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TECHNICAL NOTE

Modeling of the Capacitated Single Allocation Hub Location Problem with a Hierarchical Approch

M. Karimi [a], A. R. Eydi [b], E. Korani* [b]

[a] *Department of Industrial engineering, Najafabad Branch, Islamic Azad University, Isfahan, Iran*
[b] *Department of Industrial engineering, University of Kurdistan, Sanandaj, Iran*

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ABSTRACT

The hierarchical hub covering facility location problems are applied to distribution systems, transportation, waste disposal, treatment services, emergency services and remote communication. The problems attempt to determine the location of service providers' facilities at different levels and specify their linking directions in order to reduce costs and to establish an appropriate condition in distribution network. By utilizing these problems, the present paper attempts to allocate "capacitated" option to each provider service and consequently establish and choose the best possible condition, so that demands centers are rationally and effectively guided by service providers' centers and their request never remains without response. To do this, the model "the capacitated single allocation hierarchical hub median location problem" is developed, created and provided. In addition, considering of the increasing demand, modulating choices are addressed in order to fulfill the future needs and to impose uncertainty in decision making in the results. To validate the model, we used IAD data, which the results confirm its consistency.

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1. INTRODUCTION

Hub location problems attempt to consider hub locating and to allocate demand nods to created hub facilities. In a single allocation P-hub median problem, among potential hub nodes, P-hub frequency is chosen and each demand point (non-hub nodes) is exactly allocated to a single hub. They mainly aim to reduce routing costs and to minimize the cost of hubs' establishment. The number of the hub nodes (e.g. P) is already determined and the hubs are directly linked. This problem has a finite space and a discrete structure. Its application is summarized in two general domains transportation and communication. O'kelly [1] first provided p-hub location problem as a mathematical model by studying airline networks. Campbell [2] provided the first mathematical formulation for single allocation P-hub median problem, and then a few years later presented a linear model for solving P-hub locating problem in a network with N node. Skorin-Kapov et al. [3] presented a new mixed integer formulation for this problem. Ernst and Krishnamoorthy provided another different integer linear formulation for solving larger problems which needed less constraints and variables. In addition, it was shown that how Australian Post Office uses different values of α discount for distribution and collection network [4]. To further study, Campbell's [5] and Alumur and Kara's [6] works are suggested. Hierarchical hub problems are one issue raised in the hub location problem.

Chen et al. [7] examined the strategic design of delivery networks which could efficiently provide these services. Because of the high cost of direct connections, they focused on tree-structured networks and established the complexity of the problem. They also exploited an empirically identified solution structure to create new neighborhoods which improved solution values over more general neighborhood structures.

Labbe and Yaman [8] considered the problem of locating hubs and assigning terminals to hubs for a telecommunication network. The hubs were directly connected to a central node and each terminal node is directly connected to a hub node. Their aim were to minimize the cost of locating hubs, assigning terminals

*Corresponding Author Email: ehsan_korani@yahoo.com (E. Korani)

and routing the traffic between hubs and the central node. They presented two formulations and showed that the constraints are facet-defining inequalities in both cases.

Wagner [9] proposed new model formulations for hub covering problems and it was improved. This model discussed multiple and single allocation problems, including non-increasing quantity-dependent transport time functions for transport links for the latter case.

Yoon and Current [10] introduced a new hub location and network design formulation that considered the fixed costs of establishing the hubs and the arcs in the network. In addition, the variable costs associated with the demands on the arcs. The problem was formulated as a mixed integer programming problem embedding a multi-commodity flow model [10]. Alumur and Kara [11] focused on cargo applications of the hub location problem. They proposed a new mathematical model for the hub location problem that was relaxed the complete hub network assumption. Their model minimized the cost of establishing hubs and hub links and formulated a single-allocation hub covering model that permuted visiting at most three hubs on a route [11]. Alumur et al. [12] provided a uniform modeling treatment to all the single allocation variants of the existing hub location problems, under the incomplete hub network design. So that, they defined the single allocation incomplete p-hub median, the incomplete hub location with fixed costs, the incomplete hub covering, and the incomplete p-hub center network design problems [12]. Calik et al. [13] studied the single allocation hub covering problem over incomplete hub networks and proposed an integer programming formulation. The aim of their model was to find the location of hubs. The hub links to be established between the located hubs and the allocation of non-hub nodes to the located hub nodes such that the travel time between any origin–destination pair was within a given time bound. They presented an efficient heuristic based on tabu search and test the performance of their heuristic on some well-known data sets. Campbell [14] provided time definite models for multiple allocation p-hub median problems and hub arc location problems. Service levels were imposed by limiting the maximum travel distance via the hub network for each origin–destination pair. Contreras et al. [15] considered the tree of hub location problem and proposed a four index formulation which yields much tighter LP bounds than previously proposed formulations.

Ernst et al. [16] studied the uncapacitated hub center problems with either single or multiple allocation. Both problems were proved to be NP-hard. So, they presented the integer programming formulations for both problems and proposed a branch-and-bound approach for solving the multiple allocation case [16]. Limbourg and Jourquin [17] were used from a set of estimated potential locations as input for an iterative procedure based on the p-hub median problem that took the variation in trans-shipment costs according to the number of trans-shipped containers into account [17]. Meyer et al. [18] presented an exact 2-phase algorithm where in the first phase, and computed a set of potential optimal hub combinations used a shortest path based branch and bound. They developed a heuristic for the single allocation p-hub center problem based on an ant colony optimization approach [18]. Sim et al. [19] presented the stochastic p-hub center problem with chance constraints, which they used to model the service-level guarantees. Contreras et al. [20] presented the Tree of Hubs Location Problem. It was a network hub location problem with single assignment where a fixed number of hubs have to be located, with the particularity which it was required that the hubs were connected by means of a tree. The problem combined several aspects of location, network design and routing problems. They proposed an integer programming formulation for the problem [20]. Lin [21] studied the integrated hierarchical hub-and-spoke network design problem for dual services. It integrated otherwise mutually exclusive secondary route networks for their respective services so that the total operating cost was minimized while meeting the service time and operations restrictions. A directed network configuration and formulate a link-based integer mathematical model was proposed [21]. Meng and Wang [22] developed a mathematical program with equilibrium constraints (MPEC) model for the intermodal hub-and-spoke network design (IHSND) problem with multiple stakeholders and multi-type containers. Their model incorporated a parametric variational inequality (VI) that formulated the user equilibrium (UE) behavior of intermodal operators in route choice for any given network design decision of the network planner. In addition, this model used a cost function that was capable of reflecting the transition from scale economies to scale diseconomies in distinct flow regimes for carriers or hub operators [22]. Yaman [23] studied allocation strategies and their effects on total routing costs in hub networks. Given a set of nodes with pairwise traffic demands, the p-hub median problem was the problem of chosen p nodes as hub locations and routing traffic through these hubs at minimum cost. This problem had two versions; in single allocation problems, each node could send and receive traffic through a single hub, whereas in multiple allocation problems, there was no such restriction and a node might send and receive its traffic through all p hubs. Furthermore, models for variations of this problem with service quality considerations, flow thresholds, and non-stop service was presented [23].

Ishfaq and Sox [24] developed a mathematical model used the multiple-allocation p-hub median approach. The model encompassed the dynamics of individual modes of transportation through transportation costs, modal connectivity costs, and fixed location costs under service time requirements. A tabu search meta-heuristic was used to solve large size (100 nodes) problems.

Alumur et al. [25] introduced the multimodal hub location and hub network design problem. They approached the hub location problem from a network design perspective. In addition, to the location and allocation decisions, they also studied the decision on how the hub networks with different possible transportation modes must been designed. In this multimodal hub location and hub network design problem, they jointly considered transportation costs and travel times, which were studied separately in most hub location problems presented in the literature. They first proposed a linear mixed integer programming model for this problem and then derive variants of the problem that might arise in certain applications.

Alumur et al. [26] discussed about of Hierarchical multimodal hub location problem. This paper deals with time bound deliveries between pair origin-destination, so that they ensured each demand give the service in the time bound. In addition, they showed that the locations of airport hubs was less sensitive to the cost parameters compared to the locations of ground hubs and it was possible to improve the service quality at not much additional cost in the resulting multimodal networks [26].

