A Method for Ranking All Efficient DMUs in $T_v$

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Abstract

Introduction: In the evaluation of Decision Making Units (DMUs) using Data Envelopment Analysis (DEA) models, more than one DMU may be identified as efficient. So ranking them is necessary for Decision Makers (DM). One of the main problems in ranking DMUs, by using Data Envelopment Analysis, is the ranking of non-extreme efficient DMUs. Most of the existing methods can only rank extreme DMUs.

Aim: In this paper, we propose a method to rank non-extreme DMUs in $T_v$.

Materials and Methods: We rank non-extreme DMUs by finding a positive convex combination (with the best score) of extreme efficient DMUs located on the minimum face containing the non-extreme DMU.

Results: Our proposed method can rank all efficient DMUs in 3 steps: at first, extreme and non-extreme efficient DMUs must be identified (It can be done by AP model). Ranking related score for each extreme DMU can be attained by one of the existing ranking methods that can rank extreme DMUs. Then for obtaining ranking related score for non-extreme DMU, we consider the face with minimum dimension, containing the non-extreme DMU. At last, by solving a model or its dual, and based on the ranking related scores of extreme efficient DMUs located on the minimum face containing the non-extreme DMU, a ranking related score for non-extreme DMUs can be obtained.

Conclusion: Although, we rarely have got observed non-extreme efficient DMUs, we must theoretically be able to rank them, because, all virtual efficient DMUs are almost non-extreme. Our method can rank non-extreme efficient DMUs as same as extreme efficient DMUs.

Keywords: Ranking, Data Envelopment Analysis, Minimum Face

Introduction

Data envelopment analysis was originated in 1978 by Charnes et al.[1], the first DEA model being called CCR (Charnes, Cooper and Rhodes) model. The objective of DEA models is to evaluate the performance of decision making units (DMUs) that are responsible to convert a set of inputs into a set of outputs. Efficient DMUs are identified by an efficiency score equal to 1, and inefficient DMUs have efficiency scores less than 1. Although efficiency score can be a criterion for the ranking of inefficient DMUs, this criterion cannot rank

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efficient DMUs. During the last decade, various models were developed for the purpose of ranking DEA efficient units. Adler et al.\cite{2} classified the ranking methods into some categories. The first category involves the evaluation of a cross-efficiency matrix, in which the units are self- and peer-evaluated. The second category, generally known as the super-efficiency method, does the ranking through the exclusion of the unit from the production possibility set and through analyzing the change in the Pareto frontier. The third category is based on benchmarking, in which a unit is highly ranked if it is chosen as a useful target for many other units. The fourth category utilizes multivariate statistical techniques, which are generally applied after the DEA dichotomy classification. The fifth category ranks inefficient units through the proportional measures of inefficiency. The last category requires the collection of additional, preferential information from relevant decision-makers and combines multiple-criteria decision methodologies with the DEA approach. However, whilst each technique is useful in a specialist area, no single methodology can be prescribed here as the complete solution to the question of ranking.

Many of the ranking methods fall into the second category. First, Anderson and Peterson\cite{3} introduced super-efficiency for the ranking purposes through the exclusion of the unit from the PPS and then analyzing the change in the Pareto frontier. Since the proposed model may be infeasible and also unstable in some cases, other super efficiency methods have been proposed. For example.\cite{4-9} Although the developed models are interesting and useful, in general, they are not able to rank non-extreme efficient DMUs. Jahanshahloo et al.\cite{10} proposed a method based on Monte Carlo method that is able to rank all efficient DMUs. This method is not a DEA method, and it may seem time-consuming, and complex for computations. In this paper, a DEA-based method has been developed, which is able to rank every (extreme and non-extreme) efficient DMU.

Notice that when a DMU is evaluated, the efficiency score of the DMU is obtained by the combination of a set of efficient DMUs (reference set), which form a part of the segments on the efficiency frontier. The reference set provides information regarding which efficient DMUs are used for the performance evaluation of the other DMUs. For the ranking purposes, we consider the minimum face containing non-extreme DMU in this article. Once we find every DMU located on the minimum face, we will attempt to find a positive convex combination for every extreme efficient DMU located on this minimum face. We prove that this can be done. Obviously, we can construct non-extreme DMUs only by means of extreme DMUs. In this way, by means of this positive convex combination, we can have a score for non-extreme DMUs. We may have, however, alternative positive convex combinations, each of which corresponds to a score. We want to compare each DMU in its best score. To achieve this target, we use a two-phase model.

