On the Full-Rate Linear Complexity \(2 \times 2\) Space-Time Block Code: Application in Keyhole Channels

S. Saleh Hosseini Bidaki\(^1\), Behrouz Bagheri Ranjbar\(^1\) and Siamak Talebi\(^2\)
\(^1\) Electrical Engineering Department, University of Kerman, Kerman, Iran.
\(^2\) Advanced Communications Research Institute, Sharif University, Tehran, Iran.
Email: s.saleh.hosseini@gmail.com, behrouz.bagheri.r@gmail.com, siamak.talebi@mail.uk.ac.ir

Received: March 2012 Revised: April 2012 Accepted: May 2012

ABSTRACT:
In this paper, a new full rate, full diversity \(2 \times 2\) STBC with linear complexity in the receiver is introduced for keyhole channels, where the rank-deficiency of the channel matrix degrades system performance. This code is optimized based on the known criteria for these types of the channels. Simulation results demonstrate that the proposed code outperforms some of the well-recognized STBCs when BPSK and 4-QAM constellations are utilized.

KEYWORDS: Keyhole Channels, Multi-Input Multi-Output (MIMO), Space-Time Block Codes (STBCs).

1. INTRODUCTION
Multi input multi output (MIMO) systems such as space time codes (STCs) encounter a phenomenon, known as keyhole or pinhole, in some real environments that lead to rank-deficiency of the channel matrix. In this phenomenon, the transmitted signal must pass through a keyhole and then propagate to the receiver (Fig. 1). An example of a keyhole in a realistic environment arises in a hallway or a tunnel [1].

According to this rank-deficiency, maximum achievable diversity of a STC is the minimum number of transmit and receive antennas, so the system performance will be degraded in comparison with usual i.i.d channels.

We know that various analytical studies of space time block codes (STBCs) under this condition (keyhole) have already taken place. For example, in [2], a general analysis of orthogonal STBCs (OSTBCs) over keyhole channels is undertaken, where error probability expressions involving integration of hyper geometric function is put forward by using orthogonal property of OSTBCs. Furthermore, authors in [3] focused on closed form error probability expressions of OSTBCs based on a systematic analysis. The design criteria for STCs with two transmit antennas and one receive antenna in keyhole channels are presented in [4]. These criteria have been generalized based on the analysis of the pair wise error probability (PEP) in the asymptotic of high signal to noise ratio (SNR) [5].

![Fig. 1. Keyhole channel](image)

It is worth mentioning that the design of STCs for keyhole channels has not been studied extensively to date. Although some space time trellis codes (STTCs) such as super orthogonal STTC are modified in [4] and [5], there are no STBCs for keyhole channels that could satisfy all the design criteria in [5].

In this paper, the aim is to modify the proposed STBC in [6], which is a full rate and full diversity code with linear complexity in the receiver, for keyhole channels based on the criteria in [5]. Simulation results illustrate that our STBC’s performance is far superior to some of well-known STBCs, when BPSK and 4-QAM constellations are utilized. The remainder of this paper is organized as follows: next section describes a wireless communication system and briefly discusses the general design criteria for STCs in keyhole channels. Section 3 details the proposed STBC and also defines its properties. In section 4, simulation results are revealed and compared with results from a couple of big names in this arena to validate our approach. The paper then draws toward a close by outlining its contribution.
2. SYSTEM MODEL AND DESIGN CRITERIA

2.1. System Model

Let us consider a MIMO channel with \( N_t \) transmit antennas; \( N_r \) receive antennas and quasi-static flat fading of block length \( T \). The channel state information (CSI) is assumed to be known at the receiver, but unknown at the transmitter (perfect CSI). Moreover, we assume that the keyhole doesn’t lose the energy of captured signal. The received matrix is:

\[
Y = \sqrt{\rho} X(s) H + W \tag{1}
\]

where \( X(s) \) is the \( T \times N_t \) complex matrix of the transmitted signal, \( \rho \) is the average SNR at each receive antenna, and \( W \) denotes the \( T \times N_r \) noise matrix in which all entries are i.i.d \( \mathcal{C} \mathcal{N}(0,1) \) (zero mean, unit variance, complex Gaussian). Since each transmitted signal has to pass through the keyhole and then propagate to the receiver, \( H \) is modelled as:

\[
H = \varpi \times \varpi^T \tag{2}
\]

where \( \varpi \) (multipath gains at the transmitter) and \( \varpi^T \) (multipath gains at the receiver) are column vectors \( (N_t \times 1) \) with entries \( \varpi_i \), \( i = 1, \ldots, N_t \) and \( (N_r \times 1) \) with entries \( \varpi_r^T \), \( r = 1, \ldots, N_r \), respectively. All the entries of these vectors are also i.i.d \( \mathcal{C} \mathcal{N}(0,1) \).

2.2 Design Criteria

We now discuss the design criteria for STCs in keyhole channels [5] and differences between these criteria and the well-known design criteria for STCs in common channels [7].

According to (3), it is obvious that the maximum achievable diversity is minimum \( (N_t, N_r) \) for \( N_r \neq N_t \), but the upper bound of \( P_e \) has the form \( \frac{\log_2 e}{\sqrt{N_r}} \) which makes the diversity order to be \( N_r \) with a slightly unexpected twist in the case of \( N_r = N_t \) [5]. Thus, adding the antenna(s) to the minimum \( (N_t, N_r) \) until \( N_r \neq N_t \) is the only way to increase diversity order in keyhole channels. As a result, the relation between diversity order and the number of transmit (or receive) antennas is not always linear in the keyhole Rayleigh channels (unlike i.i.d Rayleigh channels) where the number of receive (or transmit) is constant (Fig. 2-3).

According to (3), the coding criteria for designing STCs in the keyhole Rayleigh channels is not different from the well-known rank-determinant criteria in [7] when \( N_r \geq N_t \). Hence, the codes reported for the Rayleigh fading channels would also perform well under the keyhole channel if \( N_r \geq N_t \) [5].

The main difference between design criteria for these two types of channels appears when \( N_r > N_t \), where the coding gain has a different expression. Therefore, the STCs in this case should satisfy this expression. For example, in [4] and [5] some STTCs are modified based on this expression.
3. CODE CONSTRUCTION AND ITS PROPERTIES

The structure of the STBC which is used in this paper is as follows:

\[
X(s) = \begin{bmatrix}
    a_1s_1 + a_2s_2 & b_1s_3 + b_2s_4 \\
    -a_1s_2 - a_2s_3 & b_1s_1 + b_2s_4
\end{bmatrix}
\]  

(4)

where \(s_1, s_2, s_3, \) and \(s_4\) denote the symbols chosen from a constellation such as BPSK or 4-QAM. For the FRLR STBC, coefficients \( \{a_1, a_2, b_1, b_2\} \subseteq \mathbb{C} \) have to meet the three constraints that are explained below:

**Constraint I.** Clearly, coefficients \(a_1, a_2, b_1, \) and \(b_2\) must satisfy following equations:

\[
a_1z_1 + az_2 \neq 0
\]  

(5)

\[
b_1z_1 + b_2z_2 \neq 0
\]  

(6)

except for \(z_1 = z_2 = 0, \) where \(z_1 \) and \(z_2 \in \mathbb{Z}(t).\)

**Constraint II.**

\[
|a_1|^2 + |a_2|^2 = |b_1|^2 + |b_2|^2 = 1.
\]  

(7)

The above condition guarantees that in each time slot equal average, power is transmitted.

**Constraint III.**

\[
a_1b_2 = a_2b_1.
\]  

(8)

This constraint completely distinguishes the new proposed code from the one introduced in [8], and reduces the complexity of the ML receiver to linear complexity that will be dealt with later.

This STBC transmits four symbols over two-time slots. Therefore, its symbol transmission rate is equal to two, i.e., it benefits from full-rate power. Now, let us describe two other important properties of this code.

3.1. Full-diversity

In order to prove the full-diversity feature of the specified code in (4), the codeword difference matrix \(X(s)\) and \(X'(s)\) must be full rank for any two different codewords \(X(s)\) and \(X'(s).\)

In this case, we have:

\[
|\det(X(s) - X'(s))|^2 = |a_1e_1 + a_2e_2|^2 |b_1e_1 + b_2e_2|^2
\]

\[
+ |b_1^*e_1 + b_2^*e_2|^2 + |a_1e_1 + a_2e_2|^2 |b_1^*e_1 + b_2^*e_2|^2
\]

\[
+ 2\Re[(a_1^*e_1 + a_2^*e_2)(b_1^*e_1 + b_2^*e_2)(b_1^*e_2 + b_2^*e_1)(a_1^*e_2 + a_2^*e_2)]
\]  

(9)

where \(e_k = s_k - \delta_k \in \mathbb{Z}(t)\) for \(k = 1, 2, 3, 4.\) Since \(a_1, a_2, b_1, \) and \(b_2\) satisfy **Constraint I,** sum of the first and the second terms of (9) is a positive scalar. Using **Constraint III,** the third term of (9) could be rewritten as:

\[
2\Re[(a_1^*b_1^*e_1 + a_2^*b_2^*e_2) + a_1b_2^*e_2(a_1^*e_4 + a_2^*e_3) + a_2b_1^*e_1(a_1^*e_3 + a_2^*e_4)] \times
\]

\[
[a_1^*b_1^*e_1 + a_2^*b_2^*e_2] - [a_1b_2^*e_1 + a_2b_1^*e_2]
\]  

(10)

Clearly, (10) is a non-negative scalar. Consequently, full diversity property is achieved by the specified STBC in (4).

3.2 Linear Complexity at the Receiver

The major feature of the proposed code is its linear decoder which is explained below.

For any received signal matrix \(Y,\) the coherent ML decoder finds:

\[
\arg\min_{X_e} ||Y - X_eH||_F, \quad t = 1, 2, ..., M^L
\]  

(11)

where \(\cdot ||_F\) denotes norm Frobenius norm and \(M^L\) is the constellation size. Considering (4) and (11), should minimize the following expression:

\[
F(s_1, s_2, s_3, s_4) = C + F'_1(s_1, s_2) + F'_2(s_3, s_4)
\]

\[
+ 2\Re[(a_1^*b_1^* + a_2^*b_2^*)(\sum_{k=1}^{N} \alpha_1 \alpha_2 k_1 \alpha_2 k_2 k_3 k_4) s_k s_k^*]
\]

(12)

\[
+ 2\Re((a_2^*b_1^* + a_1^*b_2^*) \sum_{k=1}^{N} \alpha_1 \alpha_2 k_1 k_2 k_3 k_4 s_k s_k^*)
\]

where \(C\) is a constant which is independent of the symbols, and \(F'_1(s_1, s_2)\) and \(F'_2(s_3, s_4)\) are functions of \(\{s_1, s_2\}\) and \(\{s_3, s_4\}\), respectively. Obviously, the two latter terms of (12) are eliminated by choosing \(a_1^*b_2^* = a_2^*b_1^*.\) Therefore, the ML decoding, as in QOSTBC for three and four transmit antennas [11], results in minimizing \(F'_1(s_1, s_2)\) for all values of \(s_1, s_2\) as well as \(F'_2(s_3, s_4)\) for all values of \(s_3, s_4.\)

Although this is a considerable reduction of the decoding complexity, it could still be decreased further by using conditional decoding for \(F'_1(s_1, s_2)\) and \(F'_2(s_3, s_4)\), (see [8]). In what follows, the conditional decoding procedures for \(F'_1(s_1, s_2)\) and \(F'_2(s_3, s_4)\) are investigated.

For simplicity, we concentrate on two receive antennas. One could imitate similar procedures decoding the received signal for other numbers of receive antennas.

By using the received matrix \(Y, \omega_1, \omega_2, \omega_3, \) and \(\omega_4\) are prepared for a given value of symbol \(s_2\) as follows:

\[
\omega_1 = y_{11} - a_2s_2 \alpha_{11}^\alpha_1
\]

(13-a)

\[
\omega_2 = y_{21} - a_2s_2 \alpha_{11}^\alpha_2
\]

(13-b)

\[
\omega_3 = y_{12} - a_2s_2 \alpha_{22}^\alpha_1
\]

(13-c)

\[
\omega_4 = y_{22} - a_2s_2 \alpha_{22}^\alpha_2
\]

(13-d)

By substituting \(\{y_{11}, y_{12}, y_{21}, y_{22}\}\) in the above equations, we have:

\[
\omega_1 = a_1s_1 \alpha_{11}^\alpha_1 + b_1s_3 \alpha_{11}^\alpha_2 + b_2s_4 \alpha_{12}^\alpha_1 + n_{11}
\]

(14-a)

\[
\omega_2 = a_1s_1 \alpha_{12}^\alpha_1 - a_2s_2 \alpha_{11}^\alpha_2 + n_{12}
\]

(14-b)

\[
\omega_3 = a_1s_1 \alpha_{12}^\alpha_2 + b_1s_3 \alpha_{12}^\alpha_2 + b_2s_4 \alpha_{12}^\alpha_1 + n_{12}
\]

(14-c)

\[
\omega_4 = b_1s_1 \alpha_{11}^\alpha_1 - a_1s_2 \alpha_{12}^\alpha_1 - a_2s_2 \alpha_{12}^\alpha_2 + n_{22}
\]

(14-d)

Now, \(\alpha[s_2]\) is computed from (14-a) to (14-d) as:

\[
\alpha[s_2] = a_2^\alpha_1 \omega_1 + b_2^\alpha_1 \omega_2 + a_2^\alpha_1 \omega_3 + b_2^\alpha_2 \omega_4
\]

\[
= (a_1 \alpha_1^\alpha_1(k_1 1^2) + (a_1 \alpha_2(k_2 1^2) + b_2 \alpha_2(k_2 2^2) + b_2 \alpha_2(k_2 2^2) + s_1)
\]

\[
+ (a_1 \alpha_2(k_1 2^2) + b_2 \alpha_2(k_1 2^2) + b_2 \alpha_2(k_1 2^2) + s_2)
\]

\[
+ (a_2 \alpha_1(k_1 1^2) + b_2 \alpha_1(k_1 2^2) + b_2 \alpha_2(k_1 2^2) + s_3)
\]

\[
+ (a_2 \alpha_1(k_1 2^2) + b_2 \alpha_1(k_1 2^2) + b_2 \alpha_2(k_1 2^2) + s_4)
\]

By applying **Constraint III** to (15) and dividing the
result by coefficient of \( s_1 \), say \( \theta \), we have:

\[
s_3^{\text{ML}}|_{s_2} = s_1 + \frac{N'}{\theta} \tag{16}
\]

where

\[
N' = a_2^2 A_{11}^* n_{11} + b_2^2 A_{21} n_{21} + a_2^2 A_{12} n_{12} + b_2^2 A_{22} n_{22}
\]

Therefore, we could obtain the ML estimate of symbol \( s_1 \) conditional on symbol \( s_2 \), say \( s_3^{\text{ML}}|_{s_2} \).

In a similar way, to minimize \( F_2(s_3, s_4) \) conditionally, we have:

\[
\beta|_{s_3} = (a_2^2 |A_{11}|^2 + |A_{12}|^2) + b_2^2 |A_{21}|^2 + |A_{22}|^2\)

\[
\beta|_{s_3} = b_2^2 A_{21} n_{21} - a_2^2 A_{11} n_{21} + b_2^2 A_{22} n_{12} - a_2^2 A_{12} n_{12}
\]

and therefore:

\[
s_3^{\text{ML}}|_{s_4} = s_3 + \frac{N'}{\theta} \tag{18}
\]

where

\[
\theta' = a_2^2 |A_{11}|^2 + |A_{12}|^2 + b_2^2 |A_{21}|^2 + |A_{22}|^2
\]

\[
N' = b_2^2 A_{21} n_{11} - a_2^2 A_{11} n_{21} + b_2^2 A_{22} n_{12} - a_2^2 A_{12} n_{12}
\]

Since \( s_3^{\text{ML}}|_{s_4} \) only depends on symbol \( s_3 \) and noise terms, minimizing procedure of \( F_2(s_3, s_4) \) is limited to minimizing the cost function \( F_2(s_3^{\text{ML}}|_{s_4}, s_4) \) for all possible values of \( s_4 \). Now, by minimizing \( F_2(s_1^{\text{ML}}|_{s_2}, s_2) \) for all possible values of \( s_2 \), estimations of symbols \( s_1 \) and \( s_2 \) and minimizing \( F_2(s_3^{\text{ML}}|_{s_4}, s_4) \) for all possible values of \( s_4 \), estimations of symbols \( s_3 \) and \( s_4 \) are obtained. As a result, the suggested STBC in (4) leads to a linear complexity at the receiver for an optimum decoder.

4. SIMULATION RESULTS

In this section, we first choose parameters of the proposed code according to criteria in section 2 by computer search and then compare its performance against two well-known STBCs, Alamouti code [9], and Golden code [10] in the keyhole Rayleigh fading channel.

For BPSK and 4-QAM constellations, parameters \( a_1 \), \( a_2 \), \( b_1 \), and \( b_2 \) are tabulated in Table 1 and 2. As these tables show, however, the allocated powers for different symbols are different, Constraint II being still observed. That means the total average power at each time slot per transmit antenna is equal.

We compare the symbol error probability (SER) performances of the new proposed code with Golden and Alamouti codes in Fig. 4 and Fig. 5, where the bandwidth efficiencies are 2 bits/sec/Hz and 4 bits/sec/Hz, respectively. Note that two transmit antennas and one receive antenna are used in both figures.

According to Fig. 4, our proposed code outperforms both Alamouti and Golden codes. For instance, the new proposed STBC has contributed almost 2dB reduction in SNR at a SER of \( 10^{-3} \) when throughput is 2 bits/sec/Hz. A similar scenario occurs in Fig. 5, i.e. nearly 2 dB reduction in SNR at a SER of \( 2 \times 10^{-3} \) with throughput equal to 4 bits/sec/Hz.

It can also be shown that similar results are obtained for two transmit and two receive antennas, as in Fig. 6-7. As it can be seen from Fig. 6, with SER at \( 10^{-5} \), the proposed code performs better than the Alamouti code by almost 1.5 dB, when bandwidth efficiency is 2 bits/sec/Hz. Moreover, the difference in SNR between this code and the Alamouti code, is approximately 1 dB with throughput equal to 4 bits/sec/Hz, when SER is \( 10^{-5} \), as shown in Fig. 7.

5. CONCLUSIONS

In this paper, we have introduced an innovative full rate, full diversity \( 2 \times 2 \) STBC with linear complexity in the receiver for the keyhole channels. This code is based on the special criteria of keyhole condition. Simulation results also confirm that the proposed code performance is outstanding for 4-QAM and BPSK constellations.

| Table 1. Optimum parameters for the proposed code (\( N_r = 1 \)). |
|------------------|------------------|------------------|------------------|
| BPSK | \( a_1 \) | \( a_2 \) | \( b_1 \) | \( b_2 \) |
| 4-QAM | \( A472e^{j1.1416} \) | \( .8944e^{j1.1416} \) | \( .472e^{-j3.0416} \) | \( .8944e^{-j3.0416} \) |

| Table 2. Optimum parameters for the proposed code (\( N_r = 2 \)). |
|------------------|------------------|------------------|------------------|
| BPSK | \( A_{11} \) | \( A_{21} \) | \( A_{12} \) | \( A_{22} \) |
| 4-QAM | \( .472e^{j1.1416} \) | \( .8944e^{j1.1416} \) | \( .472e^{-j3.0416} \) | \( .8944e^{-j3.0416} \) |
Fig. 4. Performance comparison of the proposed code with Golden and Alamouti codes with the same bandwidth efficiency 2bits/s/Hz.

Fig. 5. Performance comparison of the proposed code with Golden and Alamouti codes with the same bandwidth efficiency 4bits/s/Hz.

Fig. 6. Performance comparison of the proposed code with Golden and Alamouti codes with the same bandwidth efficiency 2bits/s/Hz.

REFERENCES