Elmastas [27] first observed a three-level cargo delivery network in Turkey and modeled its three-level structure in a hub median problem in order to minimize the cost of hubs establishment. In Elmastas's paper, two types of hub facilities are presented, so that, they performed their activities in a network with three level structure. Each level of network include some demand nodes and hub facilities, where the first level links central hubs with a complete network, the second level assigns remaining hubs to the central hubs through star networks and the third level joins the demand nodes to the hubs with star networks [28]. Then, Yaman [28] developed the model and considering the objective function of routing cost rather than establishment cost, developed single allocation P-hub median problem (SA-TH-HM). In addition, by creating a complete network linking between first level hubs, Elmastas star model was converted into a complete network structure [28]. The hub location problems can be divided into two main parts, capacitated and non-capacitated. In the literature on single allocation hub locating problems, the capacitated structure of service providers' centers and their links is considered to create a more realistic situation. Alumur et al. [6] classified and surveyed the network hub location models. They introduced some recent trends on hub location and provided a synthesis of the literature [6]. They also observed that the capacitated hub location problem has two well-known structures: single allocation and multiple allocations. They differ in how non-hub nodes are allocated to hubs. In single allocation, all the incoming and outgoing traffic of every demand center is routed through a single hub; in multiple allocations, each demand center can receive and send flow through more than one hub [6].

There are many papers for the single allocation capacitated hub location problem, some of them as follow; the first linear integer formulation for the single allocation capacitated hub location problem presented by Campbell [2].

Aykin [29] introduced the new version of the capacitated hub and spoke network problem with objective function of fixed charge costs. Then, the lower bounds are obtained by lagrangean relaxation, and presented a branch-and-bound algorithm to solve it [30].

Jaillet et al. [30] designed capacitated hub location models for airline networks. They proposed three basic integer linear programming models, each corresponding to a different service policy and presented heuristic schemes based on mathematical programming [30].

Ernst and Krishnamoorthy [31] presented two new formulations for the capacitated single allocation hub location problem. Their formulations are a modified version of the previous mixed integer formulations developed for the p-hub median problem [31].

Labbe´ et al. [32] investigated some polyhedral properties of the single allocation capacitated hub location problem and developed a branch-and-cut algorithm based on these results.

A different approach to the capacitated single allocation hub location problem is presented by Costa et al. [33]. They are introduced two bi-criteria single allocation hub location problems: in a first model, total time is considered as the second criteria and, in a second model, the maximum service time for the hubs are minimized [33].

Tavakkoli et al. [34] presented a novel multi-objective mathematical model for capacitated single allocation hub location problem and solved it by a multi-objective imperialist competitive algorithm (MOICA). Finally, to prove its efficiency, the related results are compared with the results obtained by the well-known multi-objective evolutionary algorithm, called NSGA-II [34]. However, this model has been designed for one level of service, too.

In addition, there are many papers for the multiple allocations capacitated hub location problem, some of them as follow. Ebery et al. [35] considered formulations and solution approaches for the capacitated multiple allocation hub location problems. They presented a new mixed integer linear programming formulation for the problem. They also constructed an efficient heuristic algorithm, using shortest paths [35].

Sasaki et al. [36] presented a new formulation of one-stop capacitated hub-and-spoke model as a natural extension of the uncapacitated one-stop model. They also introduced a branch-and-bound based exact solution method with Lagrangian relaxation bounding strategy, and report some results of numerical experiments using real aviation data [36].

Boland et al. [37] suggested a new formulations and solution approaches for multiple allocation hub location problems. They employed flow cover constraints for capacitated problems to improve computation times [37].

Yaman [38] studied the uncapacitated hub location problem with modular arc capacities. Then, Yaman and Carello [39] extended Yaman's [38] model, and the amount of traffic passing through the hub specified as a hub's capacity.

There are many papers in the capacitated hub location literature, but all of them are designed for systems with one level of services. So, this paper presents a new model with more than real situation in the world. Many hub location problems are designed for transportation system, cargo delivery systems, telecommunication network system, production-distribution systems and etc. [6]. However, all of them have been noticed by one level of services. Another hand, regarding the applied field of the hierarchical problem, the following cases could be mentioned: Solid waste management systems, Production-distribution systems, Education systems, Emergency medical service (EMS), Telecommunication network systems, Cargo delivery, Health care systems and other relevant ones above are examined by Sahina and Sural [40] which is a comprehensive research on hierarchical problems. Nevertheless, the capacitated hub location problems are designed without hierarchical structure for the hierarchical systems in the real world. Thus, we designed a new single allocation capacitated hub location problem with a nested hierarchical structure to be closer to the real situation.

Single allocation hub location problem in which hubs become capacitated are identified as capacitated single allocation hub locating problem (CSAHLP) with one level of service. Campbell [28] presented a mixed integer primary linear programming for CSAHLP. Ernst and Krishnamoorthy [31] extended the formulation presented by Kapov et al. [3] for problem's capacitated and non-capacitated version. They also presented a new mix integer linear programming (MILP) formulation which is an adaptation of CSAHLP related to the formulation by same authors for the non-capacitated hub median problem. Labbe et al. [32] conducted CSAHLP by imposing capacity on the flow links passing each hub and proposed a branch-and-bound algorithm. Yaman and Carrello [39] investigated hub location problem with modular link capacities. Only the implementation cost in problem was considered. In addition, in a study by Yaman [41], hub locations and links were identified to reduce costs in hub median problem with regard to capacity for arcs. He also considered two formulations and an innovative algorithm for solving problem in which the solving methods quality was compared. Correia et al. [42] revised and modified one of the most famous formulations presented in the literature on capacitated single allocation hub location problem. Presenting an example, they showed that the old formulation in the balance limitations (constraints) balance suffers from some disadvantages. Therefore, by adding cut constraints, they concluded that their formulation plays a critical role in imposing properly capacity conditions on the base problem and also it helps to reduce solution time. In the literature of the hierarchical, hub location problems are not significantly considered about of capacity. As a result, Yaman and Elloumi's [43] contribution is as the last work on this issue presented two hierarchical problems and focused on the quality of services rather than the capacity. They mainly attempted to reduce the length of the longest route and to minimize total routing cost. However, the present paper attempts to combine the hierarchical hub location problem and the capacitated hub problem. Of course, the model design has been in a manner that with the smallest variations in the costs of network establishment and solving times, it has been achieved to the optimal result, which the results are more close to real conditions of the world. These conditions allocate demand nodes to service provider facilities in a manner that they do not encounter a non-responding state at different levels and a minimized total cost is achieved. The main novelties of our works are mention as follow:

1) We attempted to combine the hierarchical hub location problem and the capacitated hub location problem. Because, many companies and complex systems are used the hierarchical structure in their activities; so, we tried to create a new model for this real situation.
2) Our model is one type of the capacitated hub location model, but the capacitated hub location models with one level of service are revised and modified in the literature by Correia et al. [42]. Thus, we extended Correia's idea from "the capacitated single allocation hub location problem with one level of service" to "the capacitated single allocation hub location problem with the hierarchical structure".

In order to achieve this, this paper presents the capacitated single allocation hub location problem in section 2. Section 3 presents the computational results of the model using Iranian Airport Data (IAD) in order to confirm the performance of the proposed model. The results of the present study and future research are presented in section 4.

2. PROBLEM FORMULATION

The present paper considers the problem of hub location in a realistic situation where different levels of services have been considered for transportation network.

In this section, a mixed-integer programming formulation for the problem with three-index (three-attribute) variables is presented. Among the works on revision of classic P-hub problem with single index characteristics, O'kelly [1] study with binary quadratic model could be mentioned. Afterwards, some works have been done by Campbell [2] and Skrin-Kapov [3]. They presented linear models with 4-index variables.