This paper has been organized as follows: Section 2 briefly introduces the background of DEA and reviews a mathematical basis used for this study. Section 3 introduces our proposed model. Two numerical examples are given in Section 4, and Section 5 contains our conclusions.

Materials and Methods  
DEA Background  
We assume n DMUs, each of which consumes m inputs to produce s outputs. Let \( X \in \mathbb{R}^{m \times n} \) and \( Y \in \mathbb{R}^{s \times n} \) be matrices containing the observed inputs and outputs for n DMUs. By \( X_j \) (the j th column of \( X \)), we denote the vector of inputs consumed by DMU \( j \), and by \( Y_j \) (the j th column of \( Y \)) the vector of outputs produced by DMU \( j \). The production possibility set \( T_v \) is defined as follows:
The frontier of $T_v$, which is a piecewise linear surface, is called the efficiency frontier. On this frontier, each DMU is relatively efficient, and that the others are inefficient. In the input orientation for the evaluation of $oDMU$ in $T_v$, the envelopment form of the BCC model is:

$$\begin{align*}
\text{Min} & \left\{ \theta \sum_{j=1}^{n} \lambda_j X_j + S^- = 0 X_o, \sum_{j=1}^{n} \lambda_j Y_j - S^+ = Y_o, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \ldots, n \right\} \\
\sum_{j=1}^{n} \lambda_j & = 1, \lambda_j \geq 0, j = 1, \ldots, n, S^- \geq 0, S^+ \geq 0
\end{align*}$$

(1)

Now, let’s consider the following two-phase model for the complete classification of DMUs:

Phase I:

$$\begin{align*}
\text{Min} & \left\{ \theta \sum_{j=1}^{n} \lambda_j X_j + S^- = 0 X_o, \sum_{j=1}^{n} \lambda_j Y_j - S^+ = Y_o, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \ldots, n \right\} \\
\sum_{j=1}^{n} \lambda_j & = 1, \lambda_j \geq 0, j = 1, \ldots, n, S^- \geq 0, S^+ \geq 0
\end{align*}$$

(2)

Phase II:

$$\begin{align*}
\text{Max} & \left\{ 1^T S^- + 1^T S^+ \right\} \\
\sum_{j=1}^{n} \lambda_j X_j + S^- = & \theta^* X_o, \sum_{j=1}^{n} \lambda_j Y_j - S^+ = Y_o, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \ldots, n, S^- \geq 0, S^+ \geq 0
\end{align*}$$

(3)

Where $\theta^*$ is the optimal solution obtained from (2). The reference set (RF) for $DMU_o$ can be conventionally defined as follows:

$$\text{RF}_o = \{ j \in E \cup E' \mid \lambda_j > 0 \text{ on } (\theta^*, \lambda^*) \text{ of } (1) \}$$

where $E \cup E'$ is the set of Pareto-Koopmans efficient DMUs, and its elements can be characterized with $\theta^* = 1$ and $1^T S^- + 1^T S^+ = 0$ in the optimal solution of the two-phase model. $E$ is the set of extreme efficient DMUs, and $E'$ is the set of non-extreme efficient DMUs. Sueyoshi [12] indicates that we can uniquely determine a reference set containing the maximum number of efficient DMUs. He shows that the reference set can be identified by a pair of solutions of (1) and it’s dual which satisfy Strong Complementary Slackness Conditions (SCSC). He proposes the following DEA model to find such a reference set for $DMU_o$. 

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DMUs. Suppose that $E$ DMU is always feasible, but it can give us the ranking-related score only for the extreme efficient DMUs. Sueyoshi[12] also shows that the identification of a reference set containing the maximum number of efficient DMUs, can be considered as a computation process to find the minimum face containing $\theta^*X_o, Y_o$. Therefore, if DMU is an efficient DMU, the previous model can find the minimum face containing DMU.

