4 - \eta_1 \kappa_1 \sin \theta_1 Z_1 - \eta_2 \kappa_2 \sin \theta_2 Z_2 - \eta_3 \kappa_3 \cos \theta_3 Z_3 - \eta_4 \kappa_4 \cos \theta_4 Z_4) \nu A_{02} \exp \{ \nu \kappa_0 x_1 \} \)

where,

\( \kappa_2 \sin \theta_{02} = \kappa_1 \sin \theta_1 = \kappa_2 \sin \theta_2 = \kappa_3 \sin \theta_3 = \kappa_4 \sin \theta_4 = \kappa_1 \sin \theta_1 = \kappa_2 \sin \theta_2 = \kappa_3 \sin \theta_3 = \kappa_4 \sin \theta_4 = \kappa_{02} \)

### 6.3 For incident SVS-wave:

#### 6.3.1 Solid constituent:

The expressions for the displacements are given by

1) \( u_3^0 = \overline{w}_3 \) at \( x_3 = 0 \) i.e.

\((\kappa_1 \cos \theta_1 Z_1 + \kappa_2 \cos \theta_2 Z_2 + \kappa_3 \sin \theta_3 (1 + Z_3) + \kappa_4 \sin \theta_4 Z_4 + \kappa_1 \cos \theta_1 Z_1 + \kappa_2 \cos \theta_2 Z_2 - \kappa_3 \sin \theta_3 Z_3 - \kappa_4 \sin \theta_4 Z_4) \nu A_{03} \exp \{ \nu \kappa_0 x_1 \} \)

2) \( u_1^0 = \overline{w}_1 \) at \( x_3 = 0 \) i.e.

\((\kappa_1 Z_1 \sin \theta_1 + \kappa_2 \sin \theta_2 Z_2 + \kappa_3 \cos \theta_3 (1 - Z_3) - \kappa_4 \cos \theta_4 Z_4 - \kappa_1 \sin \theta_1 Z_1 - \kappa_2 \sin \theta_2 Z_2 - \kappa_3 \cos \theta_3 Z_3 - \kappa_4 \cos \theta_4 Z_4) \nu A_{03} \exp \{ \nu \kappa_0 x_1 \} \)
6.3.2 Liquid constituent:

The expression for the displacement of the liquid constituent are given by
1) \( u_3^I = \bar{w}_3^I \) at \( x_3 = 0 \) i.e.
\[
(\kappa_1 \eta_1 \cos \theta_1 Z_1 + \kappa_2 \eta_2 \cos \theta_2 Z_2 + \kappa_3 \eta_3 \sin \theta_3 (1 + Z_3) + \kappa_4 \eta_4 \sin \theta_4 Z_4 + \kappa_1 \eta_1 \cos \theta_1 Z_1 + \kappa_2 \eta_2 \cos \theta_2 Z_2 - \kappa_3 \eta_3 \sin \theta_3 Z_3 - \kappa_4 \eta_4 \sin \theta_4 Z_4) \exp(i \kappa_0 x_1)
\]
2) \( u_4^I = \bar{w}_4^I \) at \( x_3 = 0 \) i.e.
\[
(\eta_1 \kappa_1 \sin \theta_1 Z_1 + \eta_2 \kappa_2 \sin \theta_2 Z_2 + \eta_3 \kappa_3 \cos \theta_3 (1 - Z_3) - \eta_4 \kappa_4 \cos \theta_4 Z_4 - \eta_1 \kappa_1 \sin \theta_1 Z_1 - \eta_2 \kappa_2 \sin \theta_2 Z_2 - \eta_3 \kappa_3 \cos \theta_3 Z_3 - \eta_4 \kappa_4 \cos \theta_4 Z_4) \exp(i \kappa_0 x_1)
\]
where,
\[
\kappa_3 \sin \theta_03 = \kappa_1 \sin \theta_1 = \kappa_2 \sin \theta_2 = \kappa_3 \sin \theta_3 = \kappa_4 \sin \theta_4 = \kappa_1 \sin \theta_1 = \kappa_2 \sin \theta_2 = \kappa_3 \sin \theta_3 = \kappa_4 \sin \theta_4 = \kappa_03
\]

