Journal of Theoretical and Applied Physics, **4**-2, 39-43 (2010)

Applications of Nijmegen potentials for elastic neutron-deuteron low energy scattering

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Received: 12 July 2010/Accepted: 18 September 2010/ Published: 20 September 2010

Abstract:

Differential cross sections for neutron-deuteron elastic scattering at low energies (E_n =1, 2.45 and 3.27 MeV) were calculated using realistic potentials (Nijm 93, Nijm I and Nijm II) which had been calculated numerically. Our calculations which based on Faddeev's equations show a satisfactory agreement with modern experimental data. We also compared our calculations with recent one's that based on CD-Bonn realistic potential. Comparison shows a good agreement as too.

***PACs*:** 28.41.Ak; 28.52.Ab; 25.40.Dn; 29.85.Ca; 25.40.Lw
Keywords: *Realistic potential; Nijm 93; Nijm I; Nijm II; Faddeev's equations.*

1. Introduction

Three-nucleon system is the simplest non-trivial testing ground in which the quality of modern nucleon-nucleon interaction models can be probed quantitatively by means of rigorous technique of various ways, among them solving the famous Faddeev equations [1], that became the base of the most known methods of calculations of three-particle wave function.

One of the ways that had been constructed to solve Faddeev's equations is the expansion of the full wave function [2], where the most complicated part of the full wave function of Faddeev equation that describe the three-nucleon motion in the nucleon-nucleon interaction region is separated and expanded into a series in hyperspherical polynomials.

We can provide quantitative descriptions for three-body interactions by using nucleon-nucleon potentials in the Faddeev equations. In the past decades, new nucleon-nucleon potentials, which are based on meson-exchange theories, have been developed. These models are able to provide a very well description of proton-proton and neutron-proton data base below 350 MeV, like the Nijmegen group [3], Argonne group [4], and Bonn group [5, 6], these potentials reproduce the nucleon-nucleon scattering data, and fitted to the properties of the deuteron and phase shifts with high precision.

Nijmegen group made large effort in the past years that concentrated on the study of the baryon-baryon [7-9], as well as the antibaryon-baryon interaction [10,11], later partial-wave analyses (PWA) of the experimental scattering data [12,13] were performed and helped in the construction of new, improved potentials [11,14-16].

The aim of this work is to obtain the differential cross sections for neutron-deuteron elastic scattering at low energy, based on the expansion of the full wave function in Faddeev's equations and using the realistic nucleon-nucleon Nijmegen potentials (93, I and II) which can be calculated numerically.

2. Formalism

Our calculations are based on the well known Faddeev equation 1, which can be written in T-matrix formalism as [17],

$$T_{ij}(z) = t_{ij}(Z) + t_{ij}(Z)G_0(Z)\left[T_{ik}(z) + T_{jk}(z)\right]. \quad (1)$$

In wave function formalism:

$$\begin{aligned} \Psi^{(1)} &= \Phi + G_0(Z)t_{23}(Z)\left(\Psi^{(2)} + \Psi^{(3)}\right), \\ \Psi^{(2)} &= G_0(Z)t_{31}(Z)\left(\Psi^{(3)} + \Psi^{(1)}\right), \\ \Psi^{(3)} &= G_0(Z)t_{12}(Z)\left(\Psi^{(1)} + \Psi^{(2)}\right), \end{aligned} \quad (2)$$

$$\Psi = \Psi^{(1)} + \Psi^{(2)} + \Psi^{(3)}, \quad (3)$$

where, t_{ij} are the two-particle transition operators which are connected with pair wise potentials $V_{ij}(i \neq j \neq k = 1, 2, 3)$ by the Lippman–Schwinger equation:

$$t_{ij}(Z) = V_{ij} + V_{ij}G_0(Z)t_{ij}(Z). \quad (4)$$

Consider the difference Ψ_C which is a three-nucleon function that represents the solution of the Faddeev equations 2:

