Simplified Dynamic Analysis of Sloshing in Rectangular Tanks with Multiple Vertical Baffles

Mahmood Hosseini¹*, Hamidreza Vosoughifar2, Pegah Farshadmanesh3

¹*. Associate Professor, International Institute of Earthquake Engineering and Seismology, Tehran, Iran, and Part Time Faculty Memner in Graduate School, Civil Engineering Department, Islamic Azad University- South Tehran Branch, Tehran, Iran.
2. Assistant Professor, Department of Civil Engineering, Islamic Azad University- South Tehran Branch, Tehran, Iran
3. Graduate M. Sc., Department of Civil Engineering, Islamic Azad University- South Tehran Branch, Tehran, Iran

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ABSTRACT

Sloshing is a well-known phenomenon in liquid storage tanks subjected to base or body motions. Up to now the use of multiple vertical baffles for reducing the sloshing effects in tanks subjected to earthquake has not been taken into consideration so much. On the other hand, although some of the existing computer programs are able to model sloshing phenomenon with acceptable accuracy, the full dynamic analysis subjected to random excitations, such as earthquake induced motions, is very time consuming. In this paper a method is presented for reducing the analysis duration based on first, conducting several dynamic analysis cases by using ANSYS-CFX for rectangular tanks of various dimensions, subjected to seismic excitations, and then, using neural network to create simple relationships between the dominant frequency and amplitude of the base excitations and the maximum level of liquid in the tank during the sloshing. The numerical modeling has been verified by using some existing experimental data, and several cases of time history analysis have been conducted to obtain the required numerical results for training a neural network. Finally, the predicted results of the neural network have been compared to those obtained by some other cases of analyses as control values.

Keywords
Multiple vertical baffle; Sloshing phenomenon; Time history analysis; Neural Network

1. Introduction

One of the most important phenomena in fluid storage tanks, including buried, semi-buried, above-ground, and elevated is the oscillation of fluid because of the tank motions during an external excitation, which is called ‘sloshing’. This phenomenon has been taken into consideration since mid 50s by Miles (1956).

Past earthquakes have shown that this phenomenon can result in severe damages to water storage tanks, and many researchers have studied this issue and the studies have been continued till recent years by Chen et al (2009). They have shown that partially filled tanks with fluid are prone to violent sloshing under certain dynamic condition. For example, when the frequency of the
tank motion is close to the natural frequency of sloshing, due to interaction between fluid and structure, the enhanced fluid motion creates localized high impacts to the tank wall(s) and roof which can cause structural damage.

To prevent tanks against sloshing, the use of baffles have been suggested and studied by some researchers since mid 60s such as Abramson (1965). To reduce sloshing effects due to earthquake only a few studies have been conducted. As one of the first works in this regard, Shaaban et al. (1977) studied the response of a partially filled liquid-storage circular cylindrical tank with or without an interior cylindrical baffle under seismic actions using Finite Element (FE) method.

Gedikli and Ergüven (1999) studied on the seismic analysis of a liquid storage cylindrical tank with a rigid baffle. They implemented the method of superposition of modes to compute the seismic response, and used the boundary element method to evaluate the natural modes of liquid in the tank. Linearized free surface conditions were taken into consideration in that study.

Yasuki et al. (2000) conducted a study on suppression of seismic sloshing in cylindrical tanks with baffle plates. The purpose of the study was to propose the evaluation model of damping characteristics for cylindrical tank with ring baffle plates.

Maleki and Ziyaefar (2007) studied on damping enhancement of seismic isolated cylindrical tanks using baffles. They claimed that in moving liquid containers, baffles play an important role in damping the liquid motion. They also analyzed velocity contours in a cylindrical tank to determine the most effective shape of the baffles.

Wu (2010) has conducted a thorough study on the nonlinear liquid sloshing in a 3D tank with baffles, in which the mechanism of liquid sloshing and the interaction between the fluid and internal structures have been investigated. They considered the effects of excitation angle on the characteristics of sloshing waves, especially for swirling waves. In that study the spectral analyses of sloshing displacement of various sloshing waves were examined and a clear evidence of correlation between sloshing wave patterns and resonant modes of sloshing waves were demonstrated.

