Super-Efficiency and Sensitivity Analysis Based on Input-Oriented DEA-R

M. R. Mozaffari*
Islamic Azad University, Science and Research-Fars Branch

J. Gerami
Islamic Azad University, Science and Research-Fars Branch

Abstract. This paper suggests a method of finding super-efficiency scores and modification of input-oriented models for sensitivity analysis of decision making units. First, by using DEA-R (ratio-based DEA) models in the input orientation, the models of super-efficiency and also models of super-efficiency modification are suggested. Second, the worst-case scenarios are considered where the efficiency of the test DMU is deteriorating while the efficiencies of the other DMUs are improving. Then, by combining these two ideas, a model is suggested which increases the super-efficiency score and modifies the change ranges in order to preserve the performance class. In the end, the super-efficiency and change interval of efficient decision making units for 23 branches of Zone 1 of the Islamic Azad University are calculated.

AMS Subject Classification: 90B10; 90C31.
Keywords and Phrases: Data envelopment analysis, sensitivity analysis.

1. Introduction

Data envelopment analysis (DEA), developed by Charnes et al. ([2]), is a nonparametric methodology for assessing the performances of a group
of decision making units (DMUs) which use multiple inputs to produce multiple outputs. During recent years, the issue of sensitivity and stability of DEA results has been extensively studied. The first DEA sensitivity analysis paper by Charnes et al. ([3]) examined change in a single output. In recent years, many studies have been performed on sensitivity analysis of inputs and outputs of DMUs. Among these, sensitivity analysis using super-efficiency models has received much attention. Zhu ([10]) used the worst-case scenario where the efficiency of the test DMU is worsening while the efficiencies of the other DMUs are improving. He thus determined the necessary and sufficient conditions for preserving the efficiency classification of a DMU when various data changes are applied to all DMUs. Moreover, Zhu presented the necessary and sufficient conditions for the infeasibility of the super-efficiency model. Despic et al. ([7]) proposed a new mathematical model for sensitivity analysis, which combines the DEA methodology with the idea of ratio analysis. Similar to Andersen and Petersen’s ([1]) idea, Wei et al. ([5]) studied efficiency and super-efficiency using DEA-R and also compared the optimal weights of DEA and DEA-R. In DEA-R models, the efficiency scale is greater than or equal to that in DEA models and the classification and modification of DMUs depend on the super-efficiency score of the unit. Thus, using DEA-R models in the input orientation in order to find the super-efficiency with greater or equal amounts can adjust the change ranges of DMUs. The present paper is an extension of Zhu’s works and addresses super-efficiency and sensitivity analysis. The paper is organized as follows. In section 2, we review Zhu’s modified models. Section 3 contains an introduction of DEA-R. Super-efficiency and sensitivity analysis are discussed in section 4. An application of our proposed model to the data of 23 branches of Zone 1 of the Islamic Azad University is given in section 5 together with the comparison of the results with those of Zhu’s modified models. Section 6 provides the conclusion.
2. Data Envelopment Analysis

Consider \( n \) DMUs with \( m \) inputs and \( s \) outputs. The input and output vectors of \( DMU_j \) (\( j = 1, \ldots, n \)) are \( X_j = (x_{1j}, \ldots, x_{mj})^t \), \( Y_j = (y_{1j}, \ldots, y_{sj})^t \) where \( X_j > 0 \), \( Y_j > 0 \).

By using the constant returns to scale, convexity, and possibility postulates, the non-empty production possibility set (PPS) is defined as follows:

\[
T_c = \left\{ (X, Y) : X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0, j = 1, \ldots, n \right\}. 
\]

Let \( I \) and \( O \) denote, respectively, the input and output subsets in which we are interested. That is, we consider the data changes in set \( I \) and set \( O \). The input-oriented super-eficiency model for \( DMU_p \) using the constant returns to scale (CRS) assumption is as follows.

\[
\theta^{super}_{(p)} = \text{Min}_\theta \quad \theta^{Super}_{(p)}
\]

\[
s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta^{Super}_{(p)} x_{ip}, \quad i = 1, \ldots, m \]

\[
\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}, \quad r = 1, \ldots, s \quad (1)
\]

\[
\lambda_j \geq 0, \quad j = 1, \ldots, n, j \neq p.
\]

By modifying Model (1) and separating the inputs and by using the CRS assumption, Seiford and Zhu ([8]) proposed the following model to obtain the stability region of \( DMU_p \).

\[
\theta^{I-}_{(p)} = \text{Min}_\theta \quad \theta^{I-(p)}
\]

\[
s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta^{I-(p)} x_{ip}, \quad i \in I \]

\[
\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ip}, \quad i \notin I \quad (2)
\]

\[
\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}, \quad r = 1, \ldots, s \]

\[
\lambda_j \geq 0, \quad j = 1, \ldots, n, j \neq p.
\]
3. Ratio-Based DEA

Despic et al. ([6]) proposed a new mathematical model for sensitivity analysis, which combines the DEA methodology with the idea of ratio analysis. Similar to Andersen and Petersen’s ([1]) idea, Wei et al. ([4,5,6]) studied efficiency and super-efficiency using DEA-R and also compared the optimal weights of DEA and DEA-R. The input-oriented DEA-R model for $DMU_p$ using the constant returns to scale (CRS) assumption is as follows.

$$\begin{align*}
\text{Max} & \quad \Delta \\
\text{s.t.} & \quad \sum_{r=1}^{s} \sum_{i=1}^{m} w_{ir} \left( \frac{x_{ij}}{y_{ij}} \right) \geq \Delta, \quad j = 1, \ldots, n \\
& \quad \sum_{r=1}^{s} \sum_{i=1}^{m} w_{ir} = 1, \quad w_{ir} \geq 0, \quad \Delta \geq 0 \quad i = 1, \ldots, m, \quad r = 1, \ldots, s.
\end{align*}$$

(3)

Model (3) is a linear programming problem in which dual of Model (3) by considering dual variables $\lambda_j$ for all $j$ and $\theta_R$ respectively corresponding to input and output constraints and the convex combination constraint is as follows.

$$\begin{align*}
\text{Min} & \quad \theta_R \\
\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_j \left( \frac{x_{ij}}{y_{ij}} \right) \leq \theta_R, \quad i = 1, \ldots, m \quad r = 1, \ldots, s \\
& \quad \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n.
\end{align*}$$

(4)

Definition 3.1. $DMU_p$ is R-CCR-I-efficient (input-oriented CCR-R-efficient) if and only if $\theta_{R}^{*} = 1$.

Model (3) has the following properties.

1. The efficiency and super-efficiency scores obtained by this model are greater than or equal to those of the CCR model.
2. The efficiency scores of the model in the input and output orientations are not necessarily equal.
3. In a situation involving no weight restrictions, the input-target
improvement strategy given by DEA-R-I is always better than the CCR-I model.

4. When DEA-R-I weights are concentrated on one output, the CCR-I efficiency and DEA-R-I efficiency are the same.

4. Super-Efficiency Based on DEA-R

In this section, we discuss sensitivity analysis and obtaining the super-efficiency score based on DEA-R. Considering set I for inputs and by modifying the inputs based on Zhu’s idea, we have the following relations:

\[ \pi_{ip} = \delta_i x_{ip} \quad \delta_i \geq 1, \quad i \in I, \quad \pi_{ij} = \frac{x_{ij}}{\delta_i} \quad \delta_i \geq 1, \quad i \in I, \]
\[ \pi_{ip} = x_{ip} \quad i \notin I, \quad \pi_{ij} = x_{ij} \quad i \notin I. \]

The DMUs are divided into those lying on the frontier and those that are not on the frontier. Also, extreme efficient, non-extreme efficient, and weak efficient DMUs are denoted by E, E', and F, respectively.

**Lemma 4.1.** Assume that DMU \( p \in F \) with non-zero input/output slack values associated with set I/ set O. Then DMU \( p \) with inputs \( \overline{x}_{ip} \) and outputs \( \overline{y}_{rp} \) as defined above still belongs to set F when other DMUs are fixed.

**Proof.** By the complementary slackness theorem for Models (3) and (4), we have \( w^*_r s^*_r = 0 \). Since \( s^*_r \neq 0 \) for \( i \in I, r \in O \), we have \( w^*_r = 0 \) for \( i \in I, r \in O \). Therefore, \( w^*_r \) is a feasible solution to (3) for DMU \( p \) with inputs \( \pi_{ip} \) and outputs \( \overline{y}_{rp} \). Therefore, the DMU \( p \) still belongs to set F.

We consider the super-efficiency model based on DEA-R-I as follows.

\[
\theta^*_R(I(p))_{Super} = \min \theta^*_R(I(p))_{Super} \\
\text{s.t.} \quad \sum_{j=1}^{n} j = 1 \lambda_j \left( \frac{\overline{y}_{rj}}{\overline{s}_{rp}} \right) \leq \theta^*_R(I(p))_{Super} \quad i = 1, \ldots, m \quad r = 1, \ldots, s \\
\sum_{j=1}^{n} j = 1 \lambda_j \geq 1, \quad j \neq p \\
\sum_{j=1}^{n} j = 1 \lambda_j \geq 1, \quad j \neq p \\
\lambda_j \geq 0, \quad j = 1, \ldots, n, \quad j \neq p. \quad (5)
\]
We consider the modified super-efficiency model based on DEA-R-I as follows.

\[
\theta^*_R-I(p) = \min \quad \theta_{R-I(p)}
\]

\[
s.t. \quad \sum_{j=1, j \neq p}^{n} \lambda_j \left( \frac{x_{ij}}{y_{ip}} \right) \leq \theta_{R-I(p)}, \quad i \in I, \quad r = 1, \ldots, s
\]

\[
\sum_{j=1, j \neq p}^{n} \lambda_j \left( \frac{x_{ij}}{y_{ip}} \right) \leq 1, \quad i \notin I, \quad r = 1, \ldots, s
\]

\[
\sum_{j=1, j \neq p}^{n} \lambda_j \geq 1,
\]

\[
\lambda_j \geq 0, \quad j = 1, \ldots, n, \quad j \neq p.
\]

\( (6) \)

**Example 4.1.** (Taken from Zhu [10]) The results of Models (1), (2), (5) and (6) for four DMUs with two inputs and one output are presented in Table 1, below.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>1.1538</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1.2381</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>1.5000</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>7</td>
<td>1</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 1: Result Models (1), (2), (5) and (6).

With regard to the models presented based on DEA-R, we present the following theorems and lemmas for the sensitivity analysis of data.

**Lemma 4.2.** If \( \theta^*_R-I(p) = 1 \), then \( \theta^*_R-I(p) \leq 1 \).

**Proof.** The proof is obvious from the fact that \( (\lambda^{super}, \theta^*_R-I(p)) = (\lambda^*, 1) \) is a optimal solution to (5), so \( (\lambda^*, \theta_{R-I(p)}) = (\lambda^*, \theta^{super}_R-I(p)) \) is a feasible solution to (6). \( \square \)

**Lemma 4.3.** If \( \theta^*_R-I(p) = 1 \) and \( \theta^*_R-I(p) < 1 \), then DMU \( p \in F \).

**Proof.** \( \theta^*_R-I(p) = 1 \) indicates that DMU \( p \in E' \cup F \). Moreover, \( \theta^*_R-I(p) < \)
1 indicates that there are non-zero slack values in \( x_{ip} \) for \( i \in I \). Thus, \( DMU_p \in F \).

In Table (1), DMU D with \( \theta^*_{R-I(p)} = 1 \) and \( \theta^*_{R-I(p)} = 1 \) belongs to F.

**Theorem 4.1.** If \( \theta^*_{R-I(p)} = 1 \) and \( \theta^*_{R-I(p)} < 1 \) then for any \( \delta_i \geq 1 \) and \( \tilde{\delta}_i \geq 1 \) for \( (i \in I) \) \( DMU_p \) remains in set F.

**Proof.** From Lemma 4.3, we know that \( DMU_p \in F \) with non-zero slack values in \( x_{ip} \) for \( i \in I \). By Lemma 4.1 and its proof of Lemma 4.1, we know that for any \( \delta_i \geq 1 \) and \( \tilde{\delta}_i \geq 1 \) and \( w^*_r \) is a feasible solution to (3) in which inputs are replaced by \( \pi_{ip} \) for \( i \in I \) and \( x_{ij} \) for \( i \notin I \). Thus, \( DMU_p \) remains in set F after input data changes set I in all DMUs. □

**Corollary 4.1.** Infeasibility of Model (6) can only be associated with extreme-efficient DMUs in set E.

**Proof.** Lemma 4.2 implies that Model (6) are always feasible for DMUs in set E or set F. Also, Model (6) is always feasible for non frontier DMUs. Therefore, infeasibility of Model (6) may only occur for extreme-efficient DMUs in set E. □

**Theorem 4.2.** A specific super-efficiency DEA model associated with set I is infeasible if and only if for any \( \delta_i \geq 1 \) \( (i \in I) \), \( DMU_p \) remains extreme-efficient. (See [8]).

**Lemma 4.3.** If model (6) is feasible and \( \theta^*_{R-I(p)} > 1 \) then \( \theta^*_{R-I(p)} > 1 \).

**Proof.** Suppose \( \theta^*_{R-I(p)} \leq 1 \). Then the input constraints of (6) turn into

\[
\sum_{j=1}^{n} \lambda_j \left( \frac{x_{ij}}{y_{ir}} \right) \leq \theta^*_{R-I(p)} \leq 1, \quad i \in I, \quad r = 1, \ldots, s,
\]

\[
\sum_{j=1}^{n} \lambda_j \left( \frac{x_{ij}}{y_{ir}} \right) \leq 1, \quad i \notin I, \quad r = 1, \ldots, s,
\]

which indicates that \((\lambda^*, \theta^*_{R-I(p)}^{super}) = (\lambda^*, 1)\) is an optimal solution to
(5), so \((\lambda, \theta^*_{R-I(p)}) = (\lambda^*, \theta^*_{R-I(p)}^{super})\) is a feasible solution to (6). Therefore, \(\theta^*_{R-I(p)}^{super} = 1\), which is a contradiction. □

Theorem 4.4. Suppose \(\theta^*_{R-I(p)}^{super} > 1\) then if \(1 \leq \delta_i(p) \delta_i(p) < \theta^*_{R-I(p)}\) for \((i \in I)\), then DMU\(_p\) remains extreme-efficient. Furthermore, if equality holds for \(\delta_i(p) \delta_i(p) = \theta^*_{R-I(p)}\), that is, \(1 \leq \delta_i(p) \delta_i(p) = \theta^*_{R-I}\), then DMU\(_p\) remains on the frontier where \(\theta^*_{R-I(p)}\) is the optimal value to (6).

Theorem 4.5. Suppose \(\theta^*_{R-I(p)}^{super} > 1\) then If \(\delta_i \delta_i > \theta^*_{R-I(p)}\) for \((i \in I)\), then DMU\(_p\) will not be extreme-efficient, where \(\theta^*_{R-I(p)}\) is the optimal value to (6).

5. An Application and Discussion

In this section, we consider the data of 23 branches of Zone 1 of the Islamic Azad University, with the inputs: number of scholarship receivers (I1), number of staff (I2), number of faculty members (I3), and number of students (I4), and the outputs: income (O1) and score of the branch (O2), as follows. Then, we discuss the results of the proposed models. In this section, by considering the result in Table 3, we compare the super-efficiency scores obtained by the input-oriented CCR and DEA-R models. One can see that the super-efficiency scores by the input-oriented DEA-R model are greater than or equal to those by the input-oriented CCR model, which has also been pointed out in [5]. Consider columns 2 and 3 of Table 3. DMUs 1, 3, 5, 7, 12, 16, and 23 are efficient and the scores obtained by the super-efficiency DEA-R model for these DMUs are not less than those obtained by the super-efficiency CCR model. So,

\[E = \{DMU_1, DMU_3, DMU_5, DMU_7, DMU_{12}, DMU_{16}, DMU_{23}\}.

With regard to the theorems presented in the paper, we use the super-efficiency scores to classify the efficient units. By Theorem 4.1, DMU\(_{23}\) is extreme efficient and the ranges of perturbation in the first input of this DMU for the CCR and DER-R super-efficiency models are 1.0951 and 1.1455, respectively. Both the input-oriented CCR and DEA-R
super-efficiency models are infeasible for the second input, i.e., the second input of $DMU_7$ can increase and those of other DMUs decrease by any amount while the efficiency classification of the DMU is preserved. On the other hand, for the second input have $\theta^*_I-I(1) = 1.6000$ and $\theta^*_R-I(1) = 1.9614$. That is, for CCR-I super-efficiency model for the first input we have $0 \leq \delta_1 \delta_1 \leq 1.6000$ and for the DEA-R super-efficiency model for the second input we have $0 \leq \delta_1 \delta_1 \leq 1.9614$ as the range of perturbations. The DEA-R model yields a broader range for perturbations in the second input.

6. Conclusion

In this paper, we dealt with super-efficiency and sensitivity analysis in DEA, using DEA-R models. As the DEA-R efficiency score is greater than or equal to the DEA efficiency score, it is greater than or equal to the DEA super-efficiency score, as well. Furthermore, since the variation interval of the DMUs depends on the super-efficiency score, the interval expands as the super-efficiency score increases, which is a very important point. Calculation of super-efficiency scores by using the super-efficiency SBM model in DEA-R is suggested for future studies.

Acknowledgment: This paper is based on the work done in a research project supported by the Science and Research Branch, Islamic Azad University, Fars. The authors would like to thank the anonymous reviewers for the useful comments on the pervision version of this paper.
Table 2. Inputs and Outputs.

<table>
<thead>
<tr>
<th>DMU</th>
<th>I1</th>
<th>I2</th>
<th>I3</th>
<th>I4</th>
<th>O1</th>
<th>O2</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>8</td>
<td>32</td>
<td>51</td>
<td>2569</td>
<td>1984200</td>
<td>415.39</td>
</tr>
<tr>
<td>02</td>
<td>17</td>
<td>128</td>
<td>134</td>
<td>4687</td>
<td>3543100</td>
<td>545.95</td>
</tr>
<tr>
<td>03</td>
<td>108</td>
<td>85</td>
<td>3200</td>
<td>2165000</td>
<td>596.88</td>
<td></td>
</tr>
<tr>
<td>04</td>
<td>70</td>
<td>29</td>
<td>2170</td>
<td>1671000</td>
<td>193.18</td>
<td></td>
</tr>
<tr>
<td>05</td>
<td>1</td>
<td>31</td>
<td>22</td>
<td>2540</td>
<td>1994000</td>
<td>161.66</td>
</tr>
<tr>
<td>06</td>
<td>157</td>
<td>105</td>
<td>4360</td>
<td>3301500</td>
<td>522</td>
<td></td>
</tr>
<tr>
<td>07</td>
<td>2</td>
<td>68</td>
<td>61</td>
<td>2918</td>
<td>2238400</td>
<td>339.29</td>
</tr>
<tr>
<td>08</td>
<td>5</td>
<td>28</td>
<td>15</td>
<td>1465</td>
<td>1140500</td>
<td>106.68</td>
</tr>
<tr>
<td>09</td>
<td>2</td>
<td>44</td>
<td>28</td>
<td>3200</td>
<td>2509000</td>
<td>200.46</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>53</td>
<td>36</td>
<td>2550</td>
<td>1974500</td>
<td>219.25</td>
</tr>
<tr>
<td>11</td>
<td>47</td>
<td>69</td>
<td>3650</td>
<td>2820500</td>
<td>197.37</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>80</td>
<td>24</td>
<td>3000</td>
<td>2335500</td>
<td>100</td>
</tr>
<tr>
<td>13</td>
<td>57</td>
<td>336</td>
<td>274</td>
<td>20561</td>
<td>15978300</td>
<td>1826.40</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>21</td>
<td>25</td>
<td>1372</td>
<td>1061100</td>
<td>117.05</td>
</tr>
<tr>
<td>15</td>
<td>64</td>
<td>111</td>
<td>142</td>
<td>8500</td>
<td>6570500</td>
<td>1000</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>94</td>
<td>75</td>
<td>4227</td>
<td>3255600</td>
<td>743.4</td>
</tr>
<tr>
<td>17</td>
<td>28</td>
<td>142</td>
<td>131</td>
<td>5947</td>
<td>4541600</td>
<td>907.84</td>
</tr>
<tr>
<td>18</td>
<td>23</td>
<td>17900</td>
<td>174</td>
<td>8300</td>
<td>6365000</td>
<td>1347.96</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
<td>112</td>
<td>106</td>
<td>4093</td>
<td>3109900</td>
<td>478.46</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>42</td>
<td>46</td>
<td>2235</td>
<td>1718500</td>
<td>224.37</td>
</tr>
<tr>
<td>21</td>
<td>46</td>
<td>206</td>
<td>210</td>
<td>13842</td>
<td>10737600</td>
<td>1598.12</td>
</tr>
<tr>
<td>22</td>
<td>3</td>
<td>47</td>
<td>25</td>
<td>2134</td>
<td>1657200</td>
<td>174.20</td>
</tr>
<tr>
<td>23</td>
<td>8</td>
<td>74</td>
<td>58</td>
<td>4256</td>
<td>3305800</td>
<td>508.51</td>
</tr>
</tbody>
</table>

Table 3: Results of Models (1), (2), (5), and (6).

<table>
<thead>
<tr>
<th>DMU</th>
<th>$\theta_{1}^{*}$</th>
<th>$\theta_{2}^{*}$</th>
<th>$\theta_{3}^{*}$</th>
<th>$\theta_{1(1)}^{*}$</th>
<th>$\theta_{1(2)}^{*}$</th>
<th>$\theta_{2(2)}^{*}$</th>
<th>$\theta_{3(3)}^{*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>1.5515</td>
<td>Inf</td>
<td>Inf</td>
<td>1.6232</td>
<td>1.6496</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>02</td>
<td>0.9705</td>
<td>0.9718</td>
<td>0.1901</td>
<td>0.1901</td>
<td>0.4395</td>
<td>0.4427</td>
<td>0.4175</td>
</tr>
<tr>
<td>03</td>
<td>1.0606</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>04</td>
<td>0.9815</td>
<td>0.9970</td>
<td>0.6469</td>
<td>0.9838</td>
<td>0.4116</td>
<td>0.5118</td>
<td>0.7653</td>
</tr>
<tr>
<td>05</td>
<td>1.4584</td>
<td>1.5213</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>06</td>
<td>0.9733</td>
<td>0.9781</td>
<td>0.5488</td>
<td>0.5648</td>
<td>0.4364</td>
<td>0.5027</td>
<td>0.5188</td>
</tr>
<tr>
<td>07</td>
<td>1.3602</td>
<td>1.3738</td>
<td>1.6000</td>
<td>1.9614</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
</tr>
<tr>
<td>08</td>
<td>0.9930</td>
<td>0.9942</td>
<td>0.1387</td>
<td>0.1995</td>
<td>0.6355</td>
<td>0.6736</td>
<td>0.9006</td>
</tr>
<tr>
<td>09</td>
<td>0.9988</td>
<td>0.9988</td>
<td>0.6263</td>
<td>0.6275</td>
<td>0.8865</td>
<td>0.8865</td>
<td>0.9866</td>
</tr>
<tr>
<td>10</td>
<td>0.9895</td>
<td>0.9915</td>
<td>0.2199</td>
<td>0.3156</td>
<td>0.5842</td>
<td>0.5885</td>
<td>0.7122</td>
</tr>
<tr>
<td>11</td>
<td>0.9843</td>
<td>0.9843</td>
<td>0.1002</td>
<td>0.1002</td>
<td>0.9330</td>
<td>0.9330</td>
<td>0.4492</td>
</tr>
<tr>
<td>12</td>
<td>1.1713</td>
<td>1.1713</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.9934</td>
<td>0.9954</td>
<td>0.2196</td>
<td>0.3483</td>
<td>0.7465</td>
<td>0.7854</td>
<td>0.7696</td>
</tr>
<tr>
<td>14</td>
<td>0.9882</td>
<td>0.9902</td>
<td>0.4172</td>
<td>0.6643</td>
<td>0.7922</td>
<td>0.8218</td>
<td>0.5491</td>
</tr>
<tr>
<td>15</td>
<td>0.9933</td>
<td>0.9964</td>
<td>0.2350</td>
<td>0.3071</td>
<td>0.9393</td>
<td>0.9755</td>
<td>0.8503</td>
</tr>
<tr>
<td>16</td>
<td>1.1968</td>
<td>1.1968</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.9865</td>
<td>0.9870</td>
<td>0.3021</td>
<td>0.3233</td>
<td>0.5144</td>
<td>0.5152</td>
<td>0.6824</td>
</tr>
<tr>
<td>18</td>
<td>0.9927</td>
<td>0.9932</td>
<td>0.6057</td>
<td>0.6390</td>
<td>0.6780</td>
<td>0.6833</td>
<td>0.7714</td>
</tr>
<tr>
<td>19</td>
<td>0.9765</td>
<td>0.9829</td>
<td>0.5605</td>
<td>0.5605</td>
<td>0.5866</td>
<td>0.6905</td>
<td>0.4830</td>
</tr>
<tr>
<td>20</td>
<td>0.9846</td>
<td>0.9866</td>
<td>0.3004</td>
<td>0.4200</td>
<td>0.6453</td>
<td>0.6506</td>
<td>0.5326</td>
</tr>
<tr>
<td>21</td>
<td>0.9956</td>
<td>0.9981</td>
<td>0.4235</td>
<td>0.5764</td>
<td>0.8263</td>
<td>0.9468</td>
<td>0.8291</td>
</tr>
<tr>
<td>22</td>
<td>0.9918</td>
<td>0.9936</td>
<td>0.3924</td>
<td>0.6164</td>
<td>0.5520</td>
<td>0.5979</td>
<td>0.8352</td>
</tr>
<tr>
<td>23</td>
<td>1.0951</td>
<td>1.1455</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
<td>Inf</td>
</tr>
</tbody>
</table>
References


Mohammad Reza Mozaffari
Department of Mathematics
Assistant Professor of Mathematics
Science and Research Branch
Islamic Azad University
Fars, Iran.
E-mail: Mozaffari23@yahoo.com

Javad Gerami
Department of Mathematics
Assistant Professor of Mathematics
Science and Research Branch
Islamic Azad University
Fars, Iran
E-mail: Javadgerami2002@yahoo.com