A new approach for extension of the ELECTRE I Method for decision-making under fuzzy environment

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Abstract

The ELECTRE methods have been applied to solve MCDM problems in a wide variety of fields. But to the best of our knowledge there is no considerable attempt to fuzzify the ELECTRE method in an effective and easy to use manner. Therefore the aim of this paper is to propose an extension to ELECTRE method for incorporating fuzzy data into the decision-making process. In this paper we consider triangular fuzzy numbers to account for the vagueness and impreciseness of decision-maker(s) (DM) judgments.

Keywords: MCDM; ELECTRE I; Fuzzy set theory; Triangular fuzzy number.

1. Introduction

Classical MCDM methods assume that the ratings of alternatives and the importance weights of criteria are crisp and precise, today it is recognized as unrealistic, because under many conditions, crisp data are inadequate to model real-life situations. Real-world decision making problems, including MCDM problems, usually result in uncertain, imprecise, indefinite and subjective data being present, and on the other hand human judgments including preferences are often vague and decision maker usually cannot estimate his preference with an exact numerical value. Since human judgments including preference are often vague and cannot estimate his preference with an exact numerical value linguistic variables such as “very poor”, “poor”, “medium”, “fair”, “high”, “very high”, etc. are regarded as a more realistic way and the natural representation of the preference or judgment of a decision maker to describe the desired value of alternatives and the importance weight of criteria.

In this paper we will extend the ELECTRE I method using fuzzy set theory to take this vagueness and impreciseness into account, and in which distance between two fuzzy numbers is calculated using vertex method.

2. The proposed fuzzy ELECTRE I method

Consider the fuzzy decision matrix (\( \tilde{D} \)) and importance weight of criteria (\( \tilde{W} \)) as follows:
The normalized fuzzy decision matrix can be represented as

\[ \mathbf{R} = [\tilde{r}_{ij}]_{nm}, \]

where

\[ \tilde{r}_{ij} = \left( \frac{a_{ij}}{\max_i \{a_{ij}\}}, \frac{b_{ij}}{\max_i \{b_{ij}\}}, \frac{c_{ij}}{\max_i \{c_{ij}\}} \right), \quad i = 1, 2, \ldots, m, \quad j \in B, \]

\[ \tilde{r}_{ij} = \left( \frac{\min_i \{a_{ij}\}}{c_{ij}}, \frac{\min_i \{b_{ij}\}}{b_{ij}}, \frac{\min_i \{c_{ij}\}}{a_{ij}} \right), \quad i = 1, 2, \ldots, m, \quad j \in C, \]

where \( B \) is associated with the benefit criteria, and \( C \) is associated with the cost criteria. Now we can calculate the weighted normalized fuzzy decision matrix as follows:

\[ \mathbf{V} = [\tilde{v}_{ij}]_{mn}, \]

where

\[ \tilde{v}_{ij} = \tilde{r}_{ij} \tilde{w}_{ij}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n. \]

The fuzzy concordance index \( \tilde{C}(A_k, A_l) \) for each pair of alternatives \( (A_k, A_l) \) can be calculated as

\[ \tilde{C}(A_k, A_l) = \sum_{j \in \mathcal{R}(A_k, A_l)} \tilde{w}_{kj} \]

The discordance index \( D(A_k, A_l) \) for each pair of alternatives \( (A_k, A_l) \) is calculated as

\[ D(A_k, A_l) = \max_j \frac{d(\tilde{v}_{kj}, \tilde{v}_{lj})}{\max_j d(\tilde{v}_{kj}, \tilde{v}_{lj})}, \]

where \( \mathcal{R}(A_k, A_l) \) is the set of criteria for which alternative \( A_k \) is strictly preferred to alternative \( A_l \) and \( d(\tilde{v}_{kj}, \tilde{v}_{lj}) \) is the distance between two fuzzy numbers \( \tilde{v}_{kj}, \tilde{v}_{lj} \) using the vertex method.

By comparing \( \tilde{C}(A_k, A_l) \) and \( D(A_k, A_l) \) with threshold values \( \tilde{C}^* \) and \( D^* \) respectively we can obtain \( f(A_k, A_l) \) and \( g(A_k, A_l) \) for each pair of alternatives \( (A_k, A_l) \) as follows:

- if \( \tilde{C}(A_k, A_l) \geq \tilde{C}^* \) and \( \tilde{C}(A_k, A_l) \geq \tilde{C}(A_k, A_l) \), then \( f(A_k, A_l) = 1 \) otherwise \( f(A_k, A_l) = 0 \).
- if \( D(A_k, A_l) \leq D^* \), then \( g(A_k, A_l) = 1 \) otherwise \( g(A_k, A_l) = 0 \).

Then \( h(A_k, A_l) \) will be calculated as

\[ h(A_k, A_l) = f(A_k, A_l) \cdot g(A_k, A_l), \]

finally the outranking relations are built by as follows:

- if \( h(A_k, A_l) = 1 \) then alternative \( A_k \) outranking alternative \( A_l \).
References