Magnetic susceptibility as a tool for mineral exploration
(Case study: Southern of Zagros Mountains)

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Abstract
Magnetic susceptibility has been extensively used to determine the magnetic properties of rocks for different applications, such as hydrocarbon or mineral explorations. This magnetic quantity can be directly measured in an accurate but time-consuming operation, or it can be mathematically approximated using a reliable procedure to achieve a desired accuracy. The Poisson theory is one of the most well-known approaches which provide a meaningful relationship between the earth’s gravity and magnetic fields to derive the magnetic susceptibility. In this approach, the reliability and efficiency of the derived magnetic susceptibility depends on the method of computation of the gravity gradient tensor. We investigated two different methods of determination of gradient tensor; different distance method and Fourier transform technique. From the investigation, the Fourier transform method was more consistent with the geological features which led to more reliable information required for mineral explorations. The performance of the Poisson theory, the different distance method, and the Fourier transform was investigated in the coastal Fars, in Iran. This was highly disposing for geological and mineral features. Salt domes in the study area were detected and results compared with the available geological map.

Keywords: fourier transforms, gravity gradient tensor, magnetic susceptibility, salt glacier.

1. Introduction
Studying the physical structure of the Earth is one of the most important purposes in geophysical or geological applications, such as mineral or hydrocarbon explorations. Due to the specific characteristics of the Earth’s magnetic field, it provides an appropriate tool for identifying disposed areas for mineral and hydro-carbonic explorations. Magnetic susceptibility which is defined as the degree of magnetization of materials in an applied magnetic field [1], has always been used as a practical tool for solving geological, geophysical and mineral problems [2,3,4,5]. Many studies have been done to evaluate the
performance of magnetic susceptibility in different applications. Müllerová and Müller (1972) in their study applied magnetic susceptibility so as to determine the type of rocks or minerals. Ellwood and Wenner (1981), Rochette (1987a), Hrouda and Kahan (1991), Heller (1998) utilized magnetic susceptibility for magnetic explorations. Plimer (1985) in his study detected magnetic changes of the minerals based on magnetic susceptibility, oxidation and phase transition. In addition, Evans and Heller (2003) also used magnetic susceptibility to detect the structural changes caused by climate changes.

Magnetic susceptibility was commonly measured in the laboratories or in the field using the Kappa-meter. Regardless of the measurement errors, although, direct measurement of magnetic susceptibility can provide more accurate information about geological surface structures of an area, but these measurements are time-consuming and as such requires expensive equipment. As a result, different approximation methods of the magnetic susceptibility have been investigated.

One of the most widely used methods of approximating magnetic susceptibility is based on the Poisson equation. The density of the Earth provides a relationship between magnetic and gravitational potentials which is called Poisson equation. Based on Poisson equation, different magnetic parameters including ratio of magnetization to density, magnetic susceptibility, and molar magnetic susceptibility can be approximated. Furthermore, the gravity and magnetic anomalies can be interpreted simultaneously [13,14,15]. The Poisson equation was satisfied when the gravity and magnetic anomalies are caused by the same mass [16,17], and magnetization and density have uniform distribution in the generating source [18,19,20]. This equation is independent of the shape and position of the source [17]. Estimation of the ratio of magnetization to density for the sources of potential anomalies is one of the applications of the Poisson equation. The most disposing rocks producing magnetic anomalies can be identified by using this ratio. Furthermore, the ratio of magnetization to density can be utilized for classification of the different types of rocks, and estimation of the Koenigsberger ratio. In general, Koenigsberger ratio is the ratio between the residual magnetization and induction magnetization of the mass generating anomalies. This ratio was widely used in historical magnetometer researches. The ratio of magnetization to density can also be used to approximate the magnetic susceptibility, especially when the Kappa-meter can not assess the sample directly. Many studies have been done to estimate the ratio of magnetization to density using the Poisson equation [16, 15, 18, 19, 20, 21, 22]. Jekeli et al. (2010) estimated the changes in magnetic anomalies using Poisson equation [23].

One of the most important components of the Poisson equation to be determined is the gravity gradient tensor. Gravity gradient tensor is the second order derivative of the gravitational potential in different directions. The gravity gradient tensor can be measured using air and ground gradiometry equipment, or can be estimated using computational methods such as mathematical smoothing operator [24], convolution method [25], Fourier transform technique [26], and the different distance method [27]. In this study, magnetic susceptibility was estimated using the Poisson equation and available terrestrial data including gravimetric and magnetic observations. The different distance technique and Fourier transform were used to determine the gravity gradient tensor. Simplification of the Poisson equation and estimation of magnetic susceptibility in a wide area simultaneously is also another advantage of the suggested method in this paper. Thus, by having the magnetic susceptibility changes in a specified region, it is possible to present appropriate interpretation of geological and mineralogical composition which can be widely used in mineral and exploration studies.