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$$\Psi_C = \Psi - \psi\Psi_s(\overrightarrow{x_1}), \qquad (5)$$

and expand it into a series of hyper spherical polynomials, where, $\psi = \phi_d(r_{23})$ is the deuteron wave function, $\vec{r}_{12} = \vec{x}_1 - \frac{\vec{r}_{23}}{2}$ and $\vec{r}_{31} = -\vec{x}_1 - \frac{\vec{r}_{23}}{2}$, $\Psi_s(\overrightarrow{x_1})$ describes the scattering of a neutron by a deuteron, and satisfies the Schrödinger equation:

$$\frac{\partial^2}{\partial\vec{x}^2}\Psi_s(\overrightarrow{x_1}) + k^2\Psi_s(\overrightarrow{x_1}) = U\Psi_s(\overrightarrow{x_1}), \qquad (6)$$

$$U = \frac{4m(V_{12}+V_{31})}{3}, \qquad k^2 = \frac{4mE}{3}, \qquad E = \frac{2E_n}{3}$$

m is the nucleon mass, $V_{ij} \equiv V(r_{ij})$ with ij =(12, 23, 31) is the pair nucleon-nucleon potential, and x_1 is the distance between the incident neutron and the deuteron center of mass.
At $x_1 \to \infty$, we have:

$$\Psi_s(\overrightarrow{x_1}) = \exp(i\vec{k}\vec{x}_1) + f(\theta)\frac{\exp(ikx_1)}{x_1}, \qquad (7)$$

where,

$$f(\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}(2\ell+1)(\exp(2i\delta_\ell)-1)P_\ell(\cos\theta) \qquad (8)$$

We can rewrite Eq. 5 as:

$$\Psi_C = (\Psi - \Phi) - \phi_d(r_{23})f(\theta)\frac{\exp(ikx_1)}{x_1}, \qquad (9)$$

where Φ is the Faddeev asymptotic wave function which is a product of the internal wave function of the deuteron and a plane wave.
The general solution of Eq. 6 in the whole space of the variable $\vec{x}_1$ can be presented in the form

$$\Psi_s(\overrightarrow{x_1}) = \exp(i\vec{k}\vec{x}_1) + \frac{1}{kx_1},$$
$$\sum_{\ell=0}^{\infty}[(2\ell+1)\ i^\ell\psi_{\ell k}(x_1)\ P_\ell(\cos\theta)], \qquad (10)$$

where,

$$\psi_{\ell k}(x_1) = \exp(i\delta_\ell)\left[cos\delta_\ell F_\ell(kx_1) + sin\delta_\ell G_\ell(kx_1)\right], \qquad (11)$$

and

$F_\ell(kx_1) = kx_1 j_\ell(kx_1)$, where $j_\ell(kx_1)$ is the Bessel spherical function, $G_\ell(kx_1) = kx_1 n_\ell(kx_1)$, where $n_\ell(kx_1)$ is the Neumann spherical function.
The three-nucleon function 5 which is the solution of the Faddeev equations, can be found by expanding it in a series of hyper spherical polynomials

$$\Psi_C = \frac{B(\rho)}{\sqrt{\pi^3}}, \qquad (12)$$
$$B(\rho) = \pi^{-\frac{1}{2}}\rho^{-2}m\int d\rho'\rho'^3 P(\kappa,\rho,\rho')\int d\Omega(V_{12} + V_{31})\Phi$$
$$+\pi^{-2}\rho^{-2}m\int d\rho'\rho'^3 B(\rho')P(\kappa,\rho,\rho')\int d\Omega(V_{12} + V_{31} + V_{23}), \qquad (13)$$

where the function $P(\kappa,\rho,\rho')$ is defined as

$$P(\kappa,\rho,\rho') = -i\left[J_2(\kappa\rho)H_2^{(1)}(\kappa\rho')\Theta(\rho'-\rho) + J_2(\kappa\rho')H_2^{(1)}(\kappa\rho)\Theta(\rho-\rho')\right], \qquad (14)$$

where

$$\kappa = \sqrt{2m(E-\varepsilon)},$$

ε is deuteron binding energy,
$J_2(x)$ is the Bessel function of the second order,
$H_2^{(1)}(x)$ is the Hankel function of the second order,
$\Theta(x)$ is the Heaviside function.

The calculations [2] showed that the function $B(\rho)$ is different from zero only for ρ-values smaller than about 4 fm (in the region of nuclear force action between all the three nucleons); beyond this region, $B(\rho) \to 0$.

