Loss Minimization Sliding Mode Control of IPM Synchronous Motor Drives

Mehran Zamanifar(1) - Sadegh Vaez-Zadeh(2)
(1) Department of Electrical Engineering - Isfahan University of Technology, Isfahan, Iran
(2) Department of Electrical Engineering - Tehran University, Tehran, Iran

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Abstract: In this paper, a nonlinear loss minimization control strategy for an interior permanent magnet synchronous motor (IPMSM) based on a newly developed sliding mode approach is presented. This control method sets force the speed control of the IPMSM drives and simultaneously ensures the minimization of the losses besides the uncertainties exist in the system such as parameter variations which have undesirable effects on the controller performance except at near nominal conditions. Simulation results are presented to show the effectiveness of the proposed controller.

Index Terms: Permanent magnet motors, speed control, loss minimization control, sliding mode.

I. Introduction

Interior permanent magnet synchronous (IPMS) motors are widely used in high performance drive applications such as robotics, aerospace and electric vehicles (EV) [1,2]. These applications require demanding still practical control methods [4]. Thus much effort has been directed towards the efficiency optimization control (EOC) of the IPMS motors by minimizing machine copper and iron losses [5-8]. Model-based EOC methods are very fast and do not produce torque ripples; but they are not robust against machine parameter variations [5-7]. The stator resistance may vary due to the skin effects, temperature variations, etc. Core losses also vary due to the variations of motor flux and speed and become an important issue at high speeds [8]. Also permanent magnet (PM) flux may vary due to temperature variations and excessive flux weakening. Machine inductance is known to depend on air-gap flux [9]. The adverse effect of motor parameter variations on minimum loss operation of IPMS motors is analyzed in [10,11] and it is shown that these parameter variations except at near nominal conditions have undesirable effects on the controller performance. Thus, it is vital to compensate the variation of the mentioned parameters in the high performance IPMS motor drives especially when an EOC is used.

Sliding mode control (SMC) has been studied by many researchers due to its favorable advantages, such as insensitivity to parameter variations and external load disturbances [12-13]. Only the bounds of the uncertainties are needed. The robustness of this controller is guaranteed, but the worst drawback is the chattering, which limits the applications of SMC. The chattering phenomenon is greatly considered as motion which oscillates about the sliding surface. There are two possible mechanisms which produce such a motion. First, the presence of switching device non-idealities such as delays; second, the presence of parasitic dynamics (actuator and sensor dynamics) in series with the plant. Using a discrete-time control design approach, or high sample rate, we can reduce the chattering due to the switching device non-idealities. Several methods, observer-based sliding mode control, sliding mode based on disturbance compensation and boundary layer sliding mode control have been used to reduce the chattering due to parasitic dynamics. The most common and simplest approach to reduce the effects of chattering has been the boundary layer [14].

In this paper, a newly designed sliding mode controller for an IPMS motor drive is introduced to both minimize the machine losses and track a reference speed command. Finally, extensive simulation results are presented to show robust, efficient and high performance motor drive operation under the proposed control system.

II. Machine Model

Under certain assumptions a widely used model for an IPM synchronous motor including copper and iron losses in a synchronously rotating reference frame is presented in Fig. (1) [5,10]:

The model equations are given as follows:

\[
\frac{di_q}{dt} = \frac{R_{q}}{L_{q}}(-R_{i_q}i_{q}+v_{q})+\frac{L_{q}}{L_{q}}\omega_c i_{q} \\
\frac{di_d}{dt} = \frac{R_{d}}{L_{d}}(-R_{i_d}i_{d}+v_{d})+\frac{L_{q}}{L_{q}}\left(\omega_{c}i_{q}+\lambda_{m}\right) \\
T_e = \frac{3P}{2}\left[p_{m}+(L_{d}-L_{q})i_{q}i_{d}\right]
\]