Then, Ernst and Krishnamoorthy [4] created a multi-commodity flow model with three-index variables, which each origin of a single commodity flows along the network direction. A model based on three-index variables was developed in order to minimize routing cost [28]. The current work attempts to present model hypotheses, relevant parameters, indices and decision making variables to develop the Capacitated Time Restricted Hierarchical Hub Median Poblem With Single Assignment (SA-TH-CHM). The symbols are specified as follows:

I is the demand points, $H \subseteq I$ is potential points for generating second level service providing hubs, and $C \subseteq H$ is potential points for first level service providing central hubs. P_H and P_C specify the number of hubs and central hubs to be generated, respectively. F_{ij} specifies exchanges from node $i \in I$ to $j \in I$. Normally, the flow from a node to itself is equal to zero (i.e. $F_{ii} = 0; \forall i \in I$). It is assumed in the model that $F_{ii} = 0$ works for all $i \in I$. C_{ij} is the routing cost of a unit of flow from node $i \in I$ to $j \in I$. In the model, $C_{ij} = C_{ji}$ works for all pair nodes i and j and also $C_{ii} = 0$, $i \in I$. α_H and α_C are reduction factors and their value always ranging zero and one, in the case $\alpha_H \geq \alpha_C$ [28]. α_H and α_C are the discount coefficients for route generation cost between hubs and central hubs and high level hubs. If node $i \in I$ is allocated to hub $j \in H$ and hub j is allocated to central hub $l \in C$, then variable x_{ijl} is equal to 1, otherwise is equal to zero. w^i_{jl} is the traffic demands between hub $j \in H$ and central hub $l \in C$, due to node $i \in I$ as origin or destination. v^i_{jl} specifies exchanges rate passing central hub $k \in C$ to central hub $l \in C \setminus \{k\}$, due to $i \in I$ as origin. t_{ij} is the passing time from node $i \in I$ to node $j \in I$, and $t_{ij} = t_{ji}$, also $t_{ii} = 0$. It is possible to make the duration of movement between hubs, central hubs and two central hubs shorter because more advanced and special vehicles could be used. This is possible by discount factors $\overline{\alpha}_H$ and $\overline{\alpha}_C$. The value of these parameters are limited to (0, 1) and $\overline{\alpha}_H \geq \overline{\alpha}_C$. The following variables could be defined by the Wagner's extended idea [2]: If $l \in C$, then $\hat{D}_l$ shows the time when all the flows caused by demand nodes and hubs allocated to the central hub l reach the node l. D_l shows the time when vehicles toward the demand nodes and their hubs linked to the central hubs l, leave l for destination. With these variables, it is possible to define cargo delivery upper limit with determined time bound of β for the problem as follows. For each $i \in I$, then TFi calculates all exchanges from node i to other points (i.e. $\sum_{m \in I} F_{im} = TF_i; \forall i \in I$).

For each $i \in I$ if it is selected as hub, its service capacity for the first and second levels are shown by Γc_i and Γh_i, respectively. If x_{lll} for each $l \in C$ is equal to 1, it could be said that central hub is established in the node, and also if x_{jjl} for each $j \in H, l \in C$ is equal to 1, it could be said that the second level hub is established. According to the above, the problem is as follows.

2. 1. Objective Function The Equation (1) shows the objective of this problem which is equal to total routing costs due to exchanges between their relevant hubs, hubs, and central hubs, and also exchanges between central hubs themselves.

$$\min \quad z = \sum_{i \in I} \sum_{r \in I} (F_{ir} + F_{ri}) \sum_{j \in H} C_{ij} \sum_{l \in C} x_{ijl} + \sum_{i \in I} \sum_{j \in H} \sum_{l \in C \setminus \{j\}} \alpha_H C_{jl} w^i{}_{jl} + \sum_{i \in I} \sum_{j \in C} \sum_{l \in C \setminus \{j\}} \alpha_C C_{jl} v^i{}_{jl} \qquad (1)$$

2.2. Hierarchical Hub Location Equations Equation (2) guarantee that each demand node is allocated to one hub and one central hub. Equation (3) show that if node i links to hub j and central hub of l, then node j must be a hub linked to the central hub l. Equation (4) guarantees if node j is allocated to the central hub l, then l becomes necessarily central hub. The number of hubs and central hubs are specified by P_H and P_C, respectively, which these Equations are mentioned in Equations (5) and (6).

$$\sum_{j \in H} \sum_{l \in C} x_{ijl} = 1 \quad \forall i \in I \qquad (2)$$

$$x_{ijl} \leq x_{jjl} \quad \forall i \in I, j \in H \setminus \{i\}, l \in C \qquad (3)$$

$$\sum_{m \in H} x_{jml} \leq x_{lll} \quad \forall j \in H, l \in C \setminus \{j\} \qquad (4)$$

$$\sum_{j \in H} \sum_{l \in C} x_{jjl} = p_H \qquad (5)$$

$$\sum_{l \in C} x_{lll} = p_C \qquad (6)$$

If node i is linked to a hub which in turn it is linked to central hub i, then traffic happens from node i to the nodes which are linked to other central hubs and leave the node. If node i is not linked to central hub l, traffic happens from node i to the nodes which are linked to the central hub i and move toward the node. These flows are assumed in Equations (7) and (12). In Equation (8) (i.e. cut equations) which is taken from Correia's idea [42], an upper bound for traffic volume between the hub and the central hub, and the central hub with another the central hub are considered. Equation (8) gives a complete description of capacitated problem. In fact, considering these equations in the model formulations, variables v_{lk}^{j} are allowed to be contrary to zero, regardless of $x_{ijl}=0$. The Equation (8) are actually a preventing action to this problem.

In Equations (9) and (11), amount of $w^{i}{}_{jl}$ is determined by allocation variables. Traffic in proximity of node i and between hub node j and central node l, is the same traffic between node i and the nodes which are linked to hub j. In this case, node i is linked to hub j and central hub l, otherwise amount is zero. Note that the traffic between nodes i and j will not be happened from hub j toward the central hub l, when i links to j. The Equation (10) is for modification of LP. Constraints 13 enable the presence or the absence of three-level allocations.

$$\sum_{k\in C\setminus\{l\}} v_{lk}{}^{i} - \sum_{k\in C\setminus\{l\}} v^{i}{}_{kl} = \sum_{r\in I} F_{ir} \sum_{j\in H} (x_{ijl} - x_{rjl}) \quad \forall i \in I, l \in C \tag{7}$$

$$\sum_{k\in C, l\neq k} v_{lk}^{i} \leq \sum_{m\in I} F_{im} \sum_{j\in H} x_{ijl} \quad \forall i \in I, l \in C \tag{8}$$

$$w^{i}{}_{jl} \geq \sum_{r\in I\setminus\{j\}} (F_{ir} + F_{ri})(x_{ijl} - x_{rjl}) \quad \forall i \in I, j \in H, l \in C\setminus\{j\} \tag{9}$$

$$x_{ljl} = 0 \quad \forall j \in H, l \in C\setminus\{j\} \tag{10}$$

$$w^{i}{}_{jl} \geq 0 \quad \forall i \in I, j \in H, l \in C \tag{11}$$

$$v^{i}{}_{kl} \geq 0 \quad \forall i \in I, k \in C, l \in C\setminus\{k\} \tag{12}$$

$$x_{ijl} \in \{0,1\} \quad \forall i \in I, j \in H, l \in C \tag{13}$$

2. 3. Capacity Equations In this model, Equation (14) specifies the maximum ability for estimating second level demands for each location $j \in H$ which is same Γh_j, i.e. the capacity for providing services of second level in location j. Equation (15) for each location $l \in C$ determines maximum ability for estimating first level demands which is same Γc_l, i.e. the capacity for providing first level services in location l.

$$\sum_{l\in C}\sum_{i\in I} TF_i x_{ijl} \leq \Gamma h_j \quad \forall j \in H \tag{14}$$

$$\sum_{j\in H}\sum_{i\in I} (TF_i - f_{ij}) x_{ijl} \leq \Gamma c_l \quad \forall l \in C \tag{15}$$

