Optimal Reactive Power Dispatch using Improved Differential Evolution Algorithm

Hamid Falaghi¹, Arsalan Najafi²

¹- Department of Electrical Engineering, University of Birjand, Birjand, Iran  
Email: falaghi@birjand.ac.ir 
²- Department of Electrical Engineering, University of Birjand, Birjand, Iran  
Email: arsalan.najafi@birjand.ac.ir (corresponding author)

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ABSTRACT:
Reactive power dispatch plays a key role in secure and economic operation of power systems. Optimal reactive power dispatch (ORPD) is a non-linear optimization problem which includes both continues and discrete variables. Due to complex characteristics, heuristic and evolutionary based optimization approaches have become effective tools to solve the ORPD problem. In this paper, a new optimization approach based on improved differential evolution (IDE) has been proposed to solve the ORPD problem. IDE is an improved version of differential evolution optimization algorithm in which new solutions are produced in respect to global best solution. In the proposed approach, IDE determines the optimal combination of control variables including generator voltages, transformer taps and setting of VAR compensation devices to obtain minimum real power losses. In order to demonstrate the applicability and efficiency of the proposed IDE based approach, it has been tested on the IEEE 14 and 57-bus test systems and obtained results are compared with those obtained using other existing methods. Simulation results show that the proposed approach is superior to the other existing methods.


1. INTRODUCTION
Optimal reactive power dispatch (ORPD) problem is one of the most important issues in power system operation which plays a key role in secure and economic operation of power systems. This problem denotes optimal settings of controllable variables such as generator voltage magnitudes, tap ratios of transformers and setting of shunt VAR compensation devices to minimize the transmission line losses while satisfying physical and operating constraints [1].

ORPD is a non-linear and non-convex optimization problem that contains both discrete and continues variables. Several conventional and classical solution methods have been presented to deal with the ORPD problem such as gradient based search, linear programming, dynamic programming and etc. [2]-[9]. These methods are computationnally fast. However, due to complex characteristics and discrete nature of the problem, they face difficulties in solving the problem. In the recent years many meta-heuristic and evolutionary methods have been implemented to the ORPD problem. The advantages of evolutionary algorithms in terms of the modeling and search capabilities have encouraged their application to the ORPD problem. The genetic algorithm (GA) [10], particle swarm optimization (PSO) [11], [12], evolutionary programming (EP) [13], differential evolution (DE) [14]-[16], harmony search algorithm (HSA) [17], general quantum genetic algorithm (GQGA) [18], simulated annealing (SA) [19] and seeker optimization algorithm (SOA) [20] are some of the meta-heuristic methods that presented to solve the ORPD problem. In [21] a method based on hybrid PSO algorithm with mutation operator has been presented to solve the ORPD problem. In [22] a modified multi objective shuffled frog leaping algorithm has been proposed to solve the problem. Ref. [23] has proposed hybrid fuzzy multi-objective evolutionary algorithm (HFMOEA) based approach. Objectives are reactive power dispatch and voltage stability index. In HFMOEA based optimization approach, the two parameters like crossover probability and mutation probability are varied dynamically through the output of a fuzzy logic controller. In [24] a method has been suggested in which voltage stability margin is improved by managing the reactive sources. Also, in [25] a method based on a modified version of GA been presented for ORPD problem. It uses Bender’s cut to population selection and reproduction in the
The voltage angle difference between buses $ij$ bus is expressed as [17]:

$$\theta_{ij} = \theta_i - \theta_j$$

where $\theta_i$ and $\theta_j$ are the voltage magnitude at the $i$-th and $j$-th buses, respectively.