The results also show that sloshing displacement is affected distinctly by different numbers of baffles mounted vertically on the tank bottom. The more baffles mounted onto the tank bottom, the smaller the sloshing displacement is presented in both the transient and steady-state periods. Wu has categorized the processes of the evolution of vortices near the baffle tip into four phases. Results show that vortex shedding phenomenon due to stronger vertical jets occurs when the excitation frequency is close to the first natural mode of the baffled tank, and that is discussed and the size of vortex, generated near the baffle tip, is closely correlated with the baffle height. In that study two types of 3D tuned liquid dampers, a vertically tank bottom-mounted baffle and a vertical plate, were discussed for a tank under coupled surge-sway motions. Results show that the wave types of diagonal and single-directional waves switch to the swirling type due to the influence of the baffle. The phenomenon of square-like waves or irregular waves coexisting with swirling waves is found in the baffled tank under diagonal excitation. The shift of the first
natural mode of the baffled tank due to various baffle heights is remarkable. The influence of the vertical plate on the irregular waves is insignificant and several peaks appear in the spectral analysis of the sloshing displacement for the irregular waves and the numbers of peaks are more than that of the baffled tank.

To develop the three-dimensional modeling, Liu and Lin (2009) developed a 3D two-phase fluid flow model which solves the SANS equations and is developed to simulate the liquid sloshing in both baffled and unbaffled tanks.

A series of experiments have been carried out in a developed liquid sloshing set up in tanks with and without baffles by Panigrahy et al. (2009). On their study the pressures at the walls indicated a considerable fluctuation near the free surface of the liquid compared to the deeper surface in the tank. They also concluded that ring baffles are more effective as compared to the conventional horizontal baffles. Numerical simulations were carried out based on volume of the fluid techniques with arbitrary- Lagrangian- Eulerian by Eswaran et al. (2009).

During the last decade, many researchers have devoted their efforts to study sloshing analytically based on potential flow theory. For example, Choun and Yun (1999) analyzed the effects of a bottom-mounted rectangular block on the sloshing characteristics of the Liquid in rectangular tanks using the small amplitude wave theory.

Isaacson and Premasiri (2001) predicted the hydrodynamic damping due to baffles inside the tank and also estimated the total energy damping due to flow separation around the baffles. Studies of Kim et al. (2002), Akyildiz (2012) and Jung et al. (2012) are of such investigations.

It was mentioned in the literature review that the analysis of baffled tanks is generally very complicated and time consuming. This fact has been insisted by Serdar Celebi & Akyildiz (2002) and Hasheminejad and Aghabeigi (2009), respectively. So it is clear that multiple baffles make the behavior of the liquid inside the tank more complicated, and accordingly makes the analysis much more difficult and time consuming. In this study a simplified method for evaluation of sloshing effects in rectangular tanks with Multiple Vertical Baffles (MVB) is presented. The method is based on conducting several dynamic analysis cases for tanks with various dimensions subjected to seismic excitations, and the use of neural network to create simple relationships between the dominant frequency and the amplitude of the base excitations and the maximum level of liquid in the tank. Details of the study are discussed in the following section of the paper.

1. Modeling process and verification

Modeling procedure is presented briefly in this section. It should be noted that the created damping due to the existence of baffles greatly depends on the dimensions and position of the installed baffles (see Figure 1). In this article, to achieve maximum effects of reducing sloshing, baffles have been considered to be connected to the roof with equal distances (L1). Number of the baffles is the only parameter that it is needed to be considered.
For the first stage of the investigation, a rigid rectangular tank which is 1 meter high, 0.4 meter wide, and 0.96 meter long under harmonic base excitation of $x = D \sin(\omega t)$ is considered; where $D$ is horizontal displacement amplitude of the input motion, and $\omega$ is the excitation frequency.