2. 4. Time Bound Equations In case node i links to the central hub i, then it will be directly linked to a hub group j, which it will be also linked to central hub l. Equation (16) estimated all travel time of traffic demand of node i toward the central hub l, then the outcome puts into its $\hat{D}_l$. Equation (17) computed the movement time of flow traffic between two central hubs and then insert into its D_l. In addition, in case $l \in C$ and $k \in C\setminus\{l\}$, $D_l \geq \hat{D}_k$ always work. For the travel time of traffic at each origin arrives to destination at the maximum time of β, the model uses from Equation (18). Equation (19) is not-negative makers.

$$\hat{D}_l \geq \sum_{j\in H} (t_{ij} + \overline{\alpha}_H t_{jl}) x_{ijl} \quad \forall i \in I, l \in C \tag{16}$$

$$D_l \geq \hat{D}_k + \overline{\alpha}_C t_{kl} x_{kkk} \quad \forall l \in C, k \in C \tag{17}$$

$$D_l + \sum_{j\in H} (\overline{\alpha}_H t_{lj} + t_{ji}) x_{ijl} \leq \beta \quad \forall i \in I, l \in C \tag{18}$$

$$\hat{D}_l \geq 0, D_l \geq 0 \quad \forall l \in C \tag{19}$$

A problem description has been presented by an example that has a node set with n=7 and have been showed the outputs of the problem whereas P_H=3 and P_C=1. The input node set of the example and the resulting network of the model are displayed in Figures 1 and 2, respectively. According to this solution, $x_{175} = x_{275} = x_{775} = x_{355} = x_{665} = x_{465} = 1$ so the nodes of 5, 6 and 7 are chosen as hub nodes and also x_{555}=1. Therefore, the node of 5 is selected as central hub node. The notations of flow balance Equation (9) for nodes i=4, j=6 and l=5 are $(f_{41}+f_{14})(x_{465} - x_{265}) + (f_{42}+f_{24})(x_{465} - x_{265}) + (f_{43}+f_{34})(x_{465} - x_{365}) + (f_{45}+f_{54})(x_{465} - x_{565}) + \ldots + (f_{47}+f_{74})(x_{465} - x_{765}) \leq w^{4}{}_{65}$. On the other hand, the below symbolizations are capacity constraints for all hubs and central hub nodes, and the time bound constraints for the route between node 2 and node 4. The capacity equations are $TF_4.x_{465} + TF_6.x_{665} \leq \Gamma h_6$, $TF_1.x_{175} + TF_2.x_{275} + TF_7.x_{775} \leq \Gamma h_7$ and $(TF_1-f_{17}).x_{175} + (TF_2-f_{27}).x_{275} + (TF_4-f_{46}).x_{465} + TF_7.x_{775} + TF_3.x_{355} + TF_6.x_{665} \leq \Gamma c_5$. The time bound equations contain $(t_{27}+\alpha_H t_{75})\, x_{275} \leq \hat{D}_5$, $\hat{D}_5 + (\alpha_C t_{55}\, x_{555} = 0) \leq D_5$ and $D_5 + (\alpha_H t_{56} + t_{64})\, x_{465} \leq \beta$.

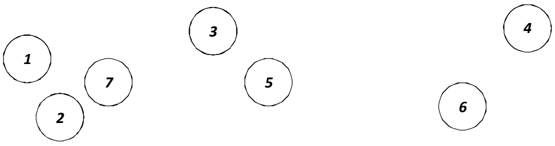

Figure 1. The node set with n=7.

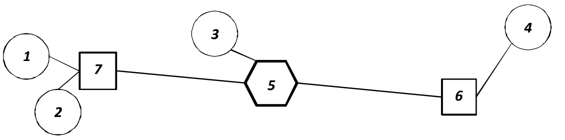

Figure 2. The output network of the model with n=7, P_H=3 and P_C=1.

3. RESULTS AND DISCUSSIONS

The Iranian Airport Data (IAD) was introduced by Karimi [44]. It includes the distances, costs, weights, capacities and fixed charge hub costs based on the hub airport location between 37 iranian cities. We assumed all the nodes as the demand set I, and we partitioned the 17 populous cities with maximum traffic among of facilities as a potential set of hubs (i.e., set of H) similar to Yaman [28] on Turkish network Data set. There are 10 populous cities in set of H, that their capacity were more than summation of all traffic; so, we chose them as a potential set of central hubs (i.e., set of C). In the next subsections, we presented the computational results and discussions on the IAD data for the model. Figure 3 shows the iran's map with 37 nodes of it. The potential nodes of central hubs and hubs are characterized by square and circle, respectively.

In order to examine the cost trends with regard to number of pre-imposed central hubs, the problem was solved with both its new and old structure. Firstly, SA-TH-CHM model with capacity and cut equations and then Time Restricted Hierarchical Hub Median Poblem With Single Assignment (SA-TH-HM) without any constraints were implemented in order to investigate the effect of capacity and cut equations in the problem. Considerable changes were observed by comparing the results. The number of hubs was assumed to be 5 and central hubs in 5 different performances were assumed to be constant from 1 to 5 (i.e., $p_C \in \{1,....,p_H\}$). In addition, according to $\alpha_C \le \alpha_H$ and $\overline{\alpha}_C \le \overline{\alpha}_H$, the magnitudes of discount coefficients in all calculations for $p_C \in \{1,....,p_H\}$ are considered to be $\alpha_H = \overline{\alpha}_H = 0.9$ and $\alpha_C = \overline{\alpha}_C \in \{0.9, 0.8\}$.

3. 1. Graphical Reports

As seen the models outputs results in Tables 1 to 6, overview for some outcomes in Figures 4 to 7, of course, the capacitated hierarchical structure has taken advantage of more significance than its past structure, i.e. by increasing the central hubs from 1 to 5, two models SA-TH-HM and SA-TH-CHM show similar results in terms of the hub and the central hub facilities location.

Thus, this mentions above are confirmed it. Mostly, when P_C=5, the location of the central hubs in both models becomes exactly the same. However, by decreasing number of the central hubs in each stage of solving direction, the changes and also the differences between location of the hubs and the central hubs in the model output become increasingly higher, so that in the stat $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C) = (0.9, 0.8, 0.9, 0.8)$, $\beta = 2990$ and P_C =1 in *SA-TH-CHM*, the 28 city and the 10-15-23-28 cities are selected as the central hub and the hubs, respectively.

In the same situation, the *SA-TH-CHM* model also choose the central hub and hubs at the 10 city and the 2-10-14-31-35 cities, respectively. Therfore, these outcome shown that more than 80% of the network structure has changed.

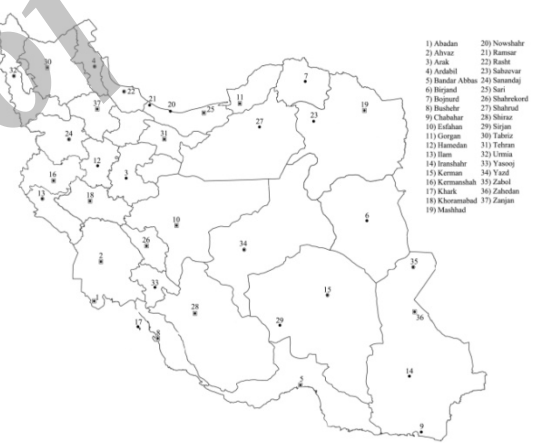

Figure 3. The map for IAD with n = 37, 17 and 10 number of potential nodes for hubs and central hubs, respectively.

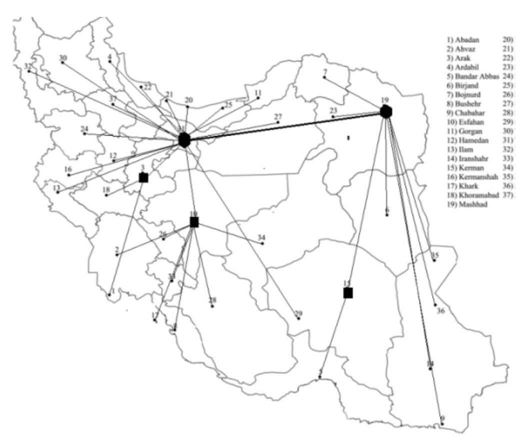

Figure 4. Grafical Veiw of hubs and central hubs locations in the SA-TH-CHM, with IAD data, when β=2990, $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C) = (.9,.8,.9,.8)$ and P_C=2.