2. Poisson Theory

Poison equation is the relationship between the gravity and magnetic potentials and can be used to estimate magnetic susceptibility. This equation had been presented by Jekeli et al. (2010) based on Poison equation:

$$\Delta B = \frac{\chi B_0}{4\pi G \rho_0} k^T \Gamma(x) k$$  \hspace{1cm} (1)$$

where \(\Delta B\) is the magnetic anomaly, \(\chi\) is the magnetic susceptibility, \(\Gamma\) is the gravity gradient tensor, \(B_0\) is the total magnetic field, \(\rho_0\) is the density, \(G\) is the global constant of gravity, and the value of \(k\) defined as follows:
\[ k = \left( \cos \eta \cos \xi \quad \sin \eta \cos \xi \quad \sin \xi \right)^T \tag{2} \]

where \( \xi \) and \( \eta \) are the components of magnetic inclination and declination obtained according to the latitude, longitude, and elevation of each point. Equation (1) can be reformed to estimate the magnetic susceptibilities:

\[ \chi = \frac{\Delta B \, \mathcal{A} \pi G \, \rho}{B_0 \, k^3 \, \Gamma k} \tag{3} \]

Equation (3) can also be used to determine the molar magnetic susceptibility and ratio of magnetization to density. In Equation (3), the most important component to be determined is the gravity gradient tensor. The gravity gradient tensor is a symmetric matrix which is defined as follows:

\[ \Gamma = \nabla g^T = \begin{pmatrix} \frac{\partial g_x}{\partial x} & \frac{\partial g_y}{\partial y} & \frac{\partial g_z}{\partial z} \\ \frac{\partial g_y}{\partial x} & \frac{\partial g_y}{\partial y} & \frac{\partial g_y}{\partial z} \\ \frac{\partial g_z}{\partial x} & \frac{\partial g_z}{\partial y} & \frac{\partial g_z}{\partial z} \end{pmatrix} = \begin{pmatrix} \Gamma_{xx} & \Gamma_{xy} & \Gamma_{xz} \\ \Gamma_{yx} & \Gamma_{yy} & \Gamma_{yz} \\ \Gamma_{zx} & \Gamma_{zy} & \Gamma_{zz} \end{pmatrix} \tag{4} \]

where \( g \) is the gravitational acceleration vector.

3. Determination of the gravity gradient tensor estimation using gravity observations

Different methods for determination of the gravity gradient tensor were investigated. The different distance method and the Fourier transform are described in subsequent sections.

3.1. Different distance method

The different distance technique is a method of calculating the gradient tensor. In this method, it is assumed that the gravitational potential is not a harmonic function. The gradient tensor is computed as follows. First of all, the positions of computation points are provided in a global Cartesian coordinate system. After that, the distance between calculation point and the points in its vicinity are calculated in detail (\( \Delta x, \Delta y, \Delta z \)) and so the matrix \( \Delta S \) is formed:

\[ \Delta S = [\Delta s_1, \Delta s_2, \ldots, \Delta s_N] \in \mathbb{R}^{3 \times N} \tag{5} \]

On the other hand, the vector of gravitational acceleration is obtained by differentiating the gravitational potential with respect to the spherical coordinates \((\varphi, \lambda, r)\) of the calculation point, and removing the effect of eccentricity acceleration. In order to unify the coordinate systems, the gravitational acceleration vector was transferred to the global Cartesian coordinate system. Then, the difference between the gravitational acceleration vector in the calculation point and the points in its neighborhood calculated (as different distance matrix) and so the matrix \( \Delta G \) is formed:

\[ \Delta G = [\Delta g_1, \Delta g_2, \ldots, \Delta g_N] \in \mathbb{R}^{3 \times N} \tag{6} \]

Finally, the gravity gradient tensor is approximated via the following equation:

\[ \Delta G = J \Delta S \tag{7} \]

where \( J \) is the gravity gradient tensor. Considering Equation (7), the gravity gradient tensor is obtained as:

\[ J \approx \Delta G \Delta S^+ \tag{8} \]