(1) (2) (3)
Conventional notation is used for machine parameters and variables. In steady state, the electrical power losses of the machine can be expressed as:

\[
P_L = \frac{3}{2} R_s \left[ (i_d^2 + i_q^2) T + \frac{3 (R_s + R_e)}{R_e} L_d \omega c i_d T \right]
\]

By differentiating (3) and (4) with respect to \(i_{dT}\) the following equations are obtained:

\[
\frac{\partial P}{\partial i_{dT}} = \frac{3}{2} R_s \left[ (i_d^2 + i_q^2) \frac{\partial T}{\partial i_{dT}} + \frac{3 (R_s + R_e)}{R_e} L_d \omega c i_q \frac{\partial T}{\partial i_{dT}} \right]
\]

III. Loss Minimization Condition

In this section, a loss minimization condition for an IPM synchronous motor is achieved. By differentiating (3) and (4) with respect to \(i_{dT}\) the following equations are obtained:

\[
\frac{\partial T}{\partial i_{dT}} = \frac{3}{2} P \left[ \lambda_m - \frac{\partial i_{dT}}{\partial i_{dT}} + \left( L_d - L_q \right) i_{dT} \right] \left( i_{dT} + i_{dT} \frac{\partial i_{dT}}{\partial i_{dT}} \right)
\]

\[
\frac{\partial P}{\partial i_{dT}} = \left( L_d - L_q \right) \frac{\partial i_{dT}}{\partial i_{dT}} + \frac{3 (R_s + R_e)}{R_e} L_d \lambda_m \omega c i_q
\]

In order to reach the loss minimization condition in every operating point, the above differentials must be equal to zero i.e.:

\[
\frac{\partial T}{\partial i_{dT}} = 0, \quad \frac{\partial P}{\partial i_{dT}} = 0
\]

then:

\[
\frac{\partial i_{dT}}{\partial i_{dT}} = -\frac{\left( L_d - L_q \right) i_{dT}}{\lambda_m + \left( L_d - L_q \right) i_{dT}}
\]

\[
\frac{\partial i_{dT}}{\partial i_{dT}} = -\frac{P}{Q}
\]

where:

\[
P = R_s i_{dT} + \frac{R_s}{R_e} \left( \lambda_m + \left( L_d - L_q \right) i_{dT} \right) \omega c
\]

\[
+ \frac{\left( R_s + R_e \right)}{R_e} \left( \lambda_m + L_d i_{dT} \right) L_d \omega c
\]

Q = \frac{R_s i_{dT}}{R_e} \left( \lambda_m + \left( L_d - L_q \right) i_{dT} \right) \omega c

+ \frac{\left( R_s + R_e \right)}{R_e} \left( \lambda_m + \left( L_d - L_q \right) \right) L_d \omega c

By equaling the right hand side of (8) and (9) and rearranging the result, the mentioned condition is obtained as:

\[
\frac{\alpha^2}{R_s} \left( R_s + R_e \right) \left( L_d - L_q \right) \left( L_d - L_q \right) - R_s \lambda_m i_{dT}
\]

\[
+ \frac{\alpha^2}{R_e} \left( R_s + R_e \right) \left( L_d - L_q \right) \left( L_d - L_q \right) - \frac{\alpha^2}{R_s} \left( \lambda_m + \left( L_d - L_q \right) \right) L_d \omega c i_{dT}
\]

\[
+ \frac{\alpha^2}{R_e} \left( R_s + R_e \right) \left( L_d - L_q \right) \left( L_d - L_q \right) - \frac{\alpha^2}{R_s} \left( \lambda_m + \left( L_d - L_q \right) \right) L_d \omega c i_{dT} = 0
\]

IV. Sliding Mode Controller

In this Section a sliding mode controller is designed to provide both high performance motor drive operation and loss minimization.

