Hybrid Linear and Circular Antenna Arrays

E. Yaacoub, M. Al-Husseini, A. Chehab, A. El Hajj, and K. Y, Kabalan

Abstract—The class of Hybrid Linear and Circular Antenna Arrays is presented. The properties of linear and circular antenna arrays are combined to obtain hybrid array types: concentric circular, cylindrical, and coaxial cylindrical antenna arrays. These three types are defined, the expressions of their array factors are derived, their directivities and half power beam widths (HPBW) are simulated. The obtained plots are approximated by curve fitting. Different combinations of excitation currents (e.g. uniform, Chebyshev, Bessel, …) can exist on the elements of the same antenna array.

Index Terms—Antennas, antenna arrays, circular arrays, linear arrays.

I. Introduction

In many antennas applications, it is desirable that the radiated power be concentrated in a certain direction, with as little interference as possible from other directions. In other words, the design objective is to increase the directivity, lower the side lobe level, and narrow (and sometimes steer) the main beam.

The performance of single antenna elements is poor compared to that of antenna arrays [1]. In an antenna array, the number of elements, the spacing between them, their excitation coefficients, and their relative phase are parameters that can be adjusted not only to increase the gain, but also to narrow the beam (i.e. decrease the beam width), steer the beam in a given direction, and control the side lobe level (by adjusting the excitation coefficients of the antenna array, e.g. Dolph-Chebyshev method).

The factors mentioned previously determine the array factor, which is used in the calculation of the directivity (and consequently the gain). The total field of an array can be formed by multiplying the field of a single element at a selected reference point (usually the origin) and the array factor [1]. Note that this multiplication property is only valid when all the elements of the array have the same element pattern; hence, in this paper, all elements are assumed to be identical and installed in the same orientation.

The gain is equal to the directivity multiplied by a loss coefficient depending on the antenna type. When losses are negligible, the gain is equal to the directivity.

The most common antenna arrays are the linear array and the circular array, whose array factors are given in [1].

The most common types of linear and uniformly spaced antenna arrays are Uniform and Chebyshev arrays. Uniform linear arrays have equal element excitations, whereas Chebyshev arrays have the unique property that all side lobes in their radiation pattern are of equal magnitude.

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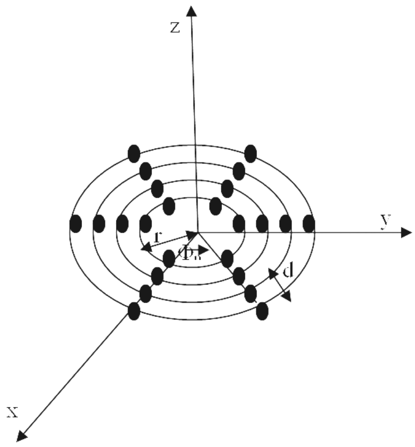

Fig. 1. Concentric circular array.

The excitation currents of the elements of a Chebyshev array are related to the coefficients of a Chebyshev polynomial [1] of degree N-1, where N is the number of the elements in the array.

In [2] and [3], another current distribution is presented where the excitation coefficients are those of a Kaiser window. It is called the Bessel current distribution and it allows fixing the maximal side lobe level.

Circular arrays are suited to provide 360 degrees of coverage in the azimuth plane [4]. Their applications include, among others, radio direction finding, air and space navigation, underground propagation, radar, and sonar [1]. The array factor of an N element circular array with uniform amplitude excitation is given in [1].

In the next sections, an attempt is made to combine the advantages of linear and circular antenna arrays. Consequently, three types of geometries involving combinations of linear and circular arrays will be studied:

- Concentric Circular Arrays, where the elements are disposed along concentric circles, as shown in Fig. 1.
- Cylindrical Arrays, where the elements are disposed along circles stacked one on top of the other to form a cylinder, as shown in Fig. 2.
- Coaxial Cylindrical Arrays, where the elements along a vertical line on the cylindrical surface form a linear array and those lying in a transversal plane cutting the cylinder constitute a concentric circular array, as shown in Fig. 3.

Particular concentric circular arrays of monopoles and dipoles are presented in [5] and [6], respectively. Concentric circular arrays with the same number of elements on each circular array are discussed in [7]. However, this work presents the originality of studying concentric circular arrays with a hybrid structure; i.e. the elements along a radial direction form a linear array.

Stacking two rings of dipoles to form a cylindrical array was discussed in [6]. The novelty in this paper is a study of

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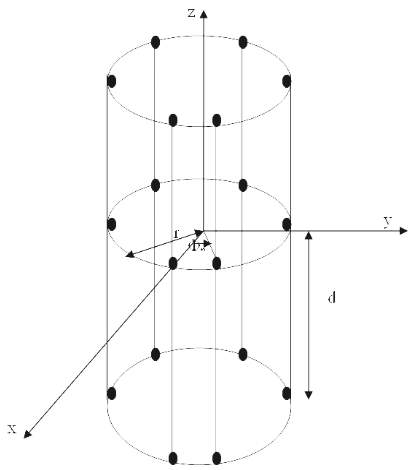

Fig. 2. Cylindrical array.