6.4 For incident SVF-wave:

6.4.1 Solid constituent:

The expressions for the displacements are given by
1) \( u_3^B = \bar{w}_3^B \) at \( x_3 = 0 \) i.e.
\[
(\kappa_1 \cos \theta_1 Z_1 + \kappa_2 \cos \theta_2 Z_2 + \kappa_3 \sin \theta_3 Z_3 + \kappa_4 \sin \theta_4 (1 + Z_4) + \kappa_1 \cos \theta_1 Z_1 + \kappa_2 \cos \theta_2 Z_2 - \kappa_3 \sin \theta_3 Z_3 - \kappa_4 \sin \theta_4 Z_4) \exp(i \kappa_0 x_1)
\]
2) \( u_4^B = \bar{w}_4^B \) at \( x_3 = 0 \) i.e.
\[
(\kappa_1 \sin \theta_1 Z_1 + \kappa_2 \sin \theta_2 Z_2 - \kappa_3 \cos \theta_3 Z_3 + \kappa_4 \cos \theta_4 (1 - Z_4) - \kappa_1 \sin \theta_1 Z_1 - \kappa_2 \sin \theta_2 Z_2 - \kappa_3 \cos \theta_3 Z_3 - \kappa_4 \cos \theta_4 Z_4) \exp(i \kappa_0 x_1)
\]

6.4.2 Liquid constituent:

The expression for the displacement of the liquid constituent are given by
1) \( u_3^B = \bar{w}_3^B \) at \( x_3 = 0 \) i.e.
\[
(\kappa_1 \eta_1 \cos \theta_1 Z_1 + \kappa_2 \eta_2 \cos \theta_2 Z_2 + \kappa_3 \eta_3 \sin \theta_3 Z_3 + \kappa_4 \eta_4 \sin \theta_4 (1 + Z_4) + \kappa_1 \eta_1 \cos \theta_1 Z_1 + \kappa_2 \eta_2 \cos \theta_2 Z_2 - \kappa_3 \eta_3 \sin \theta_3 Z_3 - \kappa_4 \eta_4 \sin \theta_4 Z_4) \exp(i \kappa_0 x_1)
\]
\[ \kappa_2 \eta_2 \cos \varphi_2 Z_2 - \kappa_3 \eta_3 \sin \varphi_3 Z_3 - \kappa_4 \eta_4 \sin \varphi_4 Z_4 \] \] \[ \alpha_{04} \exp \{ \iota \kappa_{04} x_1 \} \]

2) \[ u_1^f = \bar{u}_1^f \] at \[ x_3 = 0 \] i.e.

\[ (\eta_1 \kappa_1 \sin \vartheta_1 Z_1 + \eta_2 \kappa_2 \sin \vartheta_2 Z_2 - \eta_3 \kappa_3 \cos \vartheta_3 Z_3 + \eta_4 \kappa_4 \cos \vartheta_4 (1 - Z_4)) - \eta_1 \kappa_1 \sin \varphi_1 Z_1 - \eta_2 \kappa_2 \sin \varphi_2 Z_2 - \eta_3 \kappa_3 \cos \varphi_3 Z_3 - \eta_4 \kappa_4 \cos \varphi_4 Z_4 \] \[ \alpha_{04} \exp \{ \iota \kappa_{04} x_1 \} \]

where,

\[ \kappa_4 \sin \vartheta_{04} = \kappa_1 \sin \vartheta_1 = \kappa_2 \sin \vartheta_2 = \kappa_3 \sin \vartheta_3 = \kappa_4 \sin \varphi_1 = \kappa_2 \sin \varphi_2 = \kappa_3 \sin \varphi_3 = \kappa_4 \sin \varphi_4 = \kappa_{04} \]

and \[ V_0 = \begin{bmatrix} V_1 & \text{for incident PS - wave} \\ V_2 & \text{for incident PF - wave} \\ V_3 & \text{for incident SVS - wave} \\ V_4 & \text{for incident SVF - wave} \end{bmatrix} \]

7 Numerical results and discussion

In order to illustrate theoretical results obtained in the preceding sections, we now present some numerical results. For numerical computation, the physical data is given below:

\[ \lambda = 2.238 \times 10^{10} N/m^2, \mu = 2.992 \times 10^{10} N/m^2, \lambda_\nu = 2.05 \times 10^{10} NSec/m^2, \]
\[ \mu_\nu = 2.5 \times 10^{10} NSec/m^2, \sigma = 1.42 \times 10^{10} N/m^2, \sigma_{ff} = 1.75 \times 10^{10} N/m^2, \]
\[ \rho_0^s = 2.65 \times 10^3 NSec^2/m^4, \rho_0 f = 1.92 \times 10^3 NSec^2/m^4, \xi_{ff} = 1.745 \times 10^3 NSec/m^4, \]
\[ \bar{\lambda} = 0.91 \times 10^{10} N/m^2, \bar{\mu} = 1.11 \times 10^{10} N/m^2, \bar{\lambda}_\nu = 1.15 \times 10^{10} NSec/m^2, \]
\[ \bar{\mu}_\nu = 1.29 \times 10^{10} NSec/m^2, \bar{\sigma} = 0.7 \times 10^{10} N/m^2, \bar{\sigma}_{ff} = 0.5 \times 10^{10} N/m^2, \]
\[ \bar{\rho}_0^s = 1.25 \times 10^3 NSec^2/m^4, \bar{\rho}_0 f = 0.12 \times 10^3 NSec^2/m^4, \bar{\xi}_{ff} = 0.1 \times 10^3 NSec/m^4 \]

The solid line with centre symbols represent the swelling porous medium whereas small dashes line represent the elastic medium (EL). Variations of amplitude ratios \[ |Z_s| \] \[ s=1,2,3,4,5,6,7,8 \] with the angle of incidence \[ \theta_0 \] for different waves are shown in Figs. 2-5. Fig. 6-9 depicts the surface response at the interface \[ x_3 = 0 \]
7.1 Reflection and Transmission

7.1.1 PS Wave

Fig. 2 depicts the variation of amplitude ratio $|Z_s|$ due to the incidence of PS wave in SP and EL media. From the figure we notice that amplitude ratio $|Z_s|$ are of oscillatory behaviour in SP medium. In EL medium amplitude ratio of reflected part of PS and SVS waves remains less than the amplitude ratio $|Z_5|$ and $|Z_7|$. In region $1 \leq \theta_0 \leq 64$ amplitude ratio $|Z_1|$ is less than the amplitude ratio of reflected part of PS wave in EL medium. For $\theta_0 \geq 65$ amplitude ratio $|Z_1|$ becomes greater than the amplitude ratio of reflected part of PS wave in EL medium. In SP medium the amplitude ratio $|Z_1|$ of reflected part of PS wave remains less than the reflected part of PS wave in EL medium. In SP medium amplitude ratio $|Z_5|$ and $|Z_6|$ remains greater than the amplitude ratio $|Z_1|$ and $|Z_2|$ respectively.

7.1.2 PF Wave

Fig. 3 depicts the variation of amplitude ratio due to the incidence of PF wave. From the figure, we notice that amplitude ratio $|Z_1|$ decrease with angle of incidence. Amplitude ratio $|Z_2|$, $|Z_3|$, $|Z_4|$ is of oscillatory behaviour. The amplitude ratio $|Z_5|$ decreases with increase in angle of incidence. The amplitude ratio $|Z_6|$ initially increase then decrease with increase in angle of incidence and remains greater than the amplitude ratio $|Z_2|$ in region $1 \leq \theta_0 \leq 77$. For $1 \leq \theta_0 \leq 33$ amplitude ratio $|Z_7|$ remains less than the amplitude ratio $|Z_3|$. The amplitude ratio $|Z_8|$ remains less than the amplitude ratio $|Z_6|$ in whole region.

7.1.3 SVS Wave

In SP medium amplitude ratio $|Z_s|$ decrease with angle of incidence. Amplitude ratio $|Z_2|$ remains less than the amplitude ratio $|Z_1|$ in whole region. For $\theta_0 \geq 13$, amplitude ratio $|Z_4|$ remains less than that of $|Z_3|$. Amplitude ratio $|Z_5|$ remains greater than
the reflected part $|Z_1|$. Amplitude ratio $|Z_6|$ remains less than the $|Z_2|$. In EL medium amplitude ratio of reflected part of PS wave initially increase then decrease with angle of incidence, whereas amplitude ratio of reflected part of SVS waves increase with angle of incidence. In EL medium amplitude ratio of transmitted part of SVS wave increase with angle of incidence and remains greater than the amplitude ratio of transmitted part of PS wave and reflected part of SVS wave.

### 7.1.4 SVF Wave

Amplitude ratio of reflected part of PS, PF, SVS, SVF waves initially oscillates then increase with angle of incidence. Amplitude ratio $|Z_7|$ remains greater than the amplitude ratio $|Z_3|$. Initially amplitude ratio $|Z_1|$ is less than amplitude ratio $|Z_5|$, then for $\theta \geq 6$ it becomes greater than the amplitude ratio of transmitted part of PS wave i.e. $|Z_5|$. Amplitude ratio $|Z_3|, |Z_4|$ remains greater than the amplitude ratio $|Z_1|, |Z_2|$ respectively.