In order to verify the numerical modeling, results of some experimental tests conducted at the Hydraulic Institute of Stuttgart University on shake-table done by Goudarzi et al. are considered. The length, height, and width of the liquid volume inside the tank were 0.96, 0.64, and 0.40 meters, respectively (see Figure 2). To verify the results of the numerical analysis, results of the experimental modeling studied by Goudarzi and his colleagues, subjected to sinusoidal base excitations $U_b(t) = U_0 \sin(\omega_1 t), U_0 = 5cm$ are considered.

Comparing the results of the experimental and numerical study, good agreement between numerical results obtained by the CFX model, and the experimental results can be seen. $\omega_1$ is the excitation frequency in the first sloshing mode (see Figure 3), and the frequency less than first sloshing mode in the second verification (see Figure 4). Based on this verification, the employed ANSYS-CFX modeling process could be used for more a detailed analysis of sloshing in tanks as is explained in next sections.

![Fig. 1. The scaled-down tank model on the shake-table](image1)

![Fig. 2. The scaled-down tank model on the shake-table](image2)
2. Analytical solution

The mathematical formulation of the lateral sloshing for a rigid tank subjected to harmonic excitation in irrotational, incompressible and inviscid fluid flow is developed by satisfying Laplace equation, Eq. (1):

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$  \hspace{1cm} (1)

Where $\Phi$ is a velocity potential function. This equation is needed to satisfy the boundary conditions. By solving Eq. (1), the slosh wave shape and the natural frequencies are found from Eq. (2) as follow:

$$\vartheta(x, t) = -\frac{2BF}{a\omega_n}(2n - 1)$$  \hspace{1cm} (2)

$$\sinh \left[ \pi(2n - 1) \left( \frac{h}{a} \right) \right]$$

$$\sin \left[ \pi(2n - 1) \left( \frac{z}{a} \right) \right]$$

Where $h$ and $a$ are water depth and tank width, respectively (Dodge & Franklin, 2000).

3. Selected Tanks for the CFD Analyses

The typical double-compartment above ground water tanks which are used in water supply systems in Iran were used in this study.

To have the minimum length of the tank’s wall (to minimize the amount of the required construction material) for a given tank area in the case of double-compartment
tanks (see Figure 5), it can be easily shown that "b" parameter should be around "1.5a". Also the water depth in the scaled tank, "h" is usually considered around 5 to 6 meters in real size. It should not be so small in the scaled model to prevent special forces such as surface tension.

The common specifications of the tanks with different water capacities based on the above conditions are as shown in Table 1.

The values of the fundamental sloshing periods of the tanks in Table 1 have been calculated based on the Equation (3). \( \omega_n \) is the natural angular frequencies of sloshing modes in tanks:

\[
\omega_n^2 = \pi(2n - 1) \left( \frac{2}{a} \right) \tanh[\pi(2n - 1) \left( \frac{h}{a} \right)]
\]

(3)

Where \( n \) is the sloshing mode number and \( g \) is the gravity acceleration. Based on the above explanations and considering the exponentially growth of the required computational time with the number of elements and the time step size in the time history analysis, it was decided to use scaled models. By using some appro-priate scaling factors, reduced dimensions can be used for tanks of real size.

The scaling requirements are explained here in after, along with the presentation of numerical results. Regarding that the effects of using MVB is the main concern in this study; the base excitation and accordingly the induced sloshing have been assumed to occur in just one main direction of the tank length. On this basis, it was important to know if the tank's width, which is the dimension in the direction perpendicular to the excitation direction, has any effect on the analyses results. For this purpose, various values were considered for the "b" parameter, and by using a specific excita-tion the analysis was repeated. Studies show that the tank dimension perpendicular to the excitation direction does not have any major effect on the response values if the excitation is just in one main direction of the tank plan (see Figure 6). On this basis, in all analyses cases a constant value of 0.1 m was used for the tank’s width to reduce the required time for the analysis.