3.2. Fourier transform

Assuming the gravitational potential as a harmonic function, the Fourier transform method can be applied to determine the gradient tensor. In this method, the gravity gradient vector is computed as: First, the gravity acceleration vector is computed in a global Cartesian coordinate system. Then, the gravity acceleration vector transferred to the local Cartesian coordinate system. Finally, the Fourier transform is applied to obtain the gravity gradient tensor based on the vertical component of gravity acceleration vector. Considering \( k_x, k_y, k_z \) as the wave numbers in Fourier transform and assuming the gravitational potential as a harmonic function, the Fourier transform of the potential function is obtained as:

\[ (k_x^2 + k_y^2 + k_z^2) \Phi(k) = 0 \tag{9} \]

where \( \Phi(k) \) is the Fourier transform of the potential function. Assuming \( \nabla \times g = 0 \):

\[ \begin{align*}
\frac{\partial g_x}{\partial y} &\equiv (-ik_y) G_x = |k| G_y \\
\frac{\partial g_y}{\partial z} &\equiv (-ik_z) G_y = |k| G_z \\
\frac{\partial g_z}{\partial x} &\equiv (-ik_x) G_z = |k| G_x
\end{align*} \tag{10} \]
Therefore, the aforementioned equations lead to 3D components of gravity accelerations as:

\[
g_x \Leftrightarrow G_x = \frac{(-ik_x)}{|k|} G_z
\]

\[
g_y \Leftrightarrow G_y = \frac{(-ik_y)}{|k|} G_z
\]

\[
g_z \Leftrightarrow G_z
\]

Finally, gravity gradient tensor is computed through the following equation:

\[
\Gamma_y = F^{-1}\{[K(k) G_z(k)]\}
\]

\[
K(k) = \begin{pmatrix}
-k_x^2 & -k_x k_y & -ik_x \\
-k_x k_y & -k_y^2 & -ik_y \\
-ik_x & -ik_y & |k|
\end{pmatrix}
\] (12)

4. Case study: Estimating magnetic susceptibility in FARS area

The performance of Poisson theory for approximating the magnetic susceptibility, different distance technique and Fourier transform method for computing the gravity gradient tensor was investigated. The study area was situated in the coastal Fars, in the southern part of Iran between the longitudes from 53/41 to 55/58 and latitudes from 26/5 to 27/27 (Fig. 1). The southern boundary of coastal Fars is between Bandar Abbas (Bandar Pol) and Bandar Magham. It is located in the northern border parts of Zagros Mountains.

The study area was about 10000 square kilometers. About 8500 square kilometers of the study area was located in moderate mountainous regions and about 1500 square kilometers located in the flat coastal areas.

Geologically, the study area was located at the end of the southeast belt of Zagros Mountains. It longitudinally extends between Khamir anticline in the east and Khalfani anticline in the west. Its latitudinal extension varies in different places, but the maximum width was located from Gavbast Mountain in northwest to Kork anticline in the central south. There are numerous salt domes with thick sedimentary columns, either buried under ground or on the surface. This region was highly probable for deposing hydrocarbon resources (such as oil and gas), which are concentrated in the anticlinal structures (Fig. 1).

Fig. 1. Geological map of the study area in coastal Fars; This area includes geologic formations such as Aghajari (orange), Bakhtyari (yellow), Gachsaran (grey), Mishan (purple), Asmari (green), Bangestan (blue), and Salt dome (red).
The dataset was gathered from two data sources, terrestrial gravity and magnetic data which are measured in 6350 magnetometers and gravimetry stations. The distance between the surveying lines was about 1.5 to 2.5 km, and the distance between the points on each line was about 500 m. There are 93 surveying lines in this geophysical operation which have different length in the area. The properties of the devices used in the geophysical operation are given in Table1.

The different distance technique and Fourier transform are applied to determine the gravity gradient tensor. In the first method, gravity gradient tensor was estimated by assuming the gravity potential as non-harmonic. In this method, the gravity gradient tensor in the computation point is a function of the difference between the gravitational acceleration vector of that point and its neighboring points and the spatial distance between the mentioned points. As it was said before, gravity gradient tensor is a symmetric matrix which contains six (6) independent elements. The variation of the independent elements of gravity gradient tensor obtained from different distance method is shown in Figure 2.