The differential cross section for neutron-deuteron scattering can be written as:

$$\sigma(\theta) \equiv \frac{d\sigma}{d\theta} = \frac{2}{3}|a_4|^2 + \frac{1}{3}|a_2|^2, \qquad (15)$$

where a_4 is the quartet scattering length and a_2 is the doublet one.
For simplicity, we assumed:

$$\sigma(\theta) \cong \frac{2}{3}|a_4|^2. \qquad (16)$$

Thus, we can calculate $\sigma(\theta)$ by the following way

$$\sigma(\theta) = \frac{2}{3}|A(\theta)|^2, \qquad (17)$$

where,

$$A(\theta) = \frac{m}{3\pi}\langle\Phi|V_{12} + V_{31}|\Psi\rangle.$$

3. Results and discussion

Our calculations were based on modern realistic Nijmegen 93, Nijmegen I and Nijmegen II potentials [3], which fit well the nucleon-nucleon scattering data. We used Fortran 77 codes to calculate the Nijmegen potentials which are available in. The basic functions of Nijmegen potentials are the one-boson-exchange (OBE) potential functions with momentum-dependent central terms and exponential form factors.

The meson exchanges it include are those due to pseudoscalar mesons (π,η,η'), vector mesons (ρ,ω,ϕ), and scalar mesons (a_0,f_0,ϵ). For proton-proton scattering the potential consists of only neutral-meson exchange, $V_{pp} = V(\text{neutral})$, whereas for neutron-proton scattering it consists of neutral meson and charged meson exchange, depending on the total isospin [3],

$$V_{np} = -V(\text{neutral}) \pm 2V(\text{charged}).$$

In Nijmegen potentials, the quadratic spin-orbit operator of the potential in momentum space was adjusted so that it leads to equivalent description of the potential in both the momentum and configuration space, which is a unique feature of these Nijmegen potentials.

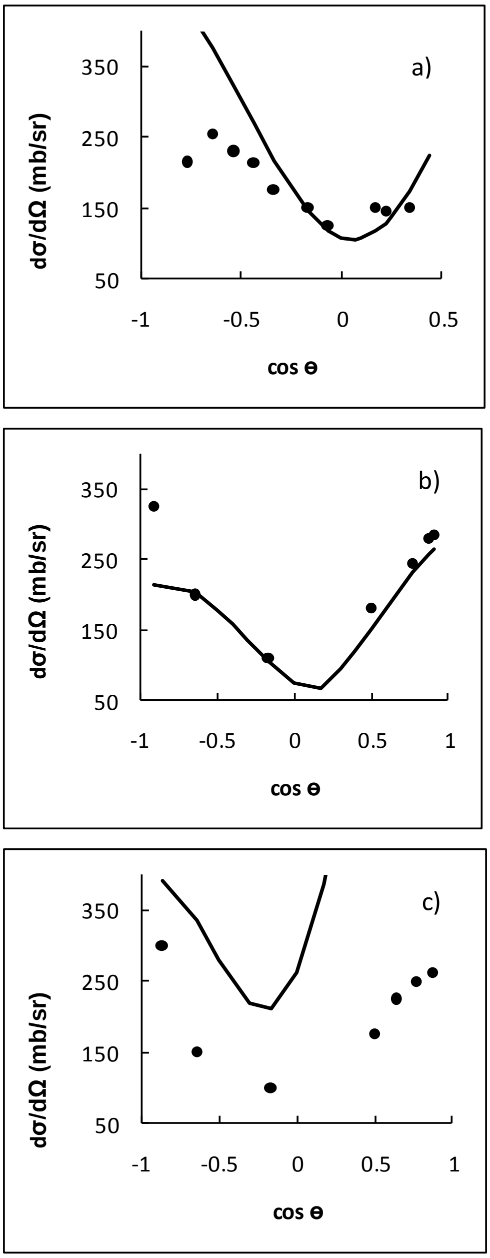

Fig. 1. Differential cross sections of neutron-deuteron scattering at $E_n = 1$ MeV (a) 2.45 MeV (b) 3.27 MeV (c). The curves were computed using the Nijm93 potential (solid line). Experimental data (bold points) were taken a) from [18] b) and c) from [19].

Nijm93 is the model that extends the update of the previous Nijmegen 78 model to include the fit to neutron-proton scattering data. The np $^1S_0$ partial wave has to be parameterized separately because of breaking of charge independence in the $^1S_0$ scattering lengths a_{pp} and a_{np}.

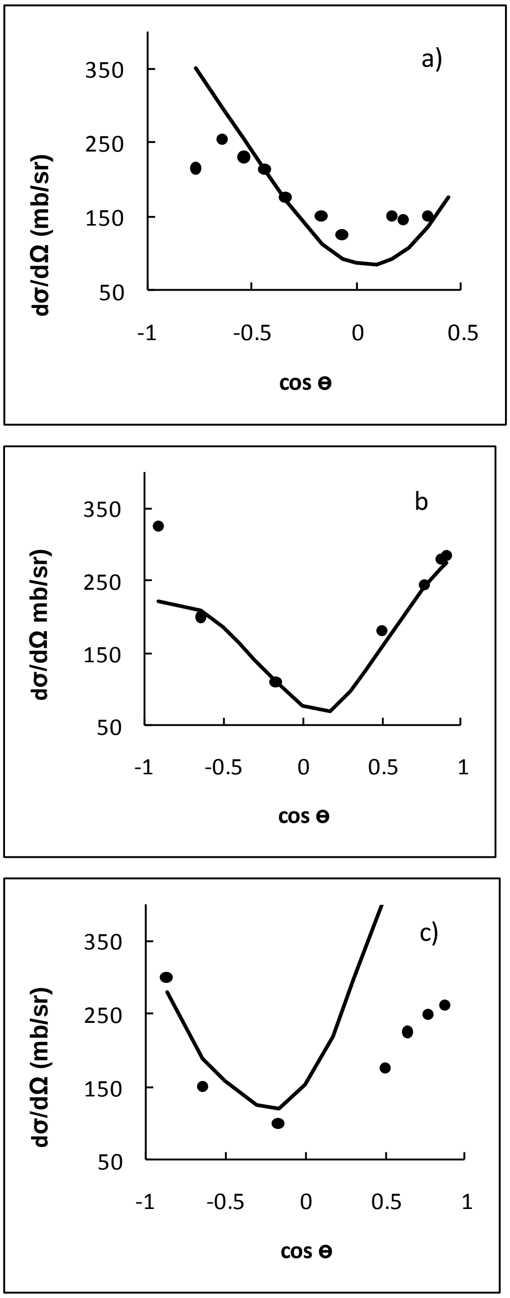

Fig. 2. Differential cross sections of neutron-deuteron scattering at $E_n = 1$ MeV (a) 2.45 MeV (b) 3.27 MeV (c). Experimental data are presented by bold points while Nijm I potential is presented by solid line.

We calculate the neutron-deuteron differential cross section using Nijm93, and plot the results in Fig. 1. The figure shows kind of good fitting at 1 MeV with the experimental data, especially at the range where the differential cross sections have low values, and even it gives a better agreement at 2.45 MeV with experiment, we can see discrepancies at 3.27 MeV, we will see almost the same behavior for the rest of Nijmegen potentials that we used in our calculations, the reason of that could be related to our assumption that $\Psi_C \to 0$, or because our use of purely nucleon-nucleon potential without mixing it up with three-nucleon potential without mixing it up with three-nucleon force.In Nijmegen I, the parameters of the potential are adjusted in each partial wave separately, that make the description of the data more accurate. This potential contains momentum-dependent terms (as do Nijm93), which in configuration space give rise to a non local structure to the potential.