<table>
<thead>
<tr>
<th>Tank Capacity (m³)</th>
<th>Water Height h (m)</th>
<th>a (m)</th>
<th>b=1.5a</th>
<th>h/a</th>
<th>h/b</th>
<th>T (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>3.0</td>
<td>5.270</td>
<td>7.905</td>
<td>0.75</td>
<td>0.569</td>
<td>2.619</td>
</tr>
<tr>
<td>250</td>
<td>3.0</td>
<td>7.453</td>
<td>11.180</td>
<td>0.53</td>
<td>0.402</td>
<td>3.197</td>
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<td>1000</td>
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<td>22.360</td>
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<td>10000</td>
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<td>0.136</td>
<td>9.502</td>
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<td>15000</td>
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<td>63.960</td>
<td>0.17</td>
<td>0.128</td>
<td>10.523</td>
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<td>73.854</td>
<td>0.14</td>
<td>0.111</td>
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<td>30000</td>
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<td>57.73</td>
<td>86.602</td>
<td>0.13</td>
<td>0.103</td>
<td>13.434</td>
</tr>
</tbody>
</table>
Another important factor which affects the required time for the response analysis is the size of the tank. In fact, the experience gained in this study showed that the required analysis time for a scaled-down model of a tank is several times less than that of the real tank.

The main reason behind this fact lies on the size of the time step, which should be used for a scaled-down tank. Actually, considering that based on Eq. (4), the sloshing frequencies vary inversely with variation of the square root of the tank’s length, shown in the equation by "a". It can be easily seen that the sloshing period in a scaled model, \( T_m \), is related to the sloshing period in the prototype tank, \( T_p \), by:

\[
\frac{T_m}{T_p} = \frac{L_m}{L_p} \sqrt{L_m \cdot L_p}
\]  

(4)

Where \( L_m \) and \( L_p \) are the length of the scaled model tank and that of the prototype tank, respectively. On this basis it is clear that the sloshing period in a scaled-down model with the length of 1/36 (for example) of the real size tank will be 6 times shorter than the sloshing period in the prototype. This means that the size of the time step of the earthquake digitized record, considered for analyzing the scaled-down model should be also scaled down by the same factor of 6 to keep the proportions of the excitation periods with respect to the sloshing period in the prototype. Accordingly, the duration of the record used for the scaled-down model will be 6 times shorter than the real record, although the number of the time steps is the same as the original record. It is clear that using a much shorter time step in time history analysis leads to a much higher convergence rate, which in turn reduces the required analysis time to a great extent. On this basis, it was decided to use a scaled-down model tank with the length of 1.0 meter, which is almost 1/36 of a tank with 10000 m\(^3\) capacity, as shown in Table 1. This capacity is related to a very common set of tank features, as shown in Table 1, with 5.0 m water height and plan dimensions of 36.5 m by 54.8 m.

4. Sloshing Response to Seismic Excitations and the Effect of Using MVB

To investigate the effect of using MVB in the sloshing response in tanks when subjected to seismic excitations, and establishing a
reasonable relationship between the number of the baffles and their features as well as the seismic excitation’s characteristics as the input, and the maximum water level variation as the output, some earthquake records were considered based on their frequency content and were applied with various scales. The earthquake records were selected by considering the sloshing frequencies of tanks with real size, which are generally low for tanks with common sizes and they were considered as shown in Table1. As an example, three earthquakes used in numerical study will be shown. Kocaeli (Sakaraya), Chichi (CHY 101N), Chichi (CHY002 (N)) are three example of seismic excitation subjected to the tanks with different capacity.

It is seen that selected earthquakes have relatively high energy in the range of long periods, and can excite well the sloshing modes in relatively large tanks with sloshing periods of larger than 9 seconds (see Figs. 7 and 8). All of these earthquakes have long period oscillations in their displacement history in the period range of 6 to 10 seconds.

Fig. 7. Acceleration and displacement time histories of Kocaeli earthquake (1999)

Fig. 8. Pseudo-velocity response spectra of the Kocaeli earthquake (1999)
As mentioned before, variation of water level beside either the tank wall or the baffle(s) is a good response value for studying the sloshing phenomenon and the effect of using baffles on it. Therefore, these variations corresponding to the aforementioned earthquakes, which show response to Kocaeli earthquake in cases of using no baffle comparing to the cases of using 1, 2, or 3 baffle(s), response to Chichi (CHY 101N) earthquake in case of using no baffle compared to the cases of using 2 or 3 baffles, and finally response to Chichi (CHY002 (N)) earthquake in case of using no baffle comparing to the cases of 1 or 3 baffles, respectively. It is seen that using baffles generally leads to a decrease in the water level variations (the maximum water rising), and using more baffles results in more decrease in water level rising (see Figs. 9, 10, and 11). Therefore, 2 baffles are recommended to be used in tanks of the sizes around the size of the studied tank.