Results and Discussion
The proposed model for the ranking of non-extreme efficient DMUs
First, we need to have the ranking-related score for the extreme DMUs. We can do this by ranking the DMUs by means of methods giving us a ranking-related score (rather than the ranking status itself), for instance, the $L_1$ norm model. We know that this ranking model is always feasible, but it can give us the ranking-related score only for the extreme efficient DMUs. Suppose that $DMU_o \in E'$. Using (4), we can find every extreme efficient DMU located on the minimum face containing DMU. We define $RF^+_o$ as follows:

$$RF^+_o = \{j | DMU_j is an extreme efficient DMU existing on the minimum face containing DMU_o \}$$

Equivalently,

$$RF^+_o = \{j | DMU_j is an extreme efficient DMU located in the reference set of DMU_o \}$$

At the first stage, our purpose is to find vector $\lambda > 0$ so that

$$\begin{bmatrix} X_o \\ Y_o \end{bmatrix} = \sum_{j \in RF^+_o} \lambda_j \begin{bmatrix} X_j \\ Y_j \end{bmatrix}$$

and $\sum_{j \in RF^+_o} \lambda_j = 1$. In the following theorem, we prove that this can be done.

Theorem: Suppose that $DMU_o \in E'$ and $RF^+_o$ is the set defined in (5). Then, there exists a positive vector $\lambda \in \mathbb{R}^{\text{card}(RF^+_o)}$ so that

$$\begin{bmatrix} X_o \\ Y_o \end{bmatrix} = \sum_{j \in RF^+_o} \lambda_j \begin{bmatrix} X_j \\ Y_j \end{bmatrix}$$

Proof: Suppose that MF is the minimum face containing $DMU_o \in T_o$ and $t = \text{card}(RF^+_o)$. We can simply show that MF is a convex and bounded set and that $RF^+_o$ is the set of all of its extreme points. Now, let’s consider $\hat{X} = \sum_{j \in RF^+_o} \frac{1}{t} \begin{bmatrix} X_j \\ Y_j \end{bmatrix}$. Obviously, $\hat{X} \in MF$ and $1 > 0$. If

$$\begin{bmatrix} X_o \\ Y_o \end{bmatrix} = \begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix},$$

nothing remains for us to prove. Otherwise, assume that $d = \begin{bmatrix} X_o \\ Y_o \end{bmatrix} - \begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix}$.

Clearly, $d$ is a feasible direction for $\begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix}$. Now, let’s define $\gamma = \max \left\{ \gamma \left\| \frac{\hat{X}}{\hat{Y}} \right\| + \gamma d \in MF \right\}$,
and \( \begin{pmatrix} \tilde{X} \\ \tilde{Y} \end{pmatrix} = \begin{pmatrix} X_o \\ Y_o \end{pmatrix} + \gamma d. \) We observe that \( \gamma < \infty \) and \( \gamma > 1. \) (Otherwise, it contradicts the fact that MF is bounded and a minimum face). Since \( \begin{pmatrix} \tilde{X} \\ \tilde{Y} \end{pmatrix} \in MF, \) there is a \( \mu \geq 0 \) so that

\[
\begin{pmatrix} \tilde{X} \\ \tilde{Y} \end{pmatrix} = \sum_{j \in RF_o^+} \mu_j \begin{pmatrix} X_j \\ Y_j \end{pmatrix}, \sum_{j \in RF_o^+} \mu_j = 1, \mu_j \geq 0, j \in RF_o^+. \]

Using this, we can write \( \begin{pmatrix} X_o \\ Y_o \end{pmatrix} \) as follows:

\[
\begin{pmatrix} X_o \\ Y_o \end{pmatrix} = \frac{1}{\gamma} \begin{pmatrix} \tilde{X} \\ \tilde{Y} \end{pmatrix} + \frac{\gamma - 1}{\gamma} \begin{pmatrix} \tilde{X} \\ \tilde{Y} \end{pmatrix} = \frac{1}{\gamma} \sum_{j \in RF_o^+} \mu_j \begin{pmatrix} X_j \\ Y_j \end{pmatrix} + \frac{\gamma - 1}{\gamma} \sum_{j \in RF_o^+} \frac{1}{t} \begin{pmatrix} X_j \\ Y_j \end{pmatrix} = \sum_{j \in RF_o^+} \left( \frac{1}{\gamma} \mu_j + \frac{\gamma - 1}{\gamma} \frac{1}{t} \right) \begin{pmatrix} X_j \\ Y_j \end{pmatrix} \]

where \( \frac{1}{\gamma} \mu_j + \frac{\gamma - 1}{\gamma} \frac{1}{t} > 0, \) and \( \sum_{j \in RF_o^+} \left( \frac{1}{\gamma} \mu_j + \frac{\gamma - 1}{\gamma} \frac{1}{t} \right) = 1. \) This completes the proof.