### 7.2 Surface Response

Figure 6, depicts the variation of surface response with angle of incidence in SP and EL media, due to the incidence of PS wave. From the figure, we notice that in SP medium surface response of $u_{s1}, u_{s3}, u_{f1}, u_{f3}$ are of oscillatory behaviour. In SP medium surface response of $u_{f1}'$ remains less than the surface response of $u_{s1}'$ in whole range, whereas surface response of $u_{f3}'$ remains greater than that of $u_{s3}'$ in whole range. In EL medium surface response of $u_{s1}'$ remains less than the surface response of $u_{s1}'$ in SP medium. The surface response of $u_{s3}'$ in EL medium remains less than the surface response of $u_{s3}'$ in SP medium in range $\theta_0 \geq 26$.

Fig. 7 depicts the variation of surface response due to the incidence of PF wave. From the figure, we notice that surface response of $u_{s1}', u_{s3}', u_{f3}'$ are of oscillatory behaviour, whereas surface response of $u_{f1}'$ increase with angle of incidence. Surface response of tangential and normal fluid constituent remains greater than the surface response of
solid tangential and normal constituent respectively.

Fig. 8 depicts the variation of surface response in SP and EL media due to the incidence of SVS wave. From the figure, we notice that surface response are of oscillatory behaviour in SP and EL media. In SP medium surface response of normal constituent remains greater than the tangential constituent in case of solid and fluid. Surface response of \( u_1^f \) remains greater than the \( u_1^s \) in \( 1 \leq \theta_0 \leq 43 \) for \( \theta_0 \geq 44 \) surface response of \( u_1^f \) becomes less than the surface response of \( u_1^s \). In EL medium surface response of \( u_1^s \) remains less than the surface response of \( u_3^s \) of SP medium. The surface response of \( u_3^s \) of EL medium remains less than the surface response of \( u_3^s \) of SP medium except for the range \( 46 \leq \theta_0 \leq 64 \).

Fig. 9 depicts the variation of surface response due to the incidence of SVF wave. From the figure we notice that surface response of \( u_1^s, u_3^s, u_1^f, u_3^f \) initially oscillates then increase with angle of incidence. Surface response of \( u_1^f \) remains greater than the surface response of \( u_1^s \) in whole range. For \( 1 \leq \theta_0 \leq 34 \) surface response of \( u_3^f \) remains less than the surface response of \( u_3^s \), then for \( \theta_0 \geq 35 \), surface response of \( u_3^f \) remains greater than the surface response of \( u_3^s \). Surface response of \( u_3^f \) remains less than the surface response of \( u_1^f \) in whole range. For \( \theta_0 \geq 29 \), the surface response of \( u_3^s \) remains greater the surface response of \( u_1^s \).

8 Conclusion:

From the above figures we notice that when PS wave is incident the oscillation of amplitude ratios in EL medium is less than that of SP medium. The amplitude ratio \( |Z_1| \) remains less than the amplitude ratio of reflected part of PS wave in EL medium. When PF wave is incident amplitude ratios \( |Z_2|, |Z_3|, |Z_4| \) oscillates, whereas amplitude ratio \( |Z_1| \) decrease with angle of incidence. In SP medium amplitude ratio decreases with increase in angle of incidence, whereas in EL medium amplitude ratio of reflected part of PS and SVS wave increase with angle of incidence due to the incidence of SVS.
wave. When SVF wave is incident amplitude ratios initially oscillates and then increase rapidly with increase in angle of incidence. When PS wave is incidence surface response in SP medium is greater than EL medium. When PF wave is incident surface response of relative tangential displacement of fluid constituent at interface is greater than the relative tangential and normal displacement of solid constituent. When SVS wave is incident then surface response of relative tangential displacement in case of solid and fluid remains close to each other. When SVF wave is incident surface response of fluid constituent remains greater than the solid constituent.

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Variation of amplitude ratio with angle of incidence when PS wave is incident
Variation of amplitude ratio with angle of incidence when PF wave is incident
Variation of amplitude ratio with angle of incidence when SVS wave is incident

Variation of amplitude ratio with angle of incidence when SVF wave is incident
Variation of surface response when PS wave is incident

Variation of surface response when PF wave is incident
Variation of surface response when SVF wave is incident

Variation of surface response when SVF wave is incident