Displacement histories of these earthquakes when scaled-down by a factor of 1/6 were decreased. To reduce the required analysis time only the 6 seconds of the strong ground motion parts of the scaled records were used in time history analyses.

![Graph showing water level variations](image1)

**Fig. 9.** Water level variations beside the tank wall with different number of baffles, when subjected to the scaled seismic record of Kocaeli earthquake (1999)

![Graph showing water level variations](image2)

**Fig. 10.** Water level variations beside the tank wall with different number of baffles, when subjected to the scaled seismic record of Chi-Chi earthquake (Chy101-1999)
5. Training the Neural Network for Predicting the Sloshing Response

To train a neural network for prediction of the sloshing response to earthquake excitations, the results of six earthquakes were used. The values of pseudo-velocities corresponding to the 1\textsuperscript{st}, 2\textsuperscript{nd}, and 3\textsuperscript{rd} sloshing modes in the tank along with the number of the baffles and the length of tanks were used as the input data, and the ratio of water level variation was used as the output data.

Considering a neural network with one intermediate or hidden layer, the network was trained (see Figure 12) [21]. After training the neural network, to test its capability in response prediction, another earthquake (Chi-Chi, Chy101 station) was considered.

The results obtained by the trained neural network and the results of the numerical model are compared (see Figure 13). It can be seen that the trained neural network can predict the sloshing response with satisfactory precision.
6. Conclusions

Based on the numerical analyses of the tanks in this study, and the proposed neural network response prediction method it can be concluded that:

- Using scaled down numerical models of tanks, which results in shorter time steps and accordingly shorter durations for time history analyses, leads to significant reduction in the required analysis time.
- When the excitation exists in one of the main directions of the rectangular tank, the width of the tank model can be chosen as small as 10 cm. This also will lead to a reduction of the required analysis time.
- Using 2 to 4 vertical baffles, equally spaced along the rectangular tanks can reduce the sloshing effect to a great extent as much as 50%.
- The proposed neural network can be used for predicting the sloshing response in tanks with satisfactory precision, and therefore, it is recommended that this approach would be used in studying the sloshing problem of tanks instead of time history analysis, which is a very time consuming process.

Nomenclatures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>G</td>
<td>acceleration of gravity</td>
</tr>
<tr>
<td>T\textsubscript{m}</td>
<td>sloshing period in a model (Sec)</td>
</tr>
<tr>
<td>T\textsubscript{p}</td>
<td>sloshing period in the prototype tank (Sec)</td>
</tr>
<tr>
<td>L\textsubscript{m}</td>
<td>length of the model tank (m)</td>
</tr>
<tr>
<td>L\textsubscript{p}</td>
<td>length of the prototype tank (m)</td>
</tr>
<tr>
<td>U\textsubscript{b}(t)</td>
<td>Sinusoidal base excitation (m)</td>
</tr>
<tr>
<td>U\textsubscript{0}</td>
<td>Horizontal displacement amplitude</td>
</tr>
<tr>
<td>\omega\textsubscript{n}</td>
<td>natural angular frequencies of sloshing modes (Sec)</td>
</tr>
<tr>
<td>A</td>
<td>Width of the tank (m)</td>
</tr>
<tr>
<td>B</td>
<td>Length of the tank (m)</td>
</tr>
<tr>
<td>H</td>
<td>Water depth (m)</td>
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Subscripts

<table>
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<th>Subscript</th>
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<tr>
<td>m</td>
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<td>p</td>
<td>Prototype</td>
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<tr>
<td>n</td>
<td>Mode number</td>
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References

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