A. Coordinate System

In order to control the developed torque and efficiency, the system outputs are chosen as:

\[
y_1 = T_c = \frac{3}{2} P \left[ \lambda_m + \left( L_d - L_q \right) i_{dT} \right]
\]

\[
y_2 = -\frac{\alpha^2}{R_s} \left( R_s + R_e \right) \left( L_d - L_q \right) \left( L_d - L_q \right) - R_s \lambda_m i_{dT}
\]

\[
- \frac{\alpha^2}{R_e} \left( R_s + R_e \right) \left( L_d - L_q \right) \left( L_d - L_q \right) - \frac{\alpha^2}{R_s} \left( \lambda_m + \left( L_d - L_q \right) \right) L_d \omega c i_{dT}
\]

\[
+ \frac{\alpha^2}{R_e} \left( R_s + R_e \right) \left( L_d - L_q \right) \left( L_d - L_q \right) - \frac{\alpha^2}{R_s} \left( \lambda_m + \left( L_d - L_q \right) \right) L_d \omega c i_{dT} = 0
\]

According to feedback linearization theorem, outputs \(y_1\) and \(y_2\) have to be differentiated successively with respect to time, until one of the components of the control vector \(u=(v_d, v_q)^T\) appears [6]. By differentiating (13) and (14) and substituting from (1) and (2), a new coordinate system can be introduced as:

\[
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2
\end{bmatrix} =
\begin{bmatrix}
f_1 & f_2 \\
g_1 & g_2
\end{bmatrix}
\begin{bmatrix}
v_d \\
v_q
\end{bmatrix}
\]

where \(f_1\) and \(f_2\) and \(g_1\) and \(g_2\) are given in Appendix I. Furthermore, using the nonlinear feedback linearization control method to decouple the control inputs, the resulting system is [14]:

\[
\begin{bmatrix}
\dot{\varphi}_d \\
\dot{\varphi}_q
\end{bmatrix} =
\begin{bmatrix}
g_1 v_d + g_4 v_q \\
g_3 v_d + g_2 v_q
\end{bmatrix}
\]

and the linear feedback is defined as:

\[
\begin{bmatrix}
\varphi_d \\
\varphi_q
\end{bmatrix} =
\begin{bmatrix}
g_1 v_d + g_4 v_q \\
g_3 v_d + g_2 v_q
\end{bmatrix}
\]

Owing to (16) being a nonlinear system, the dynamics are hard to regulate by a constant state feedback gain [13]. Therefore, for the dynamic system of (16), a model-following nonlinear sliding mode controller is proposed to track a desired reference model. The reference model is designed in a linear form as:
Using feedback is a positive parameter.

\[
\begin{bmatrix}
\dot{y}_{m1} \\
\dot{y}_{m2}
\end{bmatrix} =
\begin{bmatrix}
-a_{m1} & 0 \\
0 & -a_{m2}
\end{bmatrix}
\begin{bmatrix}
y_{m1} \\
y_{m2}
\end{bmatrix} +
\begin{bmatrix}
a_{m1} & 0 \\
0 & a_{m2}
\end{bmatrix}
\begin{bmatrix}
y_{1,ref} \\
y_{2,ref}
\end{bmatrix}
\tag{18}
\]

where \(a_{m1}\) and \(a_{m2}\) are the designed positive constants with \(a_{m1}\) determining the reference torque performance and \(a_{m2}\) determining the reference loss minimization condition. Also we have:

\[
y_{1,ref} = T_{e,ref}, \quad y_{2,ref} = 0
\tag{19}
\]

The tracking errors between the plant of (16) and reference model are:

\[
\begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} =
\begin{bmatrix}
y_{1} - y_{m1} \\
y_{2} - y_{m2}
\end{bmatrix}
\tag{20}
\]

and its error dynamics are derived as follows:

\[
\begin{bmatrix}
\hat{e}_1' \\
\hat{e}_2'
\end{bmatrix} =
\begin{bmatrix}
f_1' + 0 & 1 \\
0 & f_2' + 1 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{e}_1' \\
\hat{e}_2'
\end{bmatrix} +
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix}
\tag{21}
\]

where \(\phi_1\) and \(\phi_2\) are assumed bounded parameter uncertainties and the linear feedback is:

\[
\begin{bmatrix}
\nu_d' \\
\nu_q'
\end{bmatrix} =
\begin{bmatrix}
\phi_d + a_{m2}y_{m2} - a_{m1}y_{m1,ref} \\
\phi_q + a_{m1}y_{m1} - a_{m2}y_{m2,ref}
\end{bmatrix}
\tag{22}
\]

Now, the sliding mode design technique can be used for the error system (21) to make the errors \(e_1\) and \(e_2\) reach zero. That is the developed torque and loss minimization condition track their reference values. The control algorithm consists of three steps:

- Calculating coordinate controller, \(\nu_d'\) and \(\nu_q'\), according to the sliding mode method.
- Calculating \(\hat{\nu}_d\) and \(\hat{\nu}_q\) using feedback linearization control of (22).
- Calculating \(\nu_d\) and \(\nu_q\) using feedback linearization control of (16).

**B. Sliding Mode Surface**

In order that \(e_1\) and \(e_2\) converge to zero, two sliding surfaces are introduced as:

\[
s_1 = e_1 + \lambda_1 \int e_1 \, dt
\tag{23}
\]

\[
s_2 = e_2 + \lambda_2 \int e_2 \, dt
\tag{24}
\]

where the parameters \(\lambda_1\) and \(\lambda_2\) are positive constants which determine the convergence rate. A Lyapunov function is then proposed as:

\[
V = \frac{1}{2} s_1^2 + \frac{1}{2} s_2^2 \Rightarrow \dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2
\tag{25}
\]

By differentiating sliding surfaces and substituting from (22), we will have:

\[
\dot{s}_1 = f_1 + \nu_q + \phi_1 + \lambda e_1
\tag{26}
\]

\[
\dot{s}_2 = f_2 + \nu_d + \phi_2 + \lambda e_2
\tag{27}
\]

then:

\[
\dot{V} = s_1 [f_1 + \nu_q + \phi_1 + \lambda e_1] + s_2 [f_2 + \nu_d + \phi_2 + \lambda e_2]
\tag{28}
\]

Assuming \(\nu_d'\) and \(\nu_q'\) as:

\[
\nu_d' = -f_2 - \lambda e_2 - Q sgn(s_2), \quad Q > |\phi_2|
\tag{29}
\]

\[
\nu_q' = -f_1 - \lambda e_1 - Q sgn(s_1), \quad Q > |\phi_1|
\tag{30}
\]

and substituting them in (28):

\[
\dot{V} = s_1 [-Q sgn(s_1) + \phi_1] + s_2 [-Q sgn(s_2) + \phi_2]
\tag{31}
\]

Hence, using the proposed controller (29) - (30), the reachability of sliding mode control of the controlled system (21) is guaranteed. After reaching the surfaces \(S_1\) and \(S_2\), the system output errors will converge to zero, that is:

\[
y_1 = T_{e,ref}, \quad y_2 = y_{2,ref} = 0
\tag{32}
\]

The proposed sliding mode control is robust with respect to matched and mismatched uncertainties. In addition, with the use of the feedback linearization control, the complete decoupled control of torque and indirectly flux can be obtained.

The chattering associated with sliding mode is the main drawback of the sliding mode application. The sliding mode control chattering is substantially reduced if in equations (28) and (29), \(sgn(s_1)\) and \(sgn(s_2)\) are replaced by the following saturation function or by \(\tan^{-1}\) function [15]:

\[
Sat\left(\frac{s}{\varphi}\right) = \begin{cases} 
+1 & \text{if } s > \varphi \\
\frac{s}{\varphi} & \text{if } -\varphi \leq s \leq \varphi \\
-1 & \text{if } s < -\varphi 
\end{cases}
\tag{33}
\]

where \(\varphi\) is a positive parameter.

![Fig. (2): IPM motor drive system block diagram](www.SID.ir)
V. Simulation Results
The performance of an IPM motor under the proposed model-following nonlinear sliding mode control system has been investigated by extensive simulation. The IPM synchronous motor specifications are listed in Appendix II. The overall motor drive block diagram is shown in Fig. (2). Many simulation runs have been carried out to examine the proposed SMC scheme. Firstly, an unloaded motor, $T_L=0$, is simulated. The motor is started by an exponential speed command reference with the final value of machine rated speed as in Fig. (3). Then at the time $t=1$ (sec) a rated load torque is applied to the motor. Reference Speed command is reduced exponentially to the half rated speed at $t=1.5$ (sec). Subsequently, at $t=2.5$ (sec) the rated load is removed from the motor.

As it is expected, SMC rejects the external load disturbance. Also it can be seen that the chattering associated with the sliding mode exists with low amplitude in Fig. (4) which shows the load and the motor torque plots.

The d-q axis currents are also shown in Fig. (5). The simulation results confirm desirable motor performance under the proposed efficiency optimization controller.

Figs. (6-9) show the phase plot of the SMC. In Figs. (6) and (7) the two errors of $e_1$ and $e_2$ with respect to time are plotted. Also the effect of $\lambda_1$ and $\lambda_2$ values are shown in these figures. Sliding surfaces $S_1$ and $S_2$ are shown in Figs. (8) and (9). It is seen that the two errors oscillate and finally converge the zero point. This confirms the reachability of SMC the controlled system.
The loss minimization control of IPM motor drives, introduced by Morimoto is simulated here for comparison [4]. Fig. (7) shows the speed response of both methods for two operating points. It can be seen that since the PI controllers of the Morimoto’s method are tuned in nominal operating point, dynamics of other operating points are not desirable. But in the case of SMC, a change of operating point does not affect the desirable performance of the motor drive.

In order to show the robustness of SMC against the parameter variations, another simulation is carried out where the motor parameters are assumed to vary over a certain pattern due to ambient and operating conditions. Efficiency and total electrical losses of the motor are shown in Figs. (11-12) for varying speed (at rated torque) and varying torque (at rated speed) respectively. In these figures, three different pairs (η and \( P_{\text{Loss}} \)) of plots are shown. The first pair, shown by dash-dotted line and used as a basis for comparison, is related to the method introduced by Morimoto. It can be seen that parameter variations have undesirable effect on the motor efficiency except in nominal operating point, dynamics of other operating points are not desirable. But in the case of SMC, a change of operating point does not affect the desirable performance of the motor drive.

VI. conclusion
In this paper, a high performance model-following nonlinear sliding mode controller based on a feedback linearization control system for an IPM synchronous motor has been described. This controller minimizes the motor losses besides the tracking of a reference speed. Moreover, the desirable robustness of the controlled IPM synchronous motor drive against the parametric uncertainties is improved resulting in a more energy saving over a wide motor operating range.

Appendix I
The matrix components of equation (13) are:

\[
\begin{align*}
\mathbf{f}_1 &= \frac{3\pi}{2} \left[ (L_d - L_q) \omega_c \left( \frac{L_d}{L_d} i_d^2 - \frac{L_q}{L_q} i_q^2 \right) + \frac{R}{L_d} \left( L_d - L_q \right) i_d \right] \\
\mathbf{f}_2 &= 2R \left[ (L_d - L_q) \frac{L_d}{L_d} \omega_c i_d + \frac{L_q}{L_q} \omega_c i_q \right]^2 - 2 \frac{R}{L_d} \left( L_d - L_q \right) i_d^2 \\
\mathbf{f}_3 &= 2L_d \omega_c^2 \left( R + L_d \right) \omega_c i_d i_q - \omega_c \left( 2L_d - L_q \right) i_d^2 - 2 \frac{R}{L_d} \left( L_d - L_q \right) i_d^2 \\
\mathbf{f}_4 &= \frac{L_d \omega_c^2}{R_c} \left( R + L_d \right) \left( 4L_d - 3L_q \right) \omega_c i_d i_q \\
\mathbf{f}_5 &= \frac{R}{L_d} \left( L_q - 2L_d \right) \omega_c i_d i_q \\
\mathbf{f}_6 &= \frac{3\pi}{2} \frac{R_c}{L_d} \left( L_d - L_q \right) \omega_c i_d i_q \\
\mathbf{g}_1 &= \frac{3\pi}{2} \frac{R_c}{L_d} \left( L_d - L_q \right) \omega_c i_d i_q \\
\mathbf{g}_2 &= \frac{3\pi}{2} \frac{R_c}{L_d} \left( L_d - L_q \right) \omega_c i_d i_q \\
\mathbf{g}_3 &= \frac{3\pi}{2} \frac{R_c}{L_d} \left( L_d - L_q \right) \omega_c i_d i_q \\
\mathbf{g}_4 &= \frac{3\pi}{2} \frac{R_c}{L_d} \left( L_d - L_q \right) \omega_c i_d i_q \\
\mathbf{g}_5 &= \frac{3\pi}{2} \frac{R_c}{L_d} \left( L_d - L_q \right) \omega_c i_d i_q \\
\mathbf{g}_6 &= \frac{3\pi}{2} \frac{R_c}{L_d} \left( L_d - L_q \right) \omega_c i_d i_q \\
\end{align*}
\]
Appendix II

MACHINE SPECIFICATIONS

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated speed, rpm</td>
<td>1800</td>
</tr>
<tr>
<td>Rated torque, Nm</td>
<td>3.96</td>
</tr>
<tr>
<td>Rated current, A</td>
<td>3</td>
</tr>
<tr>
<td>P, No. of pole pairs</td>
<td>2</td>
</tr>
<tr>
<td>$R_s, R_c$, $\Omega$</td>
<td>1.93, 330</td>
</tr>
<tr>
<td>$L_d, L_q$, mH</td>
<td>42.44, 79.57</td>
</tr>
<tr>
<td>$\lambda_m$, Wb</td>
<td>0.314</td>
</tr>
<tr>
<td>J, Rotor inertia constant, Kg.m$^2$</td>
<td>0.003</td>
</tr>
<tr>
<td>B, Viscous coefficient, Nm/rad/sec.</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

Reference


Biography

Mehran Zamanifar was born in Isfahan, Iran, in 1981. He received the B.Sc. degree in electrical engineering from Isfahan University of Technology, Isfahan, Iran, in 2003 and the M.Sc. degree in power engineering from the University of Tehran, Tehran, Iran, in 2006. He is currently pursuing the Ph.D. degree in electrical engineering in Isfahan University of Technology. His research interests include modeling and control of electrical machines.

Sadegh Vaez-Zadeh (S’95–M’03–SM’05) was born in Mashhad, Iran in 1959 and received a B.Sc. degree from Iran University of Science and Technology, Tehran, Iran in 1985 and M.Sc. and Ph.D. degrees from Queen’s University, Kingston, ON, Canada, in 1993 and 1997 respectively, all in Electrical Engineering. He has been with several research and educational institutions in different positions. In 1997, he joined the University of Tehran as an assistant Professor and became an associate professor in 2001 and a full professor in 2005. He served the university as the Head of Power Division from 1998 to 2000 and currently is the Director of Advanced Motion Systems Research Laboratory.
which he founded in 1998 and the Director of Electrical Engineering Laboratory since 1998. He has been an associate editor and a member of the editorial board of Iranian Journal of Electrical and Computer Engineering since 2001. He has been active in many technical conferences in different capacities, lastly as a technical program committee member of the eighth International Conference on Electrical Machines and Systems (ICEMS 2005) held in China and an international steering committee member of ICEMS 2006 held in Japan. He has published over 120 technical papers, translated a book and holds a patent. His research interests include advanced rotary and linear electric machines and drives, motion control, magnetic levitation, electric and hybrid vehicles, power system control and robust control. Prof. Vaez-Zadeh is a member of the IEEE PES EMC Motor Sub-Committee. He has received a number of awards domestically including a best paper award form Iran Ministry of Science, Research and Technology in 2001 and a best research award form the University of Tehran in 2004.