Table 1. Properties of geophysical devices used in taking data

<table>
<thead>
<tr>
<th>Device type</th>
<th>Count</th>
<th>Precision</th>
<th>Manufacturer and Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L&amp;R—G</td>
<td>8</td>
<td>0.04 mgal</td>
<td>United State, gravimetry</td>
</tr>
<tr>
<td>G-858</td>
<td>10</td>
<td>0.01 nT</td>
<td>United State, magnetometers</td>
</tr>
<tr>
<td>DM-2 Rock (Ore)</td>
<td>1</td>
<td>0.01 g/cm³ ±</td>
<td>China, for measuring rock's density</td>
</tr>
</tbody>
</table>

Fig. 2. Variation of the independent elements of the gravity gradient tensor in the study area by different distance method
In the Fourier transform method, the gravity gradient tensor was estimated by assuming the gravitational potential as a harmonic function and using the vertical component of gravity. The variation of the independent elements of the gravity gradient tensor derived from Fourier transform technique is illustrated in Figure 3.

After computing the gravity gradient tensor, the magnetic susceptibility can be determined at each point according to Poisson theory. The magnetic susceptibility map of the study area based on the different distance method is shown in Figure 4. Comparison between the geological map of the study area (Fig. 1) and magnetic susceptibility map (Fig. 4), shows that the central and southern parts of the study area consisting of the Aghajari formation (with the highest magnetic susceptibility than other formations) or surrounded with salt domes, have a high magnetic susceptibility. Therefore, the magnetic susceptibility map is consistent with the available geological characteristics.

The magnetic susceptibility map derived from Fourier transform method for estimating the gravity gradient tensor is shown in Figure 5. As shown in Figure 5, variation of the magnetic susceptibility according to the Fourier transform technique is consistent with the geological changes of the structures in the study area (the anticline and syncline can be indicated).

Comparison of the estimated magnetic susceptibility maps by two methods (Figs. 4 and 5) shows that Fourier technique is more consistent with the geological map of the study area than the different distance method. Furthermore, the Fourier transform technique in computing the gradient tensor shows more details and can detect more variations, while the solution of different distance method is smoother (Figs. 2 and 3). Thus, it can be concluded that the Fourier transform method can be more efficient in exploration applications.

Fig. 3. Variations of the independent elements of the gravity gradient tensor in the study area by Fourier transform
Comparison of geological map of the region (Fig. 1) with obtained magnetic susceptibility map derived based on Fourier transform (Fig. 5) shows more salt domes protruding in the area (which are shown by red color in the geological map), and magnetic susceptibility of points with high values. High values of the magnetic susceptibilities can be related to the Alien igneous rocks (inner and outer) which have been brought with salt domes in this part of the Zagros Mountains. These Alien rocks can be assigned to Zagros Precambrian rock with some part of this foundation rock protrusion in Arabia. Since there is no inner or outer Alien igneous rock in any part of the stratigraphic column and sedimentary row of Phanerozoic deposits available in Zagros belt and also its foreland
basin, it is therefore possible to use high valuable magnetic susceptibility's properties for determining salt domes and their remains in the study area. Recently, there have been records of exploratory studies about salt domes, especially for identifying resources of iron. Thus, it can be noted that using the proposed method is a good way for exploration of target resources. Figure 6 shows the areas that have been identified as salt domes. As it can be seen, for the most part, there are high levels of magnetic susceptibility or significant changes. For better comparison of obtained results with geological map, magnetic susceptibility's changes map was displayed on the geological map of the region in Figure 7. Furthermore, suggested method in this study can identify areas with ferromagnetism which can have many applications for mineral exploration in the region and other parts of the country.

Fig. 6. Black circles show the range of salt glaciers. As can be seen in most of them, points are red which show high levels of magnetic susceptibility

Fig. 7. Displaying magnetic susceptibility's changes on the geological map
5. Summary and conclusion
In this study, the magnetic susceptibility was estimated using Poisson equation and gravity gradient tensor in a wide region of Iran, in coastal Fars. We applied two different methods of determining the gravity gradient tensor, different distance method and Fourier transform. We compared the results of these two methods, with the available geological map of the study area. Our computations showed that the Fourier transform technique can detect even small variations in magnetic susceptibility and thus, can indicate more details about the geological structures. Finally, we could suggest a reliable approach for detecting the existing salt domes in the study area. Simultaneous estimation of magnetic susceptibility in plenty of points, providing a fast and low-cost approach for approximation of this magnetic quantity, and providing more details about the interior parts of the study area are the most important advantages of the proposed method. Although, the proposed method may not be an alternative for measuring magnetic susceptibility on the Earth’s surface, but can be an efficient tool in exploration applications and detection of masses with high magnetic susceptibility which often show existence of materials such as iron, etc.

References
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