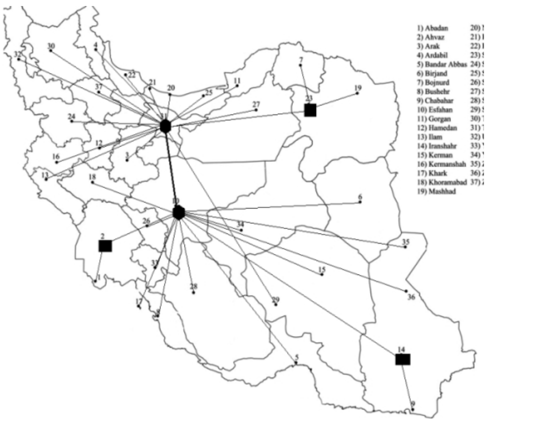

Figure 5. Grafical Veiw of hubs and central hubs locations in the SA-TH-HM, with IAD data, when β=2990, $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$ = (.9,.8,.9,.8) and P_C=2.

Figures 4 and 5 show the outputs of the two models *SA-TH-CHM* and *SA-TH-HM*, when $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C) = (0.9, 0.8, 0.9, 0.8)$,P_C=*2* and β=*2990*, respectively. In addition, Figures 6 and 7 show the outputs of *SA-TH-CHM* and *SA-TH-HM*, when $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C) = (0.9, 0.9, 0.9, 0.9)$, P_C=*2* and β=*2990,* respectively.

In these figures, square symbol indicates the hubs node and rectangle indicates the central hubs. In Figure 4, more logical state than Figure 5 could be observed. Of course, there are some similarities between these two figures; for example, city 31 in each figure is selected as a central hub which covers northern and western north area. Despite this similarity, the second central hub is selected in a more significant distance than the first significant model in Figure 4 which is the result of SA-TH-CHM. In Figure 4, the distance between two facilities at the first levels is more than the distance between the hubs and the central hub, which it is more logical because the provision capacity and services variety of the facilities increases by an increase in their level. Of course, these states could be confirmed and known by assuming that $\alpha_C \le \alpha_H$ [28]. On the other hand, if we divided vertically Figures 4 as the output of SA-TH-CHM model, the central hubs are located in two different sides, in which situation of fair distribution for the service provider's facilities are fulfilled. However, Figure 5 is results of SA-TH-HM model, if it separated into sections left and right from the middle by a vertical cut. Both of the central hubs will be also located in the left side. Thus, it is not fair situation for service delivery. The other significant difference in SA-TH-HM model opposite of SA-TH-CHM model is that demand nodes are allocated to hubs and central hubs with a long distance. However, this is rarely seen in SA-TH-HM model.

For example in the Figure 5 which is output of SA-TH-HM model, demand nodes 36, 35, 5 and 6 are located with a far distance from the hub, which do not indicate a fair distribution . In addition, the hub nodes in the output of SA-TH-HM model provide services at least for two demand nodes; but, in the Figure 4 which is a output of SA-TH-CHM, the central hub located in the node of 10 covers more than 6 demand nodes. In addition, Figures 5 and 7 confirm that hubs in SA-TH-HM provide less services for demand nodes, in which, location of the hubs have been not significant. In SA-TH-HM in both Figures 5 and 7, the hubs nodes of 2 and 14 provide services to only one of demand node and in other hubs (i.e., 23) second level of services provide only for two demand nodes. The establishment of these hubs is not practically necessary and only increases the costs.

In Figure 4, the SA-TH-CHM selected the 31 and 19 nodes as central hub that have been located in the north and northeast, when β=2990, $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$= (.9,.8,.9,.8) and P_C=2, respectively. This is occur to location of hubs have been change and chosen the 3, 10 and 19 nodes as hub, Such that, the central hubs have been away from each other. And the hubs have been nearby to their central hubs. In the event that, Figure 5 displayed unlike this status in illustrating by the SA-TH-HM. Figure 4 is a more logical structure than Figure 5 depicts for relation between hubs and central hubs, and central hubs together. In addition, relevance of among services facilities happened similar to above conditions in Figures 6 and 7.

3. 2. Effect of Capacity and Cut Equations on the Total Routing Cost To better understand capacity situation on the base structure, Tables 1 and 2 are defined for the routing costs of SA-TH-CHM and SA-TH-HM with different cases of discount coefficients, the number of the central hubs, different amounts of β and the constant number of P_H =5, respectively.

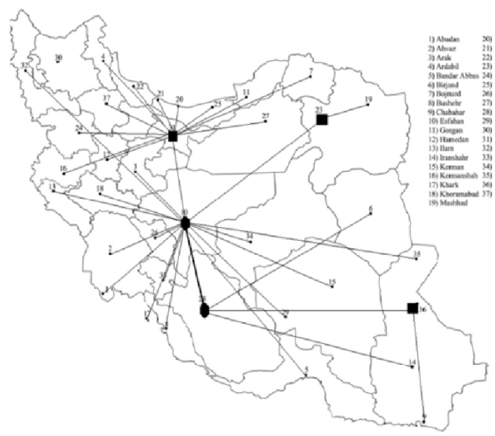

Figure 6. Grafical Veiw of hubs and central hubs locations in the SA-TH-CHM, with IAD data, when β=2990, $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$ = (.9,.9,.9,.9) and P_C=2.

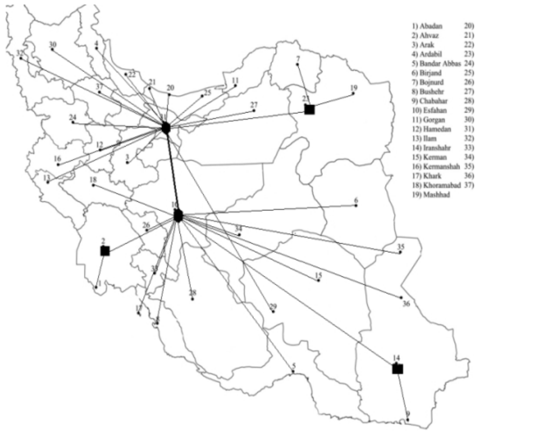

Figure 7. Grafical Veiw of hubs and central hubs locations in the SA-TH-HM, with IAD data, when β=2990, $(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)=(.9,.9,.9,.9)$ and P_C=2.

The routing costs trends are presented in Figures 8, 9 and 10 by the graphical reports in the line charts. Figures 6 and 7 are depicted for the routing costs of the models SA-TH-CHM and SA-TH-HM with equal states, i.e. P_C = 2, 3 and 4 with different types of time bound (i.e., β), respectively. Figures 8 and 9 showed that with increasing the parameter of P_C from 2 to 4, both SA-TH-CHM and SA-TH-CHM had relatively equal costs and their variation trend is declining. Of course, by decreasing the number of central hubs in each variation stage, the routing costs will be more obvious, so that the routing cost in state $(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)=(0.9,0.9,0.9,0.9)$, β=2990 and P_C=l in the *SA-TH-CHM* is 189.179506 and in the *SA-TH-HM* the routing cost is 146.965061. In addition, the routing cost in the state of $(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)=(0.9,0.8,0.9,0.8)$, β=3 and P_C=3110 for the SA-TH-CHM is 157.035483, while the routing cost in the SA-TH-HM model is 141.868391. The difference between these costs (i.e., 15.167092) is rational and indicates accuracy of the formulation; because, this cost difference is insignificant amount, opposite the capacity condition. Thus, it confirms the intelligent designing SA-TH-CHM. Figure 10 shows the routing costs of two models for particular states $P_C \in \{1,2,3,4,5\}$, β=∞ and $(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)=(0.9,0.8,0.9,0.8)$ which confirm above-mentioned suggestions. The costs have significant difference only in stringent state P_C=l as a natural and rationalized state, because the developed model has fulfilled capacity requirements in exchange for this difference. Notably, by imposing delivery time upper bound from 3110 to 2990 in the problem, the costs are increased considerably up to 0.22 percent.