Now, using following model (6), we can find a positive vector \( \lambda \in \mathbb{R}^{card(RF_o^+)} \) so that

\[
\begin{pmatrix} X_o \\ Y_o \end{pmatrix} = \sum_{j \in RF_o^+} \lambda_j \begin{pmatrix} X_j \\ Y_j \end{pmatrix}, \sum_{j \in RF_o^+} \lambda_j = 1:
\]

\[
\begin{aligned}
\text{max} & \quad \mu \sum_{j \in RF_o^+} \lambda_j X_j - X_o, \sum_{j \in RF_o^+} \lambda_j Y_j - Y_o, \\
& \sum_{j \in RF_o^+} \lambda_j = 1, \lambda_j \geq \mu, j \in RF_o^+, \mu \geq 0
\end{aligned}
\]

(6)

Given the above theorem, this model has a feasible solution, and its optimal value is positive. Suppose that \( \theta_i \) is the ranking-related score obtained from the \( L_1 \) norm model for the extreme efficient DMU \(_i\). Now, to determine the ranking-related score of DMU \(_o\), we use the following model:

\[
\begin{aligned}
\text{Max} & \quad \sum_{j \in RF_o^+} \lambda_j \theta_j, \sum_{j \in RF_o^+} \lambda_j = 1, \lambda_j \geq \varepsilon, j \in RF_o^+, \\
& \sum_{j \in RF_o^+} \lambda_j \theta_j = 0, j \in RF_o^+
\end{aligned}
\]

(7)

Where, \( \varepsilon \) is a non-Archimedean number. Because of the computational problems, instead of solving this model, we consider the dual model.

\[
\begin{aligned}
\text{Min} & \quad \sum_{i=1}^{m} v_i x_{io} + \sum_{r=1}^{s} u_r y_{or} + v_o - \varepsilon \sum_{j \in RF_o^+} \zeta_j, \sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{or} + v_o - \zeta_j = 0, j \in RF_o^+ \\
& \zeta_j \geq 0, j \in RF_o^+
\end{aligned}
\]

To solve the dual model, we solve the following two-phase model:

\[
\begin{aligned}
\text{Phase 1:} & \quad \text{Min} \quad \sum_{i=1}^{m} v_i x_{io} + \sum_{r=1}^{s} u_r y_{or} + v_o \sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{or} + v_o \geq 0, j \in RF_o^+
\end{aligned}
\]

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Phase 2:

\[
\begin{align*}
\text{Max} & \quad \sum_{j \in RF_0^+} \zeta_j \left( \sum_{i=1}^{m} v_i x_{ij} + \sum_{i=1}^{n} u_i y_{ij} + v_o - \zeta_j = \theta_j, \quad j \in RF_0^+ \right) \\
\text{Max} & \quad \sum_{j \in RF_0^+} \zeta_j \left( \sum_{i=1}^{m} v_i x_{ij} + \sum_{i=1}^{n} u_i y_{ij} + v_o = w^* \right) \\
\zeta_j & \geq 0, \quad j \in RF_0^+ 
\end{align*}
\]

(8)

Where, \( w^* \) is the optimal value of phase 1. We can find the optimal value of model (7) from the optimal value of variables and the shadow prices obtained from (8). Then, we have the optimal values of \( \lambda_j, \quad j \in RF_0^+ \). Now, we consider \( \sum_{j \in RF_0^+} \lambda_j \theta_j \) being the ranking-related score of DMU_0. By means of such scores, we can rank DMU. The greater the ranking-related score, the higher the rank.

Two Numerical Examples

Illustrative example

In this example, we use our proposed method for the ranking of 11 DMUs having two inputs and one output (Table 1). We can obtain the efficiency score and type of DMUs by means of the DEA ordinary models.