Section 2. PROBLEM FORMULATION

The objective of the reactive power optimization is minimizing the active power loss in the transmission network. The ORPD problem is formulated as an optimization problem with an objective function, expressed as [17]:

$$\min P_L = \sum_{i=1}^{nl} P_{loss,i}$$

(1)

Where $P_L$ denotes the total network line losses, $P_{loss,i}$ is real power loss of the $i$-th line and $nl$ is total number of lines. The minimization of the mentioned objective function is subjected to a number of equality and inequality constraints. The equality constraints are the power flow equations as follows:

$$PG_i - PD_i = V_i \sum_{j=1}^{nb} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij})$$

(2)

for $i = 1,\ldots,nb-1$

$$QG_i - QD_i = -V_i \sum_{j=1}^{nb} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij})$$

(3)

for $i = 1,\ldots,npq$

where $PG_i$ and $QG_i$ are the active and reactive generated powers at the $i$-th bus, $PD_i$ and $QD_i$ are the active and reactive demands at the $i$-th bus, respectively. $V_i$ is the voltage magnitude at the $i$-th bus. $\theta_{ij}$ is the voltage angle difference between buses $i$ and $j$. $nb-1$ is the total number of buses, except the slack bus. $npq$ is the total number of PQ buses, $G_{ij}$ and $B_{ij}$ are the mutual conductance and susceptance between buses $i$ and $j$, respectively.

Inequality constraints include control variable constraints and dependent variable constraints. Controllable variables are generators voltage magnitudes, transformer tap ratios and setting of shunt VAR compensation devices. Dependent variables are the bus voltages. Constraints of these variables are as follows:

- Generator constraints: Generator reactive power outputs and voltage magnitudes are restricted by their upper and lower bounds as follows:

$$QG_i^{\min} \leq QG_i \leq QG_i^{\max} \quad \text{for } i = 1,2,\ldots,ng$$

(4)

$$VG_i^{\min} \leq VG_i \leq VG_i^{\max} \quad \text{for } i = 1,2,\ldots,ng$$

(5)

where $ng$ is the number of generating units, $QG_i^{\min}$ and $QG_i^{\max}$ are the minimum and maximum reactive power outputs of generator $i$, respectively. $VG_i$ is the voltage magnitude of generator $i$. $VG_i^{\min}$ and $VG_i^{\max}$ show the minimum and maximum voltage limits of generator $i$, respectively.

- Transformer- tap constraints: transformer taps are bounded by their related minimum and maximum limits as follows:

$$T_i^{\min} \leq T_i \leq T_i^{\max} \quad \text{for } i = 1,2,\ldots,nt$$

(6)

Where $nt$ is the number of transformers. $T_i$ is the tap ratio of transformer $i$. $T_i^{\min}$ and $T_i^{\max}$ are the minimum and maximum values of the $i$-th transformer tap ratio, respectively.

- Shunt VAR compensator constraints: Setting of the shunt VAR compensation devices are restricted by their limits as follows:

$$QC_i^{\min} \leq QC_i \leq QC_i^{\max} \quad \text{for } i = 1,2,\ldots,nc$$

(7)

Where $nc$ is the number of VAR compensation devices. $QC_i$ is the generated reactive power of the $i$-th shunt VAR compensation device. $QC_i^{\min}$ and $QC_i^{\max}$ are the minimum and maximum limits of reactive power of shunt VAR compensation device $i$, respectively.

- Operating voltage constraint: Bus voltages are restricted by their maximum and minimum limits as follows:

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad \text{for } i = 1,2,\ldots,nb$$

(8)
Where \( V_i \) is the voltage of the \( i \)-th bus. \( V_i^{\text{min}} \) and \( V_i^{\text{max}} \) are the minimum and maximum limits of voltage at bus \( i \), respectively.

3. IMPROVED DIFFERENTIAL EVOLUTION

3.1. Overview of the Original DE Algorithm

Original DE algorithm is a simple population based evolutionary computational algorithm for global optimization. It is one of the accurate and fast metaheuristic optimization algorithms that was introduced in 1995s by Price and Storn \[26\]. This evolutionary algorithm begins the search process by initial random population. DE includes three main operators, namely, population size \( np \), mutation, crossover, and selection. Also, it has three control parameters, namely, population size \( np \), scaling coefficient \( F \), and crossover probability \( CR \). In the subsequent sections, the implementation details of the DE are described \[15, 27\].

1) Initialization: The DE algorithm searches in parallel using a group of members similar to the other evolutionary based heuristic optimization techniques. Each member corresponds to a candidate solution to the problem. In an \( n \)-dimensional search space, the structure of member \( k \) is represented as vector \( X_k = (x_{k,1}, x_{k,2}, \ldots, x_{k,n}) \) where the dimension represents the number of components. In the first stage of the DE optimization process, initial population contains \( np \) members should be created randomly.