3. 3. Effect of Capacity and Cut Equations on the Locations of Hubs and Central Hubs

One of the results of the present study is the location of hubs and central hubs which Tables 3 and 4 show the location of service providing location in different levels for the models SA-TH-CHM and SA-TH-HM with different states of discount coefficients $P_C \in \{1,2,3,4,5\}$, $\beta \in \{2990, 3110, \infty\}$ and constant numbers P_H=5, respectively.

In order to see the effect of discount factors on facilities location in different levels and consequently the cost of transportation between the central hubs, the results from the states P_C=2, $(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)=(0.9,0.8,0.9,0.8)$, β=2990 in both models were investigated. Thus, we found that the routing cost of SA-TH-CHM has been 157.035483 and the central hubs location of cities 19 and 31 and the hubs of cities 31, 19, 15, 10 and 3 were selected. However, the routing cost of the model SA-TH-HM has been 141.868391, and cities 10 and 31 were regarded as the central hubs and cities 14, 23, 10, 31 and 2 as the hub nodes. It is seen that the developed model of SA-TH-CHM has changed one out of two central hubs (i.e. change of city 10 to 19) under the specified states.

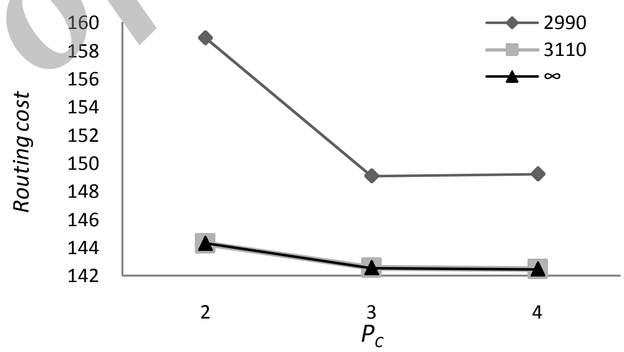

Figure 8. The routing cost of SA-TH-CHM for IAD for different amount of β, $P_C \in \{2,3,4\}$ and $(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)=(0.9, 0.9, 0.9, 0.9)$.

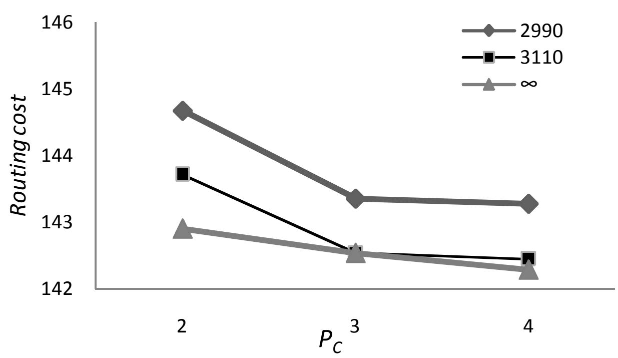

Figure 9. The routing cost of SA-TH-HM for IAD for different amount of β, $P_C \in \{2,3,4\}$ and $(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)=(0.9,0.9,0.9,0.9)$.

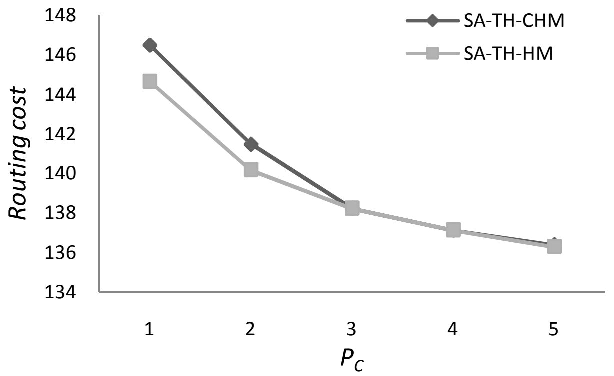

Figure 10. Compare the routing cost of SA-TH-CHM and SA-TH-HM for IAD with β=∞, $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C) = (0.9, 0.9, 0.9, 0.9)$ and $P_C \in \{1,2,3,4,5\}$.

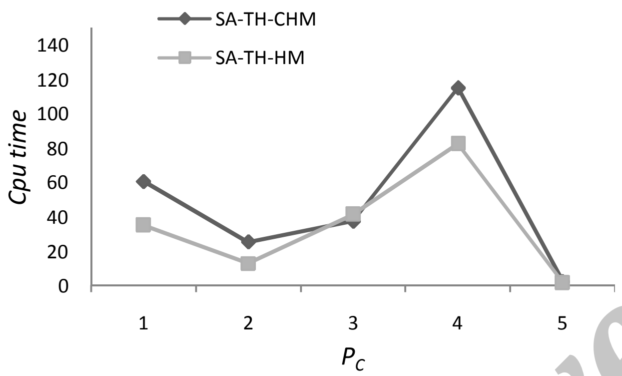

Figure 11. Compare the elapse times of SA-TH-CHM and SA-TH-HM for IAD with β=∞, $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C) = (0.9, 0.8, 0.9, 0.8)$ and $P_C \in \{1,2,3,4,5\}$.

In addition, hubs 15, 19 and 31 are replaced with 23, 14 and 2. This trend is seen at most of the cases which shows the effect of the developed model on the main body of output network. In Table 3, city 31 is selected more than 85% of times as a central hub and city 10 more than 88% of times as a central hub.

In addition, city 31 in more than 96 % of times is selected as a hub, but city 10 in 100% of times is selected as a hub and when β=∞ and P_C =1, city 31 is selected as central hub in all examples and if P_C= 2, cities 31 and 10 are the central hubs. These two cities in Iran take privilege of efficient transportation systems, indicating the presence of a correct orientation in designing the SA-TH-CHM model.

3. 4. Effect of Capacity and Cut Equations on Computation Time

In this section, the effect of different parameters on the CPU time is considered. The model has been performed by GAMS 21.7 and the solver CPLEX 11.0.0 on a 2.1 GH processor with core 2, RAM 4GB and a Windows 7. The calculation times of the unfeasible solutions are not presented. This section is considered the effect of different amount of β and discount coefficients on CPU time.

Information related to elapse time for SA-TH-CHM and SA-TH-HM on IAD data is shown in Tables 5 and 6, respectively. The increasing trend of CPU time due to decrease in the number of the central hubs could be observed in Tables 5 and 6. Furthermore, in 70% of different cases, elapse time in SA-TH-CHM model is more than run time of SA-TH-HM model, because capacity and cut equations are imposed into the SA-TH-CHM model. The maximum CPU times in SA-TH-CHM and SA-TH-HM models are 289.554 seconds and 124.353 seconds, respectively. On the other hand, the minimum elapse times in SA-TH-CHM and SA-TH-CHM are 2.376 and 1.527 seconds, respectively.

TABLE 1. Total routing cost of *SA-TH-CHM* for *IAD* data with 37 nodes and P_H=5

α_H	α_C	$\hat{\alpha}_H$	$\hat{\alpha}_C$	P_C	$\beta = \infty$	3110	2990
1	1	1	1	1	151.83	-	-
1	1	1	1	2	150.22	166.42	-
1	1	1	1	3	148.77	166.42	-
1	1	1	1	4	148.18	164.93	-
1	1	1	1	5	147.70	-	-
0.9	0.9	0.9	0.9	1	146.50	146.92	189.18
0.9	0.9	0.9	0.9	2	144.29	144.29	158.91
0.9	0.9	0.9	0.9	3	142.54	142.54	149.10
0.9	0.9	0.9	0.9	4	142.45	142.45	149.23
0.9	0.9	0.9	0.9	5	142.04	146.56	-
0.9	0.8	0.9	0.8	1	146.50	146.92	189.18
0.9	0.8	0.9	0.8	2	141.48	141.48	157.04
0.9	0.8	0.9	0.8	3	138.24	138.24	142.09
0.9	0.8	0.9	0.8	4	137.13	137.13	141.06
0.9	0.8	0.9	0.8	5	136.38	139.17	140.23
0.8	0.8	0.8	0.8	1	140.94	140.96	140.96
0.8	0.8	0.8	0.8	2	137.91	137.91	137.91
0.8	0.8	0.8	0.8	3	135.83	135.83	135.83
0.8	0.8	0.8	0.8	4	135.29	135.29	135.29
0.8	0.8	0.8	0.8	5	136.38	139.17	140.23
0.9	0.8	0.9	0.9	1	146.50	146.92	189.18
0.9	0.8	0.9	0.9	2	141.48	141.48	157.40
0.9	0.8	0.9	0.9	3	138.24	138.24	143.57
0.9	0.8	0.9	0.9	4	137.13	137.13	143.16
0.9	0.8	0.9	0.9	5	136.38	140.23	-

TABLE 2. Total routing cost of *SA-TH-HM* for *IAD* data with 37 nodes and P_H=5

α_H	α_C	$\hat{\alpha}_H$	$\hat{\alpha}_C$	P_C	β = ∞	3110	2990
1	1	1	1	1	150.62	-	-
1	1	1	1	2	149.28	166.42	-
1	1	1	1	3	148.72	166.42	-
1	1	1	1	4	148.12	164.93	-
1	1	1	1	5	147.59	-	-
0.9	0.9	0.9	0.9	1	144.65	146.92	146.97
0.9	0.9	0.9	0.9	2	142.91	143.72	144.68
0.9	0.9	0.9	0.9	3	142.54	142.54	143.36
0.9	0.9	0.9	0.9	4	142.29	142.45	143.28
0.9	0.9	0.9	0.9	5	141.94	146.38	-
0.9	0.8	0.9	0.8	1	144.65	146.92	146.97
0.9	0.8	0.9	0.8	2	140.17	143.72	141.87
0.9	0.8	0.9	0.8	3	138.24	142.54	139.05
0.9	0.8	0.9	0.8	4	137.13	142.45	138.01
0.9	0.8	0.9	0.8	5	136.29	146.38	140.05
0.8	0.8	0.8	0.8	1	138.56	140.97	146.97
0.8	0.8	0.8	0.8	2	136.16	136.16	141.87
0.8	0.8	0.8	0.8	3	135.50	135.50	139.05
0.8	0.8	0.8	0.8	4	135.29	135.29	138.01
0.8	0.8	0.8	0.8	5	136.29	139.17	140.05
0.9	0.8	0.9	0.9	1	144.65	146.92	146.97
0.9	0.8	0.9	0.9	2	140.17	140.99	141.87
0.9	0.8	0.9	0.9	3	138.24	138.24	139.05
0.9	0.8	0.9	0.9	4	137.13	137.13	138.01
0.9	0.8	0.9	0.9	5	136.29	140.05	-

CPU time of SA-TH-CHM and SA-TH-HM models for $(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C) = (0.9, 0.8, 0.9, 0.8)$, β=∞ and P_H= 5 in different cases of P_C=1,2,3,4,5 are depicted in Figure 11.

It could be seen that CPU time by increase in PC from 1 to 5 in SA-TH-CHM is exactly similar to CPU time trend for SA-TH-HM, and interestingly, these two times are similar in two states of P_C = {3,5} and there is a very small difference between three remaining times. The smallest magnitudesβ for different values of discount coefficients were calculated and 2870 was the smallest magnitude through which a justified answer for all cases could be achieved.

Then, other magnitudes with increasing trend of 120-unit were selected as well. Similar to the work by other researcher [28], we calculated the amounts of β. Therefore, we defined $\beta \in$ {2990, 3110, ∞} for the *IAD* data. For an infeasible situation, we do not have any report. For each amount of outputs of the problem with different discount coefficients have been solved (see the results in Table 1 to 6.) In order to consider a larger solution space and have feasible solutions for other magnitudes of discount coefficients.

TABLE 3. Locations of central habs and hubs in the *SA-TH-CHM* for *IAD* data with 37 nodes and P_H=5

$(\alpha_H, \alpha_C, \hat{\alpha}_H, \hat{\alpha}_C)$	β	P_c	Central hubs	Hubs
(0.9, 0.9, 0.9, 0.9)	2990	1	28	28,10,23,15,32
		2	10,28	10,28,31,36,23
		3	10,19,31	2,10,19,31,36
		4	2,10,19,31	2,10,19,31,36
		5	-	-
	3110	1	10	2,31,10,35,15
		2	10,31	2,31,10,23,15
		3	10,19,31	2,10,31,19,15
		4	10,16,19,31	10,16,31,19,15
		5	2,10,16,19,31	2,10,16,19,31
	∞	1	31	31,10,23,16,30
		2	10,31	2,31,10,23,15
		3	10,19,31	2,10,15,19,31
		4	10,16,19,31	10,16,19,31,15
		5	2,10,16,19,31	2,10,16,19,31
(0.9, 0.8, 0.9, 0.8)	2990	1	28	10,15,23,28,32
		2	19,31	3,10,15,19,31
		3	10,19,31	2,10,19,31,36
		4	10,16,19,31	10,16,19,31,2
		5	2,10,16,19,31	2,10,16,19,31
	3110	1	10	2,31,10,35,15
		2	10,31	2,31,10,23,15
		3	10,19,31	2,31,10,19,15
		4	10,16,19,31	10,16,19,31,15
		5	10,16,19,28,31	10,16,19,28,31
	∞	1	31	31,10,16,23,30
		2	10,31	2,31,10,23,15
		3	10,19,31	2,10,19,31,15
		4	10,16,19,31	10,16,19,31,15
		5	2,10,16,19,31	2,10,16,19,31
(0.8, 0.8, 0.8, 0.8)	2990	1	10	2,31,10,19,15
		2	10,31	2,31,10,23,15
		3	10,19,31	2,10,19,15,31
		4	10,16,19,31	10,16,19,31,15
		5	2,10,16,19,31	2,10,16,19,31
	3110	1	10	2,31,10,19,15
		2	10,31	2,31,10,23,15
		3	10,19,31	2,10,15,19,31
		4	10,16,19,31	10,16,19,31,15
		5	10,16,19,28,31	10,16,19,28,31
	∞	1	31	10,16,23,30,31
		2	10,31	2,10,31,15,23
		3	10,19,31	2,10,15,19,31
		4	10,16,19,31	10,16,19,31,15
		5	2,10,16,19,31	2,10,16,19,31
(0.9, 0.8, 0.9, 0.9)	2990	1	28	10,15,23,28,32
		2	10,28	10,23,28,31,36
		3	10,19,31	2,10,36,19,31
		4	2,10,19,31	2,10,19,31,36
		5	-	-
	3110	1	10	2,31,10,35,15
		2	10,31	2,10,31,23,15
		3	10,19,31	2,10,15,19,31
		4	10,16,19,31	10,15,16,19,31
		5	2,10,16,19,31	2,10,16,19,31
	∞	1	31	10,16,23,30,31
		2	10,31	2,10,15,23,31
		3	10,19,31	2,10,15,19,31
		4	10,16,19,31	10,15,16,19,31
		5	2,10,16,19,31	2,10,16,19,31

TABLE 4. Locations of central habs and hubs in the SA-TH-HM for IAD data with 37 nodes and P_H=5

$(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)$	β	P_c	Central hubs	Hubs
(0.9, 0.9, 0.9, 0.9)	2990	1	10	2,31,10,35,14
		2	10,31	2,31,10,23,14
		3	10,19,31	2,31,10,19,14
		4	10,16,19,31	16,10,31,19,14
		5	-	-
	3110	1	10	2,31,10,35,15
		2	10,31	2,31,10,23,14
		3	10,19,31	2,10,31,19,15
		4	10,16,19,31	10,16,31,19,15
		5	2,10,16,19,31	2,10,16,19,31
	∞	1	31	31,10,23,12
		2	10,31	2,31,10,23,15
		3	10,19,31	2,10,15,19,31
		4	2,10,16,31	2,10,16,23,31
		5	2,10,16,19,31	2,10,16,19,31
(0.9, 0.8, 0.9, 0.8)	2990	1	10	2,31,10,35,14
		2	10,31	2,31,10,23,14
		3	10,19,31	2,10,19,31,15
		4	10,16,19,31	10,16,19,31,14
		5	2,10,16,19,31	2,10,16,19,31
	3110	1	10	2,31,10,35,15
		2	10,31	2,31,10,23,14
		3	10,19,31	2,31,10,19,15
		4	10,16,19,31	10,16,19,31,15
		5	10,16,19,28,31	10,16,19,28,31
		1	31	31,10,23,12,30
	∞	2	10,31	2,31,10,23,15
		3	10,19,31	2,10,19,31,15
		4	10,16,19,31	10,16,19,31,15
		5	2,10,16,19,31	2,10,16,19,31
		1	10	2,31,10,19,15
(0.8, 0.8, 0.8, 0.8)	2990	2	10,31	2,31,10,23,15
		3	10,16,31	16,31,10,23,15
		4	10,16,19,31	10,16,19,31,15
		5	2,10,16,19,31	2,10,16,19,31
	3110	1	10	2,31,10,19,15
		2	10,31	2,31,10,23,15
		3	10,16,31	10,16,31,23,15
		4	10,16,19,31	10,16,19,31,15
		5	10,16,19,28,31	10,16,19,28,31
	∞	1	31	12,10,31,23,30
		2	10,31	2,31,10,23,15
		3	10,16,31	10,15,16,23,31
		4	10,16,19,31	10,16,19,31,15
		5	2,10,16,19,31	2,10,16,19,31
(0.9, 0.8, 0.9, 0.9)	2990	1	10	2,31,10,35,14
		2	10,31	2,10,14,23,31
		3	10,19,31	2,10,14,19,31
		4	10,16,19,31	10,14,16,19,31
		5	-	-
	3110	1	10	2,31,10,35,15
		2	10,31	2,10,31,23,14
		3	10,19,31	2,10,15,19,31
		4	10,16,19,31	10,15,16,19,31
		5	2,10,16,19,31	2,10,16,19,31
	∞	1	31	10,23,31,12
		2	10,31	
		3	10,19,31	2,10,15,19,31
		4	10,16,19,31	10,15,16,19,31
		5	2,10,16,19,31	2,10,16,19,31
(1, 1,1,1)	3110	3	10,28	10,28,31,23,35
		4	10,28,31	10,28,31,23,35
		5	10,16,28,31	10,16,28,31,23
			-	-
		1	31	10,12,23,31
		2	10,31	2,10,15,23,31
	∞	3	10,16,31	2,10,16,23,31
		4	2,10,16,31	2,10,16,23,31
		5	2,10,16,19,31	2,10,16,19,31

TABLE 5. Elapse times in the *SA-TH-CHM* for *IAD* data with 37 nodes and P_H=5

$(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)$	P_0	β=2990	3110	∞
(0.9, 0.9, 0.9, 0.9)	1	47.930	12.793	65.193
	2	98.555	30.297	127.177
	3	54.300	25.279	119.888
	4	73.822	121.043	154.747
	5	-	02.376	12.558
(0.9, 0.8, 0.9, 0.8)	1	45.677	15.529	60.746
	2	289.554	23.202	25.369
	3	113.418	35.696	37.560
	4	82.843	98.802	115.103
	5	02.396	02.681	02.447
(0.8, 0.8, 0.8, 0.8)	1	15.595	29.599	63.200
	2	84.361	83.214	117.142
	3	90.132	110.892	119.455
	4	65.607	103.071	93.318
	5	02.381	02.849	02.440
(0.9, 0.8, 0.9, 0.9)				
	1	67.761	13.821	62.003
	2	104.005	21.511	24.971
	3	176.258	29.782	37.160
	4	88.074	24.744	114.733
(1, 1,1,1)	5	-	02.405	02.524
	1	-	-	23.419
	2	-	55.992	131.564
	3	-	64.316	63.928
	4	-	40.398	176.355
	5	-	-	02.658

TABLE 6. Elapse times in the *SA-TH-HM* for *IAD* data with 37 nodes and P_H=5

$(\alpha_H,\alpha_C,\hat{\alpha}_H,\hat{\alpha}_C)$	*P₀*	*β=2990*	*3110*	∞
(0.9, 0.9, 0.9, 0.9)	1	17.597	47.168	39.471
	2	70.601	100.791	47.598
	3	95.803	98.666	92.591
	4	74.814	85.797	90.339
	5	-	01.899	01.740
(0.9, 0.8, 0.9, 0.8)	1	34.632	37.492	35.047
	2	55.837	79.730	12.957
	3	61.046	87.359	41.448
	4	53.481	83.467	82.562
	5	01.873	02.235	01.718
(0.8, 0.8, 0.8, 0.8)	1	66.763	53.960	12.567
	2	57.299	43.631	66.480
	3	75.546	82.854	92.106
	4	79.768	74.287	75.388
	5	01.837	02.194	01.838
(0.9, 0.8, 0.9, 0.9)				
	1	17.974	63.365	35.304
	2	48.102	91.476	13.235
	3	67.394	80.279	42.078
	4	49.123	82.605	85.278
(1, 1,1,1)	5	-	01.890	01.735
	1	-	-	11.716
	2	-	48.261	115.172
	3	-	95.957	124.353
	4	-	21.902	115.077
	5	-	-	01.527

4. CONCLUDING REMARKS

The present paper attempts to develop, create and solve a new and applied model in the same trend of the previous studies on the hub location literature. The model known as a capacitated hierarchical hub median location problem is taking advantage of some significant capabilities such as the development of the previous models with imposing capacity situations in a hierarchical structure. Thus, these significant changes have occurred with the least cost in the least possible solving time of problem. These situations are confirmed that the present model is unique in its own kind. Therefore, this paper contributes to the literature on hub location problems to move towards real situations. We study a new model with title of capacitated hierarchical hub median location problem and use the real-world dataset in hub location problem corresponded to Iranian hub airport location. Results showed that the new model has changed the structure of the network and location of services facilities (i.e., hubs and central hubs). Therefore, in practice outcome network of the previous models have not been responsive the demand of applicants of services. As a result, the network structure with the capacity on the hubs facilities has been transform into a logical framework. However, since there is no end of the knowledge for the model developed in this paper, improvement situations can be considered such as multi-objective modeling with objectives of fixed charge costs of the hubs, the central hubs and linking among them or imposing multi-allocation situations along with single allocation ones.

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Modeling of the Capacitated Single Allocation Hub Location Problem with a Hierarchical Approch

TECHNICAL NOTE

M. Karimi [a], A. R. Eydi [b], E. Korani [b]

[a] *Department of Industrial engineering, Najafabad Branch, Islamic Azad University, Isfahan, Iran*
[b] *Department of Industrial engineering, University of Kurdistan, Sanandaj, Iran*

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چکیده

مسأله مکان یابی محور سلسله مراتبی در سیستم های توزیع، حمل و نقل، دفع زباله، خدمات درمانی، خدمات اضطراری و ارتباطات راه دور کاربرد دارد.این مسائل تلاش می کنند، مکان تسهیلات ارائه دهنده خدمت در سطوح مختلف را تعیین و مسیر های ارتباطی آنان را در راستای کاهش هزینه ها و ایجاد شرایط مناسب در شبکه توزیع مشخص کنند. مسائل مکان یابی محور سلسله مراتبی یکی از مسائل کاربردی این حوزه بشمار می آیند، که در کار حاضر تلاش شده است تا با تحمیل گزینه ظرفیت به هر یک از تسهیلات ارائه دهنده خدمت، بهترین شرایط را ایجاد و انتخاب نماید بطوری که مراکز تقاضا به شکلی منطقی و هدفمند به مراکز ارائه خدمت هدایت شوند، که هیچگاه درخواست آنها بدون پاسخ نماند. به منظور تحقق ایده فوق الذکر مدل مساله مکانیابی میانه محور سلسله مراتبی ظرفیت دار طراحی،ایجاد و ارائه خواهد شد. همچنین با توجه به جریان افزایشی تقاضا گزینه های تعدیل کننده مد نظر قرار گرفته اند تا نیاز های آتی را نیز برآورده سازد و شرایط عدم قطعیت در تصمیم گیری نیز در نتایج اعمال گردد. برای اثبات درستی مدل ارائه شده از داده های *IAD* بهره برده ایم، که نتایج آن گواه بر استوار بودن آن است.

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