Table 1- The application of our proposed method for the ranking of 11 DMUs with two inputs and one output

<table>
<thead>
<tr>
<th>DMU</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2.5</td>
<td>3.6</td>
<td>3</td>
<td>2</td>
<td>1.75</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Input2</td>
<td>6</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>4.1</td>
<td>3.2</td>
<td>7.1</td>
<td>2.75</td>
<td>3.5</td>
<td>3.75</td>
</tr>
<tr>
<td>Output</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>3.3</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5.5</td>
<td>4.5</td>
<td>3</td>
<td>3.5</td>
</tr>
<tr>
<td>Eff-s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.5789</td>
<td>0.75</td>
<td>0.6352</td>
<td>0.9375</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>ext</td>
<td>ext</td>
<td>ext</td>
<td>ineff</td>
<td>ineff</td>
<td>ineff</td>
<td>ineff</td>
<td>ineff</td>
<td>n-ext</td>
<td>n-ext</td>
<td>n-ext</td>
</tr>
<tr>
<td>( L_{ij} ) norm</td>
<td>0.5</td>
<td>1</td>
<td>1.64</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(\( \text{ext} - \text{extreme efficient}, \text{n-ext} - \text{non-extreme efficient}, \text{ineff} - \text{inefficient}, \text{eff-s} - \text{efficiency score} \))

Now, using Sueyoshi’s model [12] for the non-extreme DMUs I, J, K, we have,

\( RF_I^* = \{A, B, C\}, RF_J^* = \{A, B\}, RF_K^* = \{A, B, C\} \)

The ranking-related score, for these three DMUs, obtained from the proposed method is (Table 2):

Table 2- Ranking-related score

<table>
<thead>
<tr>
<th>DMUs</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ranking-related score</td>
<td>0.825</td>
<td>0.75</td>
<td>0.725</td>
</tr>
</tbody>
</table>

Now, using the ranking-related scores, we can rank the efficient DMUs (Table 3).

Table 3- Ranking of the efficient DMUs, by using our proposed method

<table>
<thead>
<tr>
<th>DMUs</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
In this example, we can see a considerable point. Sometimes, a non-extreme efficient DMU may have a greater ranking-related score than does an extreme efficient DMU. For instance, DMU1 has the ranking-related score of 0.825, which is greater than the ranking-related score of DMU_A. This is logical. Because, in the making of the DMU1, there exists a DMU (DMU_C) with the ranking-related score of 1.64, having a considerable effect on the ranking-related score of DMU1.

Empirical Example

In this section, we use our proposed method to rank the efficient DMUs. We can see that the L_1 norm method cannot rank DMU_13 which is a non-extreme DMU. Data for 19 DMUs are given in table 4. Data are have been scaled. From table 5, we can see that non-extreme efficient DMU_13 has the rank of 2 among all the efficient DMUs.

Table 4- 19 DMUs with one input and three outputs

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Input</th>
<th>Output1</th>
<th>Output2</th>
<th>Output3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.130</td>
<td>0.080</td>
<td>0.110</td>
<td>0.060</td>
</tr>
<tr>
<td>2</td>
<td>0.110</td>
<td>0.040</td>
<td>0.030</td>
<td>0.150</td>
</tr>
<tr>
<td>3</td>
<td>0.162</td>
<td>0.120</td>
<td>0.080</td>
<td>0.130</td>
</tr>
<tr>
<td>4</td>
<td>0.190</td>
<td>0.090</td>
<td>0.040</td>
<td>0.110</td>
</tr>
<tr>
<td>5</td>
<td>0.235</td>
<td>0.230</td>
<td>0.130</td>
<td>0.230</td>
</tr>
<tr>
<td>6</td>
<td>0.275</td>
<td>0.080</td>
<td>0.166</td>
<td>0.060</td>
</tr>
<tr>
<td>7</td>
<td>0.250</td>
<td>0.050</td>
<td>0.070</td>
<td>0.150</td>
</tr>
<tr>
<td>8</td>
<td>0.172</td>
<td>0.130</td>
<td>0.090</td>
<td>0.160</td>
</tr>
<tr>
<td>9</td>
<td>0.400</td>
<td>0.260</td>
<td>0.090</td>
<td>0.060</td>
</tr>
<tr>
<td>10</td>
<td>0.380</td>
<td>0.190</td>
<td>0.080</td>
<td>0.060</td>
</tr>
<tr>
<td>11</td>
<td>0.300</td>
<td>0.140</td>
<td>0.110</td>
<td>0.280</td>
</tr>
<tr>
<td>12</td>
<td>0.610</td>
<td>0.060</td>
<td>0.120</td>
<td>0.250</td>
</tr>
<tr>
<td>13</td>
<td>0.261</td>
<td>0.194</td>
<td>0.122</td>
<td>0.250</td>
</tr>
<tr>
<td>14</td>
<td>0.307</td>
<td>0.134</td>
<td>0.107</td>
<td>0.200</td>
</tr>
<tr>
<td>15</td>
<td>0.119</td>
<td>0.032</td>
<td>0.030</td>
<td>0.151</td>
</tr>
<tr>
<td>16</td>
<td>0.158</td>
<td>0.117</td>
<td>0.081</td>
<td>0.120</td>
</tr>
<tr>
<td>17</td>
<td>0.160</td>
<td>0.093</td>
<td>0.100</td>
<td>0.072</td>
</tr>
<tr>
<td>18</td>
<td>0.182</td>
<td>0.122</td>
<td>0.072</td>
<td>0.096</td>
</tr>
<tr>
<td>19</td>
<td>0.280</td>
<td>0.139</td>
<td>0.112</td>
<td>0.250</td>
</tr>
</tbody>
</table>

Table 5- The ranking of efficient DMUs by using our proposed method

<table>
<thead>
<tr>
<th>Efficient DMUs</th>
<th>ext</th>
<th>n-ext</th>
<th>L_1 norm</th>
<th>Ranking-related score (our proposed method)</th>
<th>Rank (L_1 norm)</th>
<th>Rank (our proposed method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>*</td>
<td>-</td>
<td>0.040</td>
<td>0.040</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>*</td>
<td>-</td>
<td>0.013</td>
<td>0.013</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>*</td>
<td>-</td>
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<td>1</td>
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<tr>
<td>6</td>
<td>*</td>
<td>-</td>
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<td>9</td>
<td>*</td>
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</table>

Conclusion

In this paper, we proposed a method to find a ranking-related score for efficient non-extreme DMUs. Our approach is based on the finding of a positive convex combination of extreme efficient DMUs on the minimum face containing the DMU under evaluation. For this purpose, we can find these extreme DMUs by using model (4). Then, we obtained a score for non-extreme DMUs by using models (6), (7) (or by it’s dual). In the numerical example, we
can see that a non-extreme efficient DMU may have a greater ranking-related score than does an extreme efficient DMU. This is acceptable, because, unlike the extreme efficient DMUs, in the construction of a non-extreme efficient DMU, there is another DMU whose performance influences both the performance and the ranking-related score of the non-extreme efficient DMU.

There are some important points as follows:
1. Although, we rarely have got observed non-extreme efficient DMUs, But almost all of efficient DMUs are non-extreme. Using this approach, DM can select an appropriate target among different designated targets obtained from various target setting models. Therefore, if a DM wishes to select one of such targets with the ranking criterion in mind, our method can be used.
2. A person may ask, what should be done if the ranking-related score of two DMUs be the same? For answering this question, it can be pointed out that, practically, the ordinary ranking models like AP and $L_1$ norm model and so on, such a problem does not occur. For example in a computer with 16 digits, the probability of occurrence this kind of problem is almost $10^{-16}$.
3. In this article, we only considered the convex combination of efficient extreme points, and by doing this, we eliminate efficient non-extreme points located on the minimum face. Since, they have not any effect on making face and units.
4. One can use non-negative convex combination instead of positive convex combination. For example, to choose a target point, with higher rank rather than other candidate points, non-negative convex combination can be considered. We would want to compare units with this assumption that, all of the extreme DMUs making non-extreme DMU effect on performance and rank, not just some of them.

In this paper, we had the variable returns to scale in mind. We may, however, be able to do something similar for the constant returns to scale.

References: