Prediction of Longitudinal Dispersion Coefficient in Natural Channels Using Soft Computing Techniques

S. Adarsh¹

Abstract. Accurate estimate of longitudinal dispersion coefficient is essential in many hydraulic and environmental problems such as intake designs, modeling flow in estuaries and risk assessment of injection of hazardous pollutants into river flows. Recent research works show that in the absence of knowledge about explicit relationships concerning longitudinal dispersion coefficient and its influencing parameters, data driven techniques can be used to predict it with reasonable degree of accuracy. In this paper, the usefulness of Support Vector Machines (SVM) and Genetic Programming (GP) are examined for predicting longitudinal dispersion coefficient in natural channels. The hydraulic variables such as flow depth (H), flow velocity (U) and shear velocity (u_s) along with the width of channel (B) are used as input variables to predict longitudinal dispersion coefficient (K_x). The performance evaluation based on multiple error criteria confirm that GP shows remarkably good performance in capturing non-linear relationship between the predictors and predictant in the estimation of longitudinal dispersion coefficient when compared with empirical approaches, the traditional Artificial Neural Networks (ANN) and SVM. Hence, GP can be used as an efficient computational paradigm in the prediction of longitudinal dispersion coefficient in natural channels.

Keywords: Longitudinal dispersion coefficient; Natural channels; Artificial neural networks; Support vector machines; Genetic programming.

INTRODUCTION

Disposal of effluent from industrial factories or accidental disposal of contaminants into natural channels like streams and rivers will deteriorate the quality of water due to lack of proper mixing. Pollution of natural water bodies has received wide attention among the researchers recently, as proper water quality management is essential for public health and for preserving natural water bodies. When the effluents or contaminants are discharged into a river, it undergoes stages of mixing during the transportation to downstream by the river flow. The effluent may get dispersed longitudinally, transversely and vertically by advective and dispersive process. Ability of river and other open channel flows in dispersing additive materials in longitudinal and transverse and vertical directions is described by the dispersion coefficients \( K_x \), \( K_y \) and \( K_z \), respectively. Once the cross sectional mixing is complete, the process of longitudinal dispersion becomes the most important mechanism [1]. Thus, to know the fate of contaminant transport, the precise estimation of longitudinal dispersion coefficient is required [2,3].

Accurate estimate of longitudinal dispersion coefficient is required in many practical problems such as intake designs, modeling flow in estuaries and risk assessment of injection of hazardous pollutants into river flows [4,5]. Taylor introduced longitudinal dispersion coefficient as a measure of 1D dispersion in laminar and turbulent pipe flows [6,7]. Elder extended the dispersion in pipes to the mixing in an infinitely wide channel and concluded that vertical velocity gradient is the main governing mechanism behind mixing [8]. Fischer attributed that the velocity heterogeneity is the underlying mechanism behind mixing [9]. He proposed an integral expression for determining the longitudinal dispersion coefficient [9].

Later on, a number of investigators have proposed empirical equations based on experimental and field
data for predicting longitudinal dispersion coefficient, some among them are mentioned in [1]. However such equations are valid only in their calibrated and tested range of flow. Tayfur and Singh [10], Torpak and Cigizolu [11] used Artificial Neural Network (ANN) as a tool for estimating longitudinal dispersion coefficient. Torpak and Savić [12] applied fuzzy logic for prediction of longitudinal dispersion coefficient in natural channels. Recently, Madvar et al. [13] proposed an Adaptive Neuro-Fuzzy Inference System (ANFIS) for the prediction of longitudinal dispersion coefficient. Tayfur [14] applied Genetic Algorithm (GA) to predict longitudinal dispersion coefficient in natural channels in an optimization framework. ANN has some inherent drawbacks such as slow convergence, less generalizing performance, arriving at local minimum and overfitting problems [15]. After a detailed investigation of different empirical approaches, Madaver et al. [13] reported that these methods show varying degrees of accuracy in the estimation of longitudinal dispersion coefficient in natural channels. This opens the scope for the search for an efficient computational intelligence paradigm for accurate estimation of longitudinal dispersion coefficient in natural channels. In this study, Support Vector Machines (SVM) and Genetic Programming (GP) are applied as alternative data driven techniques (soft computing techniques) for estimating longitudinal dispersion coefficient in natural channels.

THEORETICAL BACKGROUND

The one-dimensional (1-D) Fickian type dispersion equation which is derived by Taylor [6] has been widely used to obtain reasonable estimates of the rate of longitudinal dispersion. The 1-D dispersion equation is:

$$\frac{\partial C_x}{\partial t} + u \frac{\partial C_x}{\partial x} = K_x \left( \frac{\partial^2 C_x}{\partial x^2} \right),$$

(1)

where $C_x$ is the concentration average in section, $u$ is longitudinal average velocity, $t$ is time, $x$ is longitudinal direction in flow stream. Fischer [9] proposed a triple integral expression for estimation of longitudinal dispersion coefficient in the following form:

$$K_x = \frac{1}{A} \int_0^B hu' \int_0^y \frac{1}{\varepsilon_r} \int_0^y hu' dy' dy,$$

(2)

where $A$ is the cross section area of flow, $B$ is the top width of water surface, $h$ is the local depth of flow in any transverse point, $u'$ is deviation of depth averaged flow velocity from the cross sectional mean velocity, $y$ is the transverse location from left bank and $\varepsilon_r$ is transverse mixing coefficient. It is noted that Equation 2 is a basis for several empirical equations proposed for the determination of $K_x$. Fischer et al. [3] suggested the following equation for determination of $\varepsilon_r$:

$$\varepsilon_r = 0.15 Hu_s,$$

(3)

where $H$ is the average depth of flow of the cross section, $u_s$ is the shear velocity and is given by $u_s = \sqrt{gHS}$ in which $S$ is the slope of total energy line. Fischer et al. [9] developed the following equation to predict $K_x$, which is a simplified form of Equation 2:

$$K_x = 0.011 \frac{U^2 B^2}{Hu_s},$$

(4)

where $U$ is cross sectional average flow velocity.

Seo and Cheong [11] used dimensional analysis and a non-linear multiple regression method to derive an expression for $K_x$. Deng et al. [16] followed a more theoretical approximation of Equation 2. They developed mathematical expressions for the lateral distribution over the flow depth, deviation of local velocity from mean velocity and transverse mixing coefficient and substituted in Equation 2. By using 81 sets of measured data from 30 rivers in United States, Kashefpour and Falconer [17] developed two models for predicting longitudinal dispersion coefficient in rivers. These equations were developed by considering the hydraulic and geometric parameters by using dimensional and regression analyses.

SUPPORT VECTOR MACHINES

Support Vector Machine (SVM) is a relatively recent addition to the family of soft computing techniques evolved from the concept of statistical learning theory explored by Boser et al. [18]. SVM performs the regression by using a set of non-linear functions that are defined in a high dimensional space. SVM has been used to solve non-linear regression problems by risk minimization where the risk is measured using Vapnik’s accuracy intensive loss function ($\varepsilon$) [19]. SVM uses a risk function consisting of the empirical error and a regularization term which is derived from the Structural Risk Minimization (SRM) principle. More details on SRM can be found in Cortes and Vapnik [20]. Considering a set of input-output pairs $\{x_i, y_i\}$ $(x_2, y_2) \cdots (x_i, y_i)$ $\chi \in R^N$, $y \in r$ as training dataset, where $x$ is the input, $y$ is the output, $R^N$ is the $N$-dimensional vector space and $r$ is the one dimensional vector space. In this problem $x = [B, H, U, u_s]$ and $y = [K_x]$.

The intension of SVM is to fit a function that can approximately predict the value of output on supplying a new set of predictors (input variables).

The $\varepsilon$-intensive loss function can be described as follows:

$L_\varepsilon(y) = 0,$ for $|f(x) - y| \leq \varepsilon,$

(5)
otherwise:
\[ L_\varepsilon(y) = |f(x) - y| - \varepsilon. \]  
(6)

This defines an \( \varepsilon \)-tube so that if the predicted values
is within the tube, the loss is zero, otherwise the loss is equal to the absolute value of the deviation minus \( \varepsilon \).

This concept is depicted in Figure 1.

SVM attempts to find a function \( f(x) \) which tries
to fit a given dataset keeping the deviation from the
actual output '\( e \)' as flat as possible.

Consider a linear function of the following form:
\[ f(x) = \langle w, x \rangle + b, \quad w \in \mathbb{R}^N, \quad b \in \mathbb{R}, \]  
(7)

where \( w \) is an adjustable weight vector, and \( b \) is the
scalar threshold. Fitness means the search for a small
value of '\( w \)'. It can be represented as a minimization
problem with an objective function comprising the
Euclidian norm as follows:

Minimize:
\[ \frac{1}{2} ||w||^2, \]  
(8)

subject to:
\[ y_i - \langle w, x_i \rangle - b \leq \varepsilon, \quad i = 1, 2, 3 \ldots l, \]  
(9)
\[ \langle w, x_i \rangle + b - y_i \leq \varepsilon, \quad i = 1, 2, 3 \ldots l. \]  
(10)

Some allowance for errors (\( \varepsilon \)) may also be introduced.
Two slack parameters \( \xi \) and \( \xi^* \) have been introduced to
penalize the samples with error more than '\( \varepsilon \)'. Thus the
infeasible constraints of the optimization problem are
eliminated. The modified formulation takes the form:

Minimize:
\[ \frac{1}{2} ||w||^2 + C \sum_{i=1}^{l} (\xi_i + \xi^*_i), \]  
(11)

Subject to:
\[ y_i - \langle w, x_i \rangle - b \leq \varepsilon + \xi_i, \quad i = 1, 2, 3 \ldots l, \]  
(12)
\[ \langle w, x_i \rangle + b - y_i \leq \varepsilon + \xi^*_i, \quad i = 1, 2, 3 \ldots l, \]  
(13)
\[ \xi_i \geq 0, \quad \xi^*_i \geq 0, \quad i = 1, 2, 3 \ldots l. \]  
(14)

The constant \( 0 < C < \infty \) determines the trade-off
between the flatness of \( f(x) \) and the amount upto which
the deviations larger than '\( \varepsilon \)' are tolerated [21]. The
above optimization problem is solved by Vapnik [22]
using Lagrange Multiplier method. The solution is
given by:
\[ f(x) = \sum_{i=1}^{M} (\alpha_i - \alpha_i^*) (x_i, x) + b, \]  
where:
\[ b = -\left( \frac{1}{2} \right) w_r(x_r + x_s), \]  
(15)

where \( x_s \) and \( x_r \) are known as support vectors and \( M \)
is the number of support vectors.

Some Lagrange multipliers (\( \alpha_i, \alpha^*_i \)) will be zero,
which implies that these training solutions are irrelevant
to the final solution (known as sparseness of the
solution). The training objects with non zero Lagrange
multipliers are called support vectors. When linear
regression is not appropriate, input data have to be
mapped into a high dimensional feature space through
non linear mapping and the linear regression needs to be
performed in the high dimensional feature space [18].
The mapping of input data onto the feature space
can be done by \( \Phi \). The dot product between \( \Phi(x_i) \)
and \( \Phi(x_j) \) is computed as a linear combination of
the training points. The concept of non-linear mapping is
depicted by Figure 2.

The functions which satisfies Mercer’s theorem
can be used for fitting the data reported in Boser et al. [18].
Polynomial functions, Radial Basis Function (RBF) and splines are the most commonly used
functions for data fitting using SVM. Recently, SVM
is successfully applied to many problems in water
resources and environmental problems [23-25].
GENETIC PROGRAMMING

The evolutionary computational techniques may be the better alternatives for solving regression problems as they follow an optimization strategy with progressive improvement towards the global optima. They start with possible trial solutions within a decision space and the search is guided by genetic operators and the principle of ‘survival of the fittest’ [26]. Genetic Algorithm (GA) is one of the most popular and powerful evolutionary optimization techniques [26,27], but it cannot be used to evolve complex models such as equations. This limitation is overcome by Genetic Programming (GP) introduced by Koza [28]. GP is an automatic programming technique for evolving computer programs to solve, or approximately solve, problems [28]. GP, which is basically an optimization paradigm, can also be effectively applied to the Genetic Symbolic Regression (GSR). GSR involves finding a mathematical expression in symbolic form relating finite values of set of independent variables \(x_i\) and a set of dependent variables \(y_i\) [29]. GP is a member of the Evolutionary Algorithm (EA) family, and works on Darwin’s natural selection theory in evolution. Here, a population is progressively improved by selectively discarding the not-so-fit population and breeding new children to form better populations. Like other evolutionary algorithms, the solution is started with a random population of individuals (equations or computer programs). Each possible solution set can be visualized as a ‘parse tree’ comprising the terminal set (input variables) and functions (generally operators such as +, -, *, /, logarithmic or trigonometric). The ‘fitness’ is a measure of how closely a trial solution solve the problem. The objective function – the minimization of error between estimated and observed value – is the fitness function. The solution set in a population associated with the “best fit” individuals will be reproduced more often than the less fit solution sets. It iteratively transforms a population of computer programs into a new generation of programs by applying analogs to naturally occurring genetic operators like reproduction, mutation and crossover. The different genetic operations can be found in detail in [28]. The basic procedure of GP is presented as a flow chart in Figure 3.

In the recent years, GP is effectively applied to solve a wide range of water resources problems [29-33]. GP can evolve an explicit equation or equivalent computer program relating the input and output variables which is a more understandable depiction of the cause-effect process. A program-based approach is adopted for the present study.

DATASET USED FOR MODELING

The development of mathematical model to predict longitudinal dispersion coefficient and its validation

```
Start

Generate an initial population comprising of computer programs

Evaluate each program (symbolic expressions) and assign it fitness according to how best it solves the problem

From temporary population, select the programs according to their fitness

Create new population by performing genetic operators such as reproduction, crossover and mutation

Evaluate the performance of new population

Is termination criteria satisfied?

No

Yes

Report the best computer program appeared so far as the result
```

Figure 3. Flow chart for Genetic Programming.
is accomplished by employing the data presented by Tayfur and Singh [16]. Table 1 summarizes the statistical information on the dataset. The hydraulic variables flow depth ($H$), flow velocity ($U$) and shear velocity ($u_s$) along with channel width ($B$) are used as input, whereas the longitudinal dispersion coefficient ($K_x$) is the target in this study.

During training, 51 sets of the whole data were used and the remaining was used for validation. The splitting of the dataset is made in such a way that the same 51 data points used by Tayfur and Singh [16] is used for training and remaining 20 for the validation to enable a comparison with the results reported in past based on ANN.

## THE MODEL DEVELOPMENT AND THE RESULTS

Initially four empirical models proposed by Fischer et al. [9], Seo and Cheong [1]; Deng et al. [16], and Kashefpour and Falconer [17] were applied to the complete dataset. The statistical performance evaluation measures like correlation coefficient ($R$), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE) are computed. The values of performance evaluation measures for validation dataset for the four empirical models and ANN reported in the past are presented in Table 2. These results indicate that ANN method is found to be a successful tool for the prediction of longitudinal dispersion coefficient in natural channels.

In this study an $\varepsilon$-variant of SVM ($\varepsilon$-SVM) is used for support vector regression and the loss function is fixed as 0.001. The data mining software WEKA proposed by Witten and Frank [34] is used for developing SVM model. Initially, a polynomial kernel of degree 2 is used to fit a non-linear model. A trial and error approach is followed to find the optimal value of kernel specific parameter C. The C parameter of 100 is found to be quite successful in giving satisfactory performance for validation dataset. The values of different performance evaluation measures for this model (for both training and validation dataset) are presented in Table 3. Table 3 shows that the RMSE value obtained for SVM with polynomial kernel for the validation dataset was better than those obtained by empirical approaches presented in Table 2. But the calculated $R$ and RMSE values are not as good as those calculated for the results of ANN model. Then a Radial Basis Function (RBF) kernel is used to fit a non-linear SVM

### Table 1. Statistical properties of the dataset.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>$U_{\text{min}} - U_{\text{max}}$ (m/sec)</th>
<th>$u_s_{\text{min}} - u_s_{\text{max}}$ (m/sec)</th>
<th>$B_{\text{min}} - B_{\text{max}}$ (m)</th>
<th>$H_{\text{min}} - H_{\text{max}}$ (m)</th>
<th>$K_x_{\text{min}} - K_x_{\text{max}}$ (m$^2$/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole</td>
<td>0.034-1.74</td>
<td>0.0024-0.553</td>
<td>11.9-711.2</td>
<td>0.22-19.94</td>
<td>1.9-892</td>
</tr>
<tr>
<td>Training</td>
<td>0.034-1.74</td>
<td>0.0024-0.268</td>
<td>12.2-253.6</td>
<td>0.22-3.96</td>
<td>1.9-837</td>
</tr>
<tr>
<td>Testing</td>
<td>0.130-1.53</td>
<td>0.0024-0.553</td>
<td>11.9-711.2</td>
<td>0.40-19.94</td>
<td>2.9-892</td>
</tr>
</tbody>
</table>

### Table 2. Relative performance of empirical approaches and ANN.

<table>
<thead>
<tr>
<th>Method</th>
<th>Equation</th>
<th>$R$</th>
<th>RMSE</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fischer [9]</td>
<td>$K_x = 0.011 \frac{U^2 B^2}{\pi u_s^2}$</td>
<td>0.625</td>
<td>297.21</td>
<td>1288.83</td>
</tr>
<tr>
<td>Seo and Cheong [1]</td>
<td>$K_x = 5.92 \left( \frac{U}{\varepsilon} \right)^{1.43} \left( \frac{B}{\pi} \right)^{0.62} H u_s$</td>
<td>0.684</td>
<td>818.59</td>
<td>367.69</td>
</tr>
<tr>
<td>Deng et al. [16]</td>
<td>$K_x = 0.13 \frac{U}{\varepsilon} \left( \frac{B}{\pi} \right)^{1.67} H u_s$ where $\varepsilon_t = 0.145 + \frac{1}{\Delta t}$</td>
<td>0.663</td>
<td>621.29</td>
<td>262.33</td>
</tr>
<tr>
<td>Kashefpour and Falconer [17]</td>
<td>$K_x = 7.248 + 1.775 \left( \frac{B}{\pi} \right)^{0.63} \left( \frac{U}{\varepsilon} \right)^{1.572} H U$</td>
<td>0.764</td>
<td>525.79</td>
<td>3214.68</td>
</tr>
<tr>
<td>ANN [10]</td>
<td>—</td>
<td>0.831</td>
<td>192.91</td>
<td>1851.00</td>
</tr>
</tbody>
</table>
model for the dataset. The combination of control parameters such as \( C = 100 \) and \( \sigma = 3.5 \) gives very good training performance. The different performance evaluation measures for RBF Kernel-based SVM is also (For both training and validation stages) summarized in Table 3.

DISCIPULUS software proposed by Francisco [35] is used for performing the GP-based modeling. The initial control parameters used for the problem are population size (500), crossover probability (0.95) and mutation probability (0.5). The basic arithmetical functions (such as addition, multiplication, subtraction and division (+, *, -, /) constitute the function set. The fitness function is selected as the root mean square error between the measure and predicted values of longitudinal dispersion coefficient. Based on the predicted values, three performance criteria namely \( R \), RMSE and MAE are calculated and presented in Table 3. Table 3 shows that the GP-based model is better than both the ANN and SVM models. From Table 3, it can be seen that the performance evaluation measures of SVM (RBF Kernel) is better than those of GP. But the RBF Kernel shows inferior performance for the validation dataset. Thus it can be inferred that GP is able to capture the trend in a better way and it can be more generalized than SVM. The observed values, values predicted by ANN, RBF-based SVM and GP models for the testing dataset are presented in Table 4.

The statistical properties such as maximum value, minimum value, average value, average deviation, standard deviation, coefficient of skewness, coefficient of variation etc. are computed for observed data for validation and the values predicted, using different models, are presented in Table 5.

Table 5 shows that skewness and coefficient of variation is the highest for SVM model; also both the extreme values are deviated largely from the observed extremes. This indicates that SVM is only fairly accurate in predictions. But the GP model results are better when compared with that of SVM.

The scatter plot of \( K_a \) between observed data and the prediction of the best SVM model for only testing data group is presented in Figure 4. The 5% error bar lines are plotted along with in this scatter plot. Such a plot can be used to indicate the range of standard deviation and to determine whether the differences are statistically significant [36]. A similar plot for the best GP model for the testing dataset is presented in Figure 5. From the plots also it can be inferred that for the GP model more data points in the prediction dataset falls within the 95% confidence interval when compared with those obtained by SVM.

DISCUSSION

The different performance evaluation criteria calculated for the testing data based on ANN model results by Tayfur and Singh [10] established that ANN-based modeling is superior to the existing theoretical and em-

<p>| Table 3. Performance evaluation of SVM and GP models. |
|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|</p>
<table>
<thead>
<tr>
<th>Performance Criteria</th>
<th>SVM Polynomial Kernel ((C = 100, d = 2))</th>
<th>SVM RBF Kernel ((C = 100, \gamma = 3.5))</th>
<th>GP</th>
<th>SVM Polynomial Kernel ((C = 100, d = 2))</th>
<th>SVM RBF Kernel ((C = 100, \gamma = 3.5))</th>
<th>GP</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>0.965</td>
<td>0.997</td>
<td>0.963</td>
<td>0.678</td>
<td>0.874</td>
<td>0.915</td>
</tr>
<tr>
<td>RMSE</td>
<td>38.59</td>
<td>11.30</td>
<td>41.64</td>
<td>461.40</td>
<td>106.09</td>
<td>60.44</td>
</tr>
<tr>
<td>MAE</td>
<td>20.93</td>
<td>4.33</td>
<td>24.71</td>
<td>174.99</td>
<td>79.27</td>
<td>54.42</td>
</tr>
</tbody>
</table>

<p>| Table 4. The predicted and observed values of longitudinal dispersion by different models. |
|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|</p>
<table>
<thead>
<tr>
<th>SI No</th>
<th>Observed ( K_a )</th>
<th>ANN( a )</th>
<th>SVM( b )</th>
<th>GP( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.90</td>
<td>26.80</td>
<td>28.12</td>
<td>17.14</td>
</tr>
<tr>
<td>2</td>
<td>37.80</td>
<td>27.10</td>
<td>63.86</td>
<td>33.39</td>
</tr>
<tr>
<td>3</td>
<td>41.40</td>
<td>31.40</td>
<td>167.09</td>
<td>47.53</td>
</tr>
<tr>
<td>4</td>
<td>53.30</td>
<td>43.00</td>
<td>93.18</td>
<td>89.81</td>
</tr>
<tr>
<td>5</td>
<td>88.90</td>
<td>77.60</td>
<td>141.13</td>
<td>105.54</td>
</tr>
<tr>
<td>6</td>
<td>2.900</td>
<td>39.20</td>
<td>130.824</td>
<td>51.45</td>
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<td>7</td>
<td>44.00</td>
<td>26.50</td>
<td>30.171</td>
<td>25.98</td>
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<tr>
<td>8</td>
<td>308.9</td>
<td>34.66</td>
<td>401.642</td>
<td>320.75</td>
</tr>
<tr>
<td>9</td>
<td>12.80</td>
<td>21.90</td>
<td>35.37</td>
<td>7.050</td>
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<tr>
<td>10</td>
<td>13.90</td>
<td>45.20</td>
<td>137.65</td>
<td>51.43</td>
</tr>
<tr>
<td>11</td>
<td>65.00</td>
<td>77.20</td>
<td>28.77</td>
<td>83.68</td>
</tr>
<tr>
<td>12</td>
<td>237.2</td>
<td>838.0</td>
<td>152.42</td>
<td>583.14</td>
</tr>
<tr>
<td>13</td>
<td>457.7</td>
<td>838.0</td>
<td>154.09</td>
<td>379.58</td>
</tr>
<tr>
<td>14</td>
<td>374.1</td>
<td>838.0</td>
<td>152.42</td>
<td>580.00</td>
</tr>
<tr>
<td>15</td>
<td>41.80</td>
<td>59.70</td>
<td>132.90</td>
<td>169.74</td>
</tr>
<tr>
<td>16</td>
<td>10.70</td>
<td>26.90</td>
<td>24.92</td>
<td>24.17</td>
</tr>
<tr>
<td>17</td>
<td>36.90</td>
<td>76.60</td>
<td>66.91</td>
<td>70.28</td>
</tr>
<tr>
<td>18</td>
<td>15.50</td>
<td>25.30</td>
<td>42.32</td>
<td>14.60</td>
</tr>
<tr>
<td>19</td>
<td>30.20</td>
<td>31.70</td>
<td>93.97</td>
<td>18.49</td>
</tr>
<tr>
<td>20</td>
<td>892.0</td>
<td>763.4</td>
<td>971.16</td>
<td>1017.31</td>
</tr>
</tbody>
</table>

\( a \): Tayfur and Singh [10].
\( b \): Present study.
Table 5. Statistical properties of values predicted by different models.

<table>
<thead>
<tr>
<th>Statistical Properties</th>
<th>Observed Data</th>
<th>ANN</th>
<th>SVM</th>
<th>GP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum value</td>
<td>892</td>
<td>838</td>
<td>971.16</td>
<td>1017.31</td>
</tr>
<tr>
<td>Minimum value</td>
<td>290</td>
<td>21.9</td>
<td>24.92</td>
<td>7.05</td>
</tr>
<tr>
<td>Average value</td>
<td>130.29</td>
<td>213.01</td>
<td>152.35</td>
<td>181.45</td>
</tr>
<tr>
<td>Average deviation</td>
<td>157.34</td>
<td>255.89</td>
<td>108.37</td>
<td>191.15</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>221.71</td>
<td>319.13</td>
<td>210.73</td>
<td>261.83</td>
</tr>
<tr>
<td>Coefficient of skewness</td>
<td>2.46</td>
<td>1.48</td>
<td>3.45</td>
<td>2.14</td>
</tr>
<tr>
<td>Coefficient of variation (%)</td>
<td>62.83</td>
<td>66.74</td>
<td>72.30</td>
<td>69.30</td>
</tr>
</tbody>
</table>

![Figure 4](image-url). The 5% error bar for SVM model (RBF Kernel).

![Figure 5](image-url). The 5% error bar for GP model.

Empirical equations. However, ANN modeling involves the tedious process of optimal setting of a larger number of control parameters such as number of hidden layers, learning rate, momentum rate, number of iterations, transfer function and weight initialization. The results show that SVM modeling with RBF kernel show better prediction of longitudinal dispersion coefficient than those with ANN modeling.

The results of GP-based modeling for testing dataset show that it captures the non-linearity of the dataset quite well. It gives a correlation coefficient value of 0.945 for testing dataset and the RMSE value of 60.44 and MAE of 54.42 which is the lowest when compared with the results by empirical equations, ANN and SVM. The different performance evaluation measures show that SVM predicts the values of $K_x$ very well for the training dataset but the prediction accuracy is not as good as that of GP (Table 3).

This establishes the better generalization capability of GP when compared with SVM. Moreover, the GP-based modeling follows a progressive improvement towards the global optima (i.e., minimum error) and give the output in the form of a computer program which is quite useful for the modeler to apply for a new set of input data for predicting the longitudinal dispersion coefficient. Thus the present study establishes the potential of GP in accurate prediction of longitudinal dispersion coefficient in natural channels. The sensitivity analysis with a modified procedure establishes that the bed width of the channel as the most significant parameter which affects the longitudinal dispersion and the results are on the expected lines.

CONCLUSION

In this paper the application of two relatively recent soft computing techniques - SVM and GP are investigated for the prediction of longitudinal dispersion coefficient in natural channels. SVM predicts longitudinal dispersion coefficient quite well when compared with the empirical approaches and ANN. Also it demands the optimal selection of only a few number of control parameters when compared with ANN. Performance evaluation based on multiple error criteria show that the two error criteria (RMSE and MAE) are the least and correlation coefficient ($R$) is the highest for the GP-based modeling than any other model considered in this study. The GP-based modeling is found to be superior
in terms of quality and it gives the output in the form of computer programs which enables the user to apply for a new set of input data to predict the longitudinal dispersion coefficient. Thus GP can be recommended as a robust soft computing paradigm to predict the longitudinal dispersion coefficient in natural channels.

REFERENCES


**BIOGRAPHY**

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