Sensorless Speed Control of Switched Reluctance Motor Drive Using the Binary Observer with Online Flux-Linkage Estimation

S. Sedghizadeh*, C. Lucas** and H. Ghafoori Fard***

Abstract: An adaptive online flux-linkage estimation method for the sensorless control of switched reluctance motor (SRM) drive is presented in this paper. Sensorless operation is achieved through a binary observer based algorithm. In order to avoid using the look up tables of motor characteristics, which makes the system, depends on motor parameters, an adaptive identification algorithm is used to estimate of the nonlinear flux-linkage parameters. This method makes position and speed estimation more accurate and robust towards any model uncertainty, also it is suitable replacement for a priori knowledge of motor characteristics.

Keywords: Sensorless control, Switched reluctance motor (SRM), Binary observer, Adaptive identification.

1 Introduction
Switched reluctance motors (SRMs) have been the focus of many researches over the past decades. The simple mechanical construction is one of the main attractive features because it has no windings or permanent magnets on the rotor so its manufacturing cost appears to be lower compared to other motor types. Also it has the advantages of simple structure, low cost and high efficiency. However, the rotor position sensing requirement is a disadvantage for SRM. Using the position sensors occupy extra space and add to the total cost of drive, they also reduce the reliability of the drive. So replacing such sensors by suitable estimation methods seems better. A large number of papers of sensorless control methods have been published in the last decade, and all of the indirect rotor position sensing methods is based on measurements of the periodically varying phase inductance or phase flux [1].

In this paper an online estimation of flux linkage characteristics method is proposed to achieve sensorless velocity control of the SRM. In this method actual flux linkage can be computed directly from terminal measurements: voltage and current of the phases and estimated flux linkage is obtained by the adaptive estimating algorithm. Actual and estimated flux linkages are used by a binary observer to estimate the velocity and rotor position. We must notice that all these processes are done online. The algorithm is tested on simulation of a real 6/4 motor that we manufactured and the relevant test results are presented. Although we did not actually construct the electronic drive to implement the proposed method experimentally as well, the validity of the simulation models for the electromotor under consideration has been extensively studied via both FE calculations and laboratory experimentation as reported in [24, 25] and its references. The method is suitable for velocity control applications, such as drill machines, washing machines and banknote counting machines.

The paper is organized as follows:
Section 2 is a short review of the existing sensorless methods. Section 3 introduces the dynamic model of SRM and system differential equations. Section 4 proposes an online flux-linkage estimation method. Section 5 represents the binary observer. Section 6 represents the complete block diagram of the system. Section 7 tests the proposed method by simulation on a 6/4 SRM and reports the most important results. Section 8 concludes the paper.

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2 A Short Review of Existing Sensorless Methods

Several indirect position sensing methods have been suggested and published for sensorless control of SRM drives. They can be divided into two groups.

1. Non-intrusive methods, where position information is obtained from terminal measurements of voltages and currents and associated computation. These methods rely on the machine characteristics for estimating the rotor position and the terminal measurements of phase voltage or mutual voltage and current are used as inputs for an estimator to obtain the rotor position. The wave form detection techniques [2], model-based or observer-based estimator techniques [3]-[7], the flux-current method [8] and the mutual voltage method [9, 10] are examples of methods that fall under this category. Observer based methods and flux-current methods are examples of schemes that could be used for high resolution position sensing. These methods are also suitable for position sensing at high speeds, but high speed computational requirements tend to increase the cost of these types of indirect sensors even more.

2. Intrusive methods, where low level, high frequency signals are injected into an idle phase to determine the position dependent, unsaturated phase inductance characteristics. Methods based on monitoring current waveforms or passive waveform detection [11, 12], modulation based techniques [13] and flux sensing techniques [14] are examples of methods belonging to this family. The simplicity of these methods is a definite advantage, although inherent speed limitation and generation of negative torque in the sensing phases could be a drawback in some cases.

3 System Differential Equation and Dynamic Model of SRM

The dynamic characteristics of SRM consist of its electromagnetic, torque and mechanical equations. By neglecting the mutual effect of the phases, the voltage applied to one phase of SRM can be expressed as follows:

\[
v_j = R \cdot i_j + \frac{d\lambda_j}{dt}, \quad j = 1, \cdots, n
\]

where,

\[
v_j: \text{Voltage of the phase } j
\]

\[
i_j: \text{Current of the phase } j
\]

\[
R: \text{Phase winding resistance}
\]

\[
\lambda_j: \text{Flux linkage of the phase } j
\]

\[
n: \text{Number of phases.}
\]

In the SRM, flux is a function of both the current and the rotor position,

\[
\frac{d\lambda_j}{dt} = \frac{\partial \lambda_j}{\partial i_j} \frac{di_j}{dt} + \frac{\partial \lambda_j}{\partial \theta_j} \frac{d\theta_j}{dt}
\]

where,

\[
\theta: \text{Rotor position}
\]

So the phase current can be expressed as follows,

\[
\frac{di_j}{dt} = \left( \frac{\partial \lambda_j}{\partial i_j} \right)^{-1} \left( v_j - R \cdot i_j - \frac{\partial \lambda_j}{\partial \theta_j} \frac{d\theta_j}{dt} \right)
\]

When each phase of the SRM is exited, it produces an instantaneous torque. The instantaneous torque of each phase can be shown as below:

\[
T_j = \frac{1}{2} \frac{\partial \lambda_j}{\partial \theta_j} \frac{di_j}{dt}
\]

and the total torque of the motor can be found by adding the instantaneous torques of the phases, as follow,

\[
T_n = \sum_{j=1}^{n} T_j[j, \theta]
\]

Mechanical equations can be expressed as below:

\[
T_n - T_L = J \frac{d\omega}{dt} + B \cdot \omega
\]

where,

\[
T_n: \text{Load torque}
\]

\[
\omega: \text{Rotor speed}
\]

\[
J: \text{Moment of inertia}
\]

\[
B: \text{Coefficient of friction.}
\]

When equation (6) is re-arranged in order to write the speed we found the following equation:

\[
\frac{d\omega}{dt} = \frac{1}{J} (T_n - T_L - B \cdot \omega)
\]

In this equations the phase flux linkage \( \lambda_j \), the rotor position \( \theta \) and the rotor speed \( \omega \) constitute the state variables.

We must notice that, in this modeling \( \lambda \) and \( T_n \) are the nonlinear parameters and usually they are derived using flux-current-angle \( (\lambda - i - \theta) \) and torque-current-angle \( (T - i - \theta) \) characteristics data, obtained by finite element analysis [15] or by some approximation modeling techniques, such as adaptive identification methods [16]-[17] and neural networks based or fuzzy logic based methods [18]-[22]. The flux linkage characteristic of the 6/4 SRM that we manufactured has shown in Fig. 1. As shown in Fig.1 the flux linkage characteristic is nonlinear because the lines are not the equidistance in different angles.
4 Online Flux Estimation Method

In this research, an adaptive self–tuning method is proposed to flux-linkage estimation. As shown in Fig. 1 we can divide the magnetic characteristics of the SRM into two main regions, linear and saturated, according to the level of current. The linear region is distinguished by a current level less than the so-called saturation value $I_s$. The saturated region is located beyond $I_s$. This division can be applied to all magnetic characteristics as represented in [1], [18], [20] and [23]. We put the $I_s$ as a constant value in this algorithm. Our assumption of fixed $I_s$, despite its obvious shift, for example as seen in Fig. 10, can still lead to acceptable control performance. We have also studied errors caused by the assumption of local models before and after the saturation currents else where and found them acceptable for our purposes. In this method the flux-linkage in the $j$th phase is assumed to be given by,

$$
\lambda_j (\hat{\theta}, i) = \begin{cases} 
K_j(\hat{\theta}) \cdot i, & i < I_s \\
K_j(\hat{\theta}) \cdot [1 + K_j(\hat{\theta})] \cdot i, & i \geq I_s 
\end{cases}
$$

(8)

for $j = 1, \ldots, n$ where $I_s$ is the saturated current as shown in Fig. 1 and $K_j(\hat{\theta})$ is a second order function that can be expressed as follows:

$$
K_j(\hat{\theta}) = a_i + b_i \hat{\theta} + c_i \hat{\theta}^2, \quad i = 1, 2, 3
$$

(9)

where $a_i$, $b_i$, and $c_i$ are the coefficients that estimated online by recursive least square (RLS) method for each $K_j(\hat{\theta})$. The RLS algorithms can be expressed as follows:

$$
X_{k+1} = X_k + Z_{k+1} [Y_{k+1} - \Phi^T_{k+1} X_k] \\
Z_{k+1} = P_{k+1} \Phi_k = P_k \Phi_k \left[1 + \Phi^T_k P_k \Phi_k \right] \\
P_{k+1} = \frac{1}{\gamma} [I - Z_{k+1} \Phi^T_k] P_k
$$

(10)

where,

- $P$: Covariance matrix
- $\gamma$: Learning rate, $0.95 < \gamma < 1$

The initial value of covariance matrix must be greater than zero. During the estimation procedure, the parameters are converged to values which account for the electromagnetic characteristics of the machine including the effects of magnetic saturation. This method makes the system robust towards any model uncertainty and changes in characteristics of the switched reluctance machine.

5 Binary Observer for SRM

In this work, a binary observer is used to provide the estimated value of the rotor angle $\hat{\theta}$ and speed $\hat{\omega}$. A second order binary observer can be configured as below:

$$
\frac{d\hat{\theta}}{dt} = \hat{\omega} + k_\theta e_f
$$

(11)

$$
\frac{d\hat{\omega}}{dt} = k_\omega e_f
$$

(12)

where,

- $\hat{\theta}$: Estimated rotor position
- $\hat{\omega}$: Estimated speed
- $e_f$: Error function
- $k_\theta, k_\omega$: Constant gains

The error function defined as follow:

$$
e_f = \sum_{j=1}^{n} \left| \mu_j \hat{\lambda}_j - \lambda_j \right|
$$

(13)

with,

$$
\frac{d\mu_j}{dt} = -k_j \left( \mu_j - \text{sgn}(\lambda_j - \hat{\lambda}_j) \right)
$$

(14)

where,

- $\lambda_j$: Estimated flux-linkage
- $k_j$: Constant gain

The constant $k_j$ must be selected suitably. If $k_j$ is selected very large, it makes $\mu_j$ discontinuous and if $k_j$ is selected very small, $\mu_j$ is continuous but it take a long time for the estimation error of the rotor speed to converge zero. The more detail and stability analysis of the binary observer appear in [6].

6 Complete Drive System

The general block diagram of the observer-based adaptive estimator for SRM drives has shown in Fig. 2. The system has three main parts:

1. SRM drive.
2. Binary observer.
3. Flux estimator.
Block diagram of the observer-based adaptive sensorless SRM drive.

Fig. 2. Block diagram of the observer-based adaptive sensorless SRM drive.

Flux-current-angle \((\lambda - i + \theta)\) characteristics

Fig. 3. Flux-current-angle \((\lambda - i + \theta)\) characteristics

The terminal measurements of phase current and voltages are used to calculate and estimate the flux-linkage and then the error of the calculated and estimated values are used in binary observer to estimate the speed and angle. A fuzzy PID controller was used to speed controller.

7 Simulation Results

In this section we represent the simulation results that obtained from the SRM drive. A 4KW, 3 phases, 6/4 SRM was used to test the online flux-linkage estimation technique though simulation. This motor was manufactured and we use the real parameters. Table 1. shows the parameters of SRM.

<table>
<thead>
<tr>
<th>Table 1. The parameters of SRM</th>
</tr>
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<tbody>
<tr>
<td>Unaligned inductance (L_u = 6.5\text{mH})</td>
</tr>
<tr>
<td>Aligned inductance (L_a = 126.3\text{mH})</td>
</tr>
<tr>
<td>Phase winding resistance (R = 0.5\text{ohms})</td>
</tr>
<tr>
<td>Coefficient of friction (B = 0.004)</td>
</tr>
<tr>
<td>Moment of inertia (J = 0.005\text{kgm}^2)</td>
</tr>
<tr>
<td>Nominal load torque (T_L = 13.5\text{Nm})</td>
</tr>
<tr>
<td>Nominal rotor speed (N_n = 3000\text{rpm})</td>
</tr>
</tbody>
</table>

The magnetic characteristic of this motor has shown in Fig. 3. In this simulation the motor current was limited to 30A and the motor was simulated at the nominal speed 3000rpm and nominal load torque 13.5Nm. The initial value of all estimated parameters \(a, b, c\) was chosen \(0.0001\) and the initial value of covariance matrix was chosen \(P(0) = (50.1) \cdot I_{3x3}\). The sampling time of simulation is \(15\mu s\). The saturation current level is \(I_s = 4.8A\). To obtain the better performance at the different speed level and different load torque conditions, especially in robustness test, the observer gains \(k_a\) and \(k_o\) were readjusted automatically.

7.1 Work at Nominal Condition

Fig. 4 shows the actual and estimated flux-linkage for each phase. Fig. 5 shows the speed and position estimation at 3000rpm with initial value for speed at 500rpm. Estimation error for speed \(e_o = \omega - \hat{\omega}\) and position \((e_\theta = \theta - \hat{\theta})\) is shown at Fig. 6.

Fig. 7 shows convergence dynamics of the estimated parameters for each phase.

Fig. 4. Actual and estimated flux-linkage for each phase
As we said before the method is suitable for velocity control applications, such as drill machines, washing machines and banknote counting machines and it is not suitable for position control applications, for which the robustness of our proposed method seems doubtful and already in fig. 6 steady state error seems to be much.

### 7.2 Robustness Test

This method is robust towards any disturbance and model uncertainty. Fig. 8 shows the speed tracking capability of the estimated speed in tracking the actual speed and disturbance rejection when occurs a step change in load torque from 10Nm to 15Nm at t=0.4sec in close loop system. Increasing the load torque forces the actual speed to drop, but the speed controller and binary observer can follows the change and rejects the disturbance finely. The binary observer gains were adjusted to obtain better performance. Fig. 9 shows the speed tracking of the close loop system by changing the command speed from 300rad/sec to 280rad/sec at t=0.4sec. As shown in Fig. 9 the estimated speed can follow this change.

In order to show the effect of model uncertainty we change some of the SRM modeling parameters. Fig. 10 shows effect of changing the unaligned inductance and...
aligned inductance in the flux-linkage characteristics of the SRM. The new values are \( L_u = 3 \text{ mH} \) and \( L_a = 40 \text{ mH} \).

Fig. 11 shows the actual and estimated speed and position under this condition. As shown in Fig. 11 the system works finely in this condition.

Fig. 8. The estimated and actual speeds when the load torque varied from 10Nm to 15Nm at \( t=0.4\text{sec} \) in close loop system.

Fig. 9. The estimated and actual speeds under command speed variation from 300rad/sec to 280rad/sec at \( t=0.4\text{sec} \) in close loop system.

Fig. 10. Flux-current-angle\((\theta, i, \lambda)\) characteristics of the SRM when \( L_u = 3 \text{ mH} \) and \( L_a = 40 \text{ mH} \)

8 Conclusions
An online adaptive flux-linkage estimation method is represented in this paper, in order to achieve sensorless speed control of the SRM. RLS algorithm and a second order binary observer are used to estimation and sensorless operation. The method is applied on the simulation of real 3 phases, 6/4 SRM that we manufactured and modeled by Miller model. Although, there is steady state error on angle estimation, the simulation results show that it works well for velocity estimation and control. Furthermore, the robustness of this method is tested towards any disturbance and model uncertainty by changing the load torque, command speed and aligned and unaligned inductance in the flux-linkage characteristics separately and satisfactory results are obtained and algorithm could estimate the velocity without any steady state error.

References


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