2) Mutation: After the population is initialized, the operators of mutation, crossover and selection create the population of the next generation. At the generation \( t \), the process for creation of a mutant solution \( Y_k(t) \) for each parent \( X_k \) in the population can be expressed as follows:

\[
Y_k(t) = X_k(t) + F \cdot (X_{r1}(t) - X_{r2}(t)), \quad k = 1, 2, \ldots, np
\]  

(9)

Where vector indices \( r1, r2, \) and \( r3 \) are randomly chosen, which \( r1, r2, \) and \( r3 \in \{1, \ldots, np\} \) and \( r1 \neq r2 \neq r3 \neq k \). \( X_{r1}, X_{r2}, \) and \( X_{r3} \) are selected members for each parent vector. \( F \) is a user-defined constant known as the ‘scaling factor’, which is a positive and real number. The usual choice for \( F \) is a number between 0 and 1.

3) Crossover: In order to increase the diversity of the population, the crossover process is employed. At the generation \( t \), the crossover operator creates a new solution (child) \( Z_k(t) \) using each parent \( X_k \) and its related mutant vectors \( Y_k(t) \) as follows:

\[
z_k(t) = \begin{cases} 
  y_{kj}(t) & \text{if } rand \leq CR \text{ or } j = jrand \\
  x_{kj}(t) & \text{otherwise} 
\end{cases},
\]

(10)

for \( j = 1, \ldots, n \)

Where \( CR \) is ‘crossover probability’ which is a user-defined value usually selected from within the range \([0, 1] \). \( CR \) controls the diversity of the population and helps the algorithm to escape from local optima. \( rand \) is a uniformly distributed random number within the range \((0, 1) \) generated a new for each component \( j \). Here, \( jrand \in [1, 2, \ldots, n] \) and ensures that the trial vector gets at least one parameter from the mutant vector.

4) Selection: To keep the population size constant over subsequent generations, the selection operator is applied to determine which one of the child and the parent will survive in the next generation. This operator compares the fitness of the parent and the corresponding child and the fitter of the two solutions is then allowed to advance into the next generation. The selection process can be expressed as,

\[
X_k(t+1) = \begin{cases} 
  Z_k(t) & \text{if } \text{FIT}(X_k(t)) \geq \text{FIT}(Z_k(t)) \\
  X_k(t) & \text{otherwise} 
\end{cases},
\]

(11)

Where \( \text{FIT}(\cdot) \) is the fitness function.

5) Stopping Criteria: The overall optimization process is terminated if the iteration approaches to the predefined maximum iteration or another predetermined convergence criterion is satisfied.

3.2. Improved Differential Evolution Algorithm

In the original version of DE algorithm, new solutions are created by three previous random selected solutions. In the improved version of DE algorithm, it is tried to reach better solutions by changing the crossover mechanism of the original DE algorithm. To do this, new solutions are generated in respect to the global best solution. Therefore, probability of obtaining optimum solutions will increase. Thus, instead of using three previous random solutions, two previous solutions and the global best solution \((X_g)\) are used. This process is expressed by \[24\]:

\[
Y_k(t) = X_k(t) + rand \times (X_g - X_k(t)) + \mu \times (X_{r1}(t) - X_{r2}(t))
\]

(12)

Where \( X_g \) is the best solution so far; \( rand \) is a uniformly distributed random number within the range \((0, 1) \), and \( \mu \) is the constant number between 0 and 1.

Fig. 1 illustrates the vector generation process defined by \(12\). In Fig. 1, it can be seen that this form of mutation uses the best found vector (global best solution) to push mutant vectors toward it. The algorithm eventually converges with less iteration, so it...
provides the best choice for search spaces, where the optimum solution is relatively easy to be found and its speed is higher than original DE because less iteration is required to converge. Also the figure verifies that the new solutions are created purposefully. In fact, IDE uses group experiences to create new solutions.

4. IMPLEMENTATION OF IDE FOR ORPD PROBLEM

Reactive power dispatch problem is a large-scale highly constrained non-linear and non-convex optimization problem with discrete and continuous variables. Because of non-convexity property of the problem, it is hard to be solved by classical and mathematical methods. Evolutionary meta-heuristic methods are very useful to solve this problem because they do not require derivative to solve the problem and non-convexity does not prevent from obtaining optimum or near optimum solutions. In this paper, an improved version of differential evolution optimization algorithm is utilized to solve the ORPD problem.

The process of the proposed IDE based approach for the ORPD problem is depicted in Fig. 2 and can be summarized as follows:

Step 1) Initialization of a group of solutions at random;
Step 2) Evaluation of the fitness function for each solution and determining global best solution;
Step 3) Creating new solutions;
Step 4) Constraints handling;
Step 5) Evaluation of solutions’ fitness function, selection and update the global best solution;
Step 6) Go to Step 3 until satisfying stopping criteria.

In the subsequent sections, the detailed implementation strategies of the IDE based ORPD are described.

4.1. Initializing Solutions

In the proposed approach each solution can be considered as an $n$-dimensional vector. In the ORPD problem, generator voltage magnitudes, transformer tap ratios and setting of shunt VAR compensation devices are decision/control variables. Therefore each solution $j$ should contain these items as follow:

$$X^j_k = [V_{G_k,1}, \ldots, V_{G_k,nG}, T_{k,1}, \ldots, T_{k,nT}, Q_{C_{j,1}}, \ldots, Q_{C_{j,nC}}]$$  

Note that it is very important to create a group of members satisfying the inequality constraints (5)-(7). That is, each created component of a member at random should be located within its related boundary. To do this, we can create each component of the member at random satisfying the related inequality constraint by mapping $[0, 1]$ into its related lower and upper limits, i.e. $[\text{lower limit}, \text{upper limit}]$.

4.2. Constraint Handling

There are two groups of constraints in the ORPD problem, namely, equality and inequality constraints. The inequality constraints contain controllable variable and dependent variables. To satisfy constraints (5)-(7) which are related to the controllable variables, if the corresponding components exceed from their upper bounds, they will be set at the upper bounds and if they decreases from their lower bounds they will be set on the lower bounds as follows:

$$X^j_k = [V_{G_k,1}, \ldots, V_{G_k,nG}, T_{k,1}, \ldots, T_{k,nT}, Q_{C_{j,1}}, \ldots, Q_{C_{j,nC}}]$$  

Fig. 2. Flowchart of the proposed approach
as follows:

\[ x_{k,j} = \begin{cases} 
  x_{k,j}^{\max} & \text{if } x_{k,j} > x_{k,j}^{\max} \\
  x_{k,j}^{\min} & \text{if } x_{k,j} < x_{k,j}^{\min}, \text{ for } j = 1, \ldots, n + n + nc \\
  x_{k,j} & \text{otherwise} 
\end{cases} \]

(14)

Where \( x_{k,j} \) is the component \( i \) of member \( j \), \( x_{k,j}^{\min} \) and \( x_{k,j}^{\max} \) are the related lower and upper bounds.

Tap changers and shunt VAR devices often have discrete values and different steps, therefore their constraints must be satisfied with different methods. Assume that there are \( n \) steps for tap changers or shunt devices. An integer number has been created between 1 and \( n \). This number shows the device step so that, digit 1 shows step 1, digit 2 shows step 2 and digit \( n \) illustrates the step \( n \), respectively. To produce integer numbers a random number has been created and then it has been rounded.

After, finding the independent variables, dependent variables will be calculated from AC power flow solution. Consequently, the power balance constraint represents by (2)- (3) is satisfied by using AC power flow. The AC power flow has been conducted by newton-raphson method which has been done by MATPower load flow. Also, constraints (4) which are related to reactive power outputs of generating units can be satisfied in the power flow calculation.

The inequality constraints (8) which show the operating voltage limits are incorporated in the fitness function as explained in the Section 4.3.

### 4.3. Fitness Assignment

The performance of members in the current population are assessed in the objective space and then assigned a scalar value known as fitness. Depending on the fitness values, members will be selected to form the new population. Members with high fitness value have more chance to be selected.

In this paper, the fitness function of the ORPD problem is a combination of the objective function (1) and a penalty function related to the inequality constraints (8) as follows:

\[ FIT = \frac{1}{P_L + W(V)} \]

(15)

\[ W(V) = \sum_{i=1}^{nb}(V_i - V_i^{\max}) \times p(V_i) + \sum_{i=1}^{nb}(V_i^{\min} - V_i) \times p(V_i) \]

(16)

The penalty factor \( p(V_i) \) enforces the voltage limits and is defined by [26]:

\[ p(V_i) = \begin{cases} 
  40, & \text{if } V_i > V_i^{\max} \text{ or } V_i < V_i^{\min} \\
  0, & \text{otherwise} 
\end{cases} \]

(17)

When the iteration approaches to the predefined maximum iteration, the overall optimization process is terminated and the global best solution which has the highest fitness value is introduced as final solution of the ORPD problem.

### 5. NUMERICAL RESULTS

To assess the efficiency of the proposed IDE based approach for the ORPD problem, it is applied to two different power systems. The results obtained from the IDE are compared with those of other methods. For more comparison, the original DE algorithm has been coded and implemented to the case studies by the authors same as IDE.

The proposed approach has been coded in MATLAB language and executed on a 2-GHz Pentium IV personal computer with 1-GB of RAM.

#### 5.1. 14-Bus Test System

This system consists of 5 generation units, 9 load buses, 20 lines in which three lines of 4-7, 4-9, and 5-6, have tap changing transformers. The bus and lines data are taken from [29]. Initial line loss is 13.49 MW. Limits of control variables are listed in Table 1.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Minimum (pu)</th>
<th>Maximum (pu)</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>( VG )</td>
<td>0.9</td>
<td>1.1</td>
<td>---</td>
</tr>
<tr>
<td>( V )</td>
<td>0.9</td>
<td>1.1</td>
<td>---</td>
</tr>
<tr>
<td>( T )</td>
<td>0.9</td>
<td>1.1</td>
<td>0.01</td>
</tr>
<tr>
<td>( QC_8 )</td>
<td>0</td>
<td>0.18</td>
<td>0.06</td>
</tr>
<tr>
<td>( QC_{14} )</td>
<td>0</td>
<td>0.18</td>
<td>0.06</td>
</tr>
</tbody>
</table>

There exists two parameters to be determined for the implementation of the proposed IDE based approach, namely, \( \mu \) and \( CR \). In this paper, these parameters have been determined through the sensitivity analysis for the 14-bus test system. To tune the parameters of \( \mu \) and \( CR \), the values of \( \mu \) are varied from 0.1 to 0.9 and the values of \( CR \) are also varied from 0.1 to 0.9 with increments of 0.1. In Table 2, the effects of the parameters are illustrated, where 30 random trials are performed for each parameter set. Among 17 sets of parameters in Table 2, Case 15 shows the best performance in terms of the best, average and the worst solutions. Therefore, the parameter values for Case 15 are selected for subsequent studies. Also, the maximum iteration number is set as 300.

Fig. 3 illustrates the convergence characteristics of the proposed IDE base approach in five runs with population size equal to 50 and different initialized randomly created members. It can be seen that all of the five runs have been converged to optimum solution.
This shows the search capability of the IDE to achieve the best solution with different starting points. In order to demonstrate the effect of population size on the performance of the optimization process, the proposed approach has been run with the different population sizes. Also the obtained results are given in Table 3.

![Figure 3](image)

**Fig. 3.** Five sequential runs of the proposed approach for 14-bus test system with different initial populations

### Table 2. Effects of parameters in IDE performance in 14-bus test system

<table>
<thead>
<tr>
<th>Case</th>
<th>μ</th>
<th>CR</th>
<th>Best solution (MW)</th>
<th>Average solution (MW)</th>
<th>Worst solution (MW)</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.9</td>
<td>13.29502</td>
<td>13.36937</td>
<td>13.4594</td>
<td>0.048988</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.8</td>
<td>13.27415</td>
<td>13.35301</td>
<td>13.48561</td>
<td>0.06686</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>0.7</td>
<td>13.28146</td>
<td>13.3259</td>
<td>13.4118</td>
<td>0.044938</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>0.6</td>
<td>13.27035</td>
<td>13.29209</td>
<td>13.31765</td>
<td>0.016013</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>0.5</td>
<td>13.22756</td>
<td>13.22758</td>
<td>13.22775</td>
<td>6.16E-05</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>0.4</td>
<td>13.22756</td>
<td>13.22758</td>
<td>13.22775</td>
<td>6.27E-05</td>
</tr>
<tr>
<td>7</td>
<td>0.7</td>
<td>0.3</td>
<td>13.22756</td>
<td>13.22779</td>
<td>13.22951</td>
<td>0.000608</td>
</tr>
<tr>
<td>8</td>
<td>0.8</td>
<td>0.2</td>
<td>13.22756</td>
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<td>0.020043</td>
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<tr>
<td>9</td>
<td>0.9</td>
<td>0.1</td>
<td>13.22757</td>
<td>13.26098</td>
<td>13.35206</td>
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<tr>
<td>10</td>
<td>0.1</td>
<td>0.1</td>
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<td>13.34647</td>
<td>13.37706</td>
<td>0.022252</td>
</tr>
<tr>
<td>11</td>
<td>0.2</td>
<td>0.2</td>
<td>13.28832</td>
<td>13.33102</td>
<td>13.4166</td>
<td>0.037159</td>
</tr>
<tr>
<td>12</td>
<td>0.3</td>
<td>0.3</td>
<td>13.28888</td>
<td>13.30393</td>
<td>13.32033</td>
<td>0.011641</td>
</tr>
<tr>
<td>13</td>
<td>0.4</td>
<td>0.4</td>
<td>13.26876</td>
<td>13.29027</td>
<td>13.31402</td>
<td>0.015233</td>
</tr>
<tr>
<td>14</td>
<td>0.6</td>
<td>0.6</td>
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<td>13.22762</td>
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<tr>
<td>15</td>
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<td>0.7</td>
<td>13.22756</td>
<td>13.22756</td>
<td>13.22756</td>
<td>4.36E-05</td>
</tr>
<tr>
<td>16</td>
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<td>0.8</td>
<td>13.22756</td>
<td>13.22785</td>
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<td></td>
</tr>
<tr>
<td>17</td>
<td>0.9</td>
<td>0.9</td>
<td>13.22759</td>
<td>13.22888</td>
<td>0.000395</td>
<td></td>
</tr>
</tbody>
</table>

As the results in Table 3 indicate, the IDE based approach can reach to the best solution with all population sizes. These provide a robustness of the IDE, regarding to the population size. However, the probability to obtain the best solutions lowers with the small population size.

### Table 3. Simulation results for 14-bus test system with different population sizes

<table>
<thead>
<tr>
<th>Population size</th>
<th>Best Solution (MW)</th>
<th>Average solution (MW)</th>
<th>Worst Solution (MW)</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>13.2276</td>
<td>13.2276</td>
<td>13.2276</td>
<td>9.4</td>
</tr>
<tr>
<td>75</td>
<td>13.2276</td>
<td>13.2276</td>
<td>13.2276</td>
<td>13.73</td>
</tr>
<tr>
<td>100</td>
<td>13.2276</td>
<td>13.2276</td>
<td>13.2276</td>
<td>17.02</td>
</tr>
</tbody>
</table>

The convergence curves of the IDE and the original DE to obtain the optimal solution in this case study are illustrated in Fig. 4.

From the figure, it can be concluded that the IDE have better convergence characteristic.

In Table 4, values of control variables and the corresponding losses obtained by the proposed IDE among 30 trials are provided. They’re compared with those obtained which are using other existing methods that are including: EP, PSO, interior point method (IPM) and the original DE. The IDE has shown the superiority to the existing methods as one can see in Table 4. In order to compare the speed of IDE and DE algorithm in converging to optimum solution result of 100 time runs of them are available in Table 5.
Table 4. Comparison results of the proposed approach and the other techniques for 14-bus test system

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{G2}$ (pu)</td>
<td>1.045</td>
<td>1.029</td>
<td>1.0463</td>
<td>1.0449</td>
<td>1.0449</td>
<td>1.0464</td>
</tr>
<tr>
<td>$V_{G3}$ (pu)</td>
<td>1.01</td>
<td>1.016</td>
<td>1.0165</td>
<td>1.0149</td>
<td>1.0146</td>
<td>1.0167</td>
</tr>
<tr>
<td>$V_{G6}$ (pu)</td>
<td>1.07</td>
<td>1.097</td>
<td>1.1</td>
<td>1.0971</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>$V_{G8}$ (pu)</td>
<td>1.09</td>
<td>1.053</td>
<td>1.1</td>
<td>1.0999</td>
<td>1.1</td>
<td>1.0995</td>
</tr>
<tr>
<td>$T_{4-7}$</td>
<td>0.9467</td>
<td>1.04</td>
<td>0.94</td>
<td>1.0238</td>
<td>1.06</td>
<td>0.96</td>
</tr>
<tr>
<td>$T_{4-9}$</td>
<td>0.9524</td>
<td>0.94</td>
<td>0.93</td>
<td>1.0998</td>
<td>1.04</td>
<td>0.91</td>
</tr>
<tr>
<td>$T_{5-6}$</td>
<td>0.9091</td>
<td>1.03</td>
<td>0.97</td>
<td>1.055</td>
<td>1.1</td>
<td>0.97</td>
</tr>
<tr>
<td>$QC_9$ (pu)</td>
<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
<td>0.1798</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>$QC_{14}$ (pu)</td>
<td>0.18</td>
<td>0.06</td>
<td>0.06</td>
<td>0.0739</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Average solution (MW)</td>
<td>---</td>
<td>13.371</td>
<td>13.35</td>
<td>---</td>
<td>13.251</td>
<td>13.2276</td>
</tr>
<tr>
<td>Worst solution (MW)</td>
<td>---</td>
<td>13.399</td>
<td>---</td>
<td>---</td>
<td>13.275</td>
<td>13.2276</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>---</td>
<td>0.00018</td>
<td>---</td>
<td>---</td>
<td>0.01616</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 4. Convergence diagram of DE and IDE algorithms for 14-bus test system

It shows the required iteration and time to converge of DE and IDE algorithm in 14-bus test system over the sequential runs. It is visible that IDE needs less iteration and time to converge optimum solution.

Table 5. Comparison result of converging speed between DE and IDE in 14-bus test system

<table>
<thead>
<tr>
<th>Variables</th>
<th>iteration time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Best</td>
</tr>
<tr>
<td>DE</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>23.4</td>
</tr>
<tr>
<td>IDE</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>9.66</td>
</tr>
</tbody>
</table>

5.2. 57-Bus Test System

The IDE with the same parameters determined in the first case study is applied to an ORPD problem in IEEE 57-bus test system. This system consists of 7 generating units, 80 lines in which 15 lines have tap changing transformer. Three VAR compensation devices have been considered for buses 18, 25 and 53. In this system, initial line loss is 28.462 MW. The variable limits in this system are listed in Table 6 [20]. Other input data of the system are given in [29].

The obtained values of control variables in this test system are provided in Table 7. It is visible that all of the variables are in their bounds and the discrete amounts of tap changer values have been met.

Table 6. Limitation of control variables for 57-bus test system

<table>
<thead>
<tr>
<th>Variables</th>
<th>Minimum (pu)</th>
<th>Maximum (pu)</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{G}$</td>
<td>0.94</td>
<td>1.06</td>
<td>---</td>
</tr>
<tr>
<td>$V$</td>
<td>0.94</td>
<td>1.06</td>
<td>---</td>
</tr>
<tr>
<td>$T$</td>
<td>0.9</td>
<td>1.1</td>
<td>0.01</td>
</tr>
<tr>
<td>$QC_{18}$</td>
<td>0</td>
<td>0.1</td>
<td>---</td>
</tr>
<tr>
<td>$QC_{25}$</td>
<td>0</td>
<td>0.059</td>
<td>---</td>
</tr>
<tr>
<td>$QC_{53}$</td>
<td>0</td>
<td>0.063</td>
<td>---</td>
</tr>
</tbody>
</table>

The best results from the IDE are compared with those of DE, canonical genetic algorithm (CGA) [20], adaptive genetic algorithm (AGA) [20], local search self-adaptive differential evolution (L-SaDE) [20], SOA [20], non-linear programming (NLP) method [20] and HSA [17] and given in Table 8. The IDE has converged to 24.210 MW while the best result obtained by the other methods is 24.257 MW.

The convergence graphs of the control variables in the IDE optimization process are showed in Figs. 5-7. From these figures, it can be seen that, the variables have variation at the initial iterations and all of them are converged to their final states in less than 200 iterations. Also, it is visible that all of the variables are kept in their limitations. Bus voltages in PU have also been showed in Fig. 8, graphically in which all of the voltage magnitudes satisfy their related constraints.

In order to compare the performance of the IDE with original DE, their convergence characteristics are compared and depicted in Fig. 9.
Fig. 5. Convergence diagram of generator voltages for 57-bus test system

Fig. 6. Convergence diagram of transformer tap ratios for 57-bus test system

Table 7. Simulation results for 57-bus test system

<table>
<thead>
<tr>
<th>$V_G_1$</th>
<th>$V_G_2$</th>
<th>$V_G_3$</th>
<th>$V_G_4$</th>
<th>$V_G_5$</th>
<th>$V_G_6$</th>
<th>$V_G_7$</th>
<th>$T_{1-18}$</th>
<th>$T_{4-18}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.06</td>
<td>1.058062</td>
<td>1.044567</td>
<td>1.034326</td>
<td>1.054805</td>
<td>1.040813</td>
<td>1.03785</td>
<td>0.9</td>
<td>1.05</td>
</tr>
<tr>
<td>$T_{21-20}$</td>
<td>$T_{24-25}$</td>
<td>$T_{24-26}$</td>
<td>$T_{7-29}$</td>
<td>$T_{34-32}$</td>
<td>$T_{11-41}$</td>
<td>$T_{15-46}$</td>
<td>$T_{14-46}$</td>
<td>$T_{14-46}$</td>
</tr>
<tr>
<td>1</td>
<td>0.95</td>
<td>0.95</td>
<td>1</td>
<td>0.97</td>
<td>0.93</td>
<td>0.93</td>
<td>0.97</td>
<td>0.95</td>
</tr>
<tr>
<td>$T_{10-51}$</td>
<td>$T_{13-49}$</td>
<td>$T_{11-43}$</td>
<td>$T_{40-56}$</td>
<td>$T_{39-57}$</td>
<td>$T_{9-55}$</td>
<td>$Q_C_{18}$</td>
<td>$Q_C_{25}$</td>
<td>$Q_C_{53}$</td>
</tr>
<tr>
<td>0.97</td>
<td>0.93</td>
<td>0.96</td>
<td>1</td>
<td>0.96</td>
<td>0.98</td>
<td>0.1</td>
<td>0.059</td>
<td>0.063</td>
</tr>
</tbody>
</table>
Table 8. Comparison results of the proposed approach and the other techniques for 57-bus test system

|--------|----------|----------|-------------|----------|----------|----------|----|-----|

Also the performances of the IDE and the original DE are compared in terms of the best, worst, and average values of real power losses among 30 trials and listed in Table 9. The obtained results show the superiority of the IDE over the original DE algorithm.

Table 9. Comparison results of IDE and DE in 57-bus test system

<table>
<thead>
<tr>
<th>Method</th>
<th>Best Solution (MW)</th>
<th>Average solution (MW)</th>
<th>Worst solution (MW)</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE</td>
<td>24.25735</td>
<td>24.55919</td>
<td>25.88301</td>
<td>0.49</td>
</tr>
<tr>
<td>IDE</td>
<td>24.21020</td>
<td>24.30000</td>
<td>24.62551</td>
<td>0.35</td>
</tr>
</tbody>
</table>

6. CONCLUSION

This paper presented an improved differential evolution algorithm for the complex problem of optimal reactive power dispatch in power systems. The evolutionary mechanism of the IDE is more effective than the original DE and it has the advantage of being easy to comprehend, simple to implement so that it can be utilized for a wide variety optimization problems. The efficiency of the proposed IDE based ORPD method is proved by case studies on two test systems with different sizes. Results of the proposed IDE algorithm have been compared to those reported in the literature. The comparisons clearly approved the effectiveness and the superiority of the proposed IDE approach over the original DE and the other existing techniques in terms of solution quality.

REFERENCES