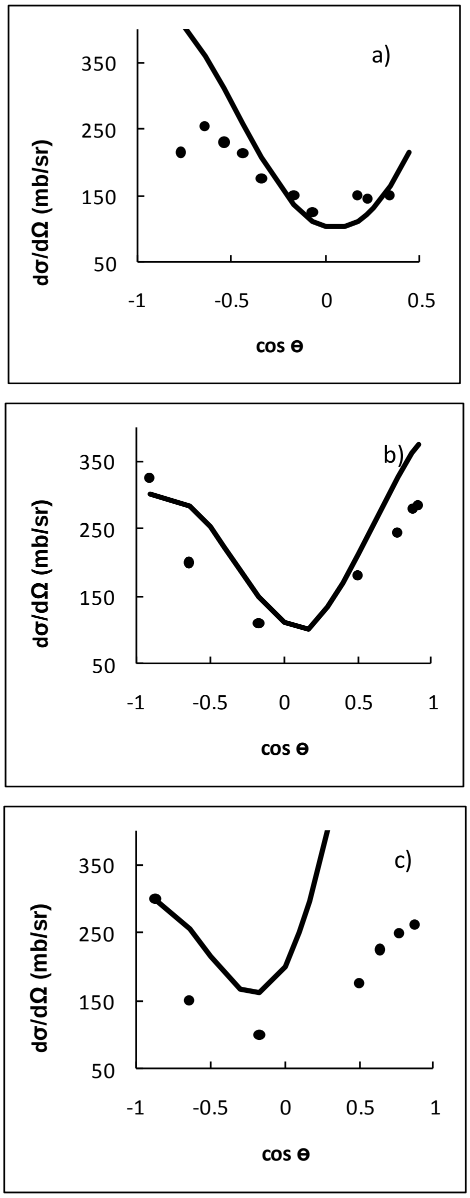

Fig. 3. Differential cross sections of neutron-deuteron scattering at $E_n = 1$ MeV (a) 2.45 MeV (b) 3.27 MeV (c). Experimental data are presented by bold points while Nijm II potential is presented by solid line.

The calculated differential cross sections for neutron-deuteron scattering using Nijm I are shown in Fig. 2, one can see from the figure that Nijm I gives a very good agreement with the experimental data, specially at 2.45 MeV. For $E_n = 3.27$ MeV, the accuracy decreases with the increases of the cosine of the scattering angle, but it is still more accurate than Nijm93 for that energy.

The Nijm II potential is similar to the Nijm I potential, but with all non-locality in each partial wave removed, where the explicit momentum-dependent terms had been left out which give rise to non-local contributions to the configuration-space potential.

In Fig. 3 one can see the differential cross sections which had been calculated using Nijm II for neutron-deuteron scattering, it is clear from the figure that Nijm II gives good agreement at 1 MeV, but it is less accurate than Nijm I and Nijm93 at 2.45 MeV. For $E_n = 3.27$ MeV we can see the same behavior that Nijm I and Nijm93 go, probably for the same reasons. Fig. 4 compares between the differential cross sections at $E_n = 1$ MeV that had been calculated by us, and the one been calculated by the CD-Bonn potential, the figure shows that the potentials starts with good agreement between each other, especially with Nijm93, Nijm II and CD-Bonn potential, but with the increases of the cosine of the scattering angle some significant differences appears between them.

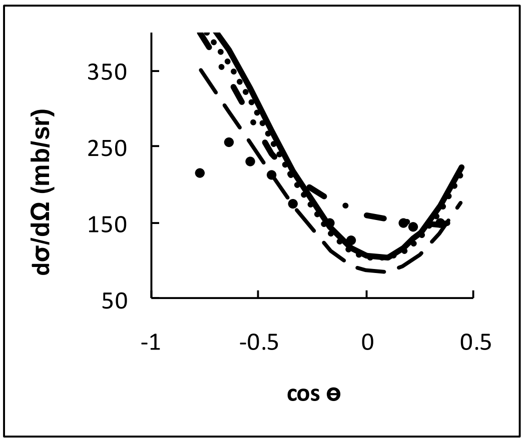

Fig. 4. Comparison between our calculations of the differential cross sections of neutron-deuteron scattering with Nijmegen potentials and the calculations with CD Bonn potential at $E_n = 1$ MeV. Nijm93 data are presented by solid line, while results of Nijm I are presented by dashed line. Nijm II date are presented by dotted line, and CD Bonn results are shown by dashed-dotted line. Experimental data are shown by bold points.

However, we should point out that the description of systems made of more than two nucleons is not complete if three-body forces (3NF) are not taken into account, so these calculations can be improved by adding 3NF effects to the NN potentials, that make it gives a better agreement between cross section data in nucleon-deuteron scattering experiments and the theoretical calculations.

4. Conclusions

We calculated that the differential cross-sections for neutron-deuteron elastic scattering, using various realistic potentials that were calculated numerically, and we conclude that the comparison of our calculations with the experiments showed a satisfactory agreement at low energy scattering. The comparison of our calculations with recent theortical calculations that based on CD-Bonn potential showed a good agreement as well.

Finally it should be noted that, in order to improve the agreements with experimental data we may need to modify the nucleon-nucleon potential parameters, and take in to account the three-nucleon force in our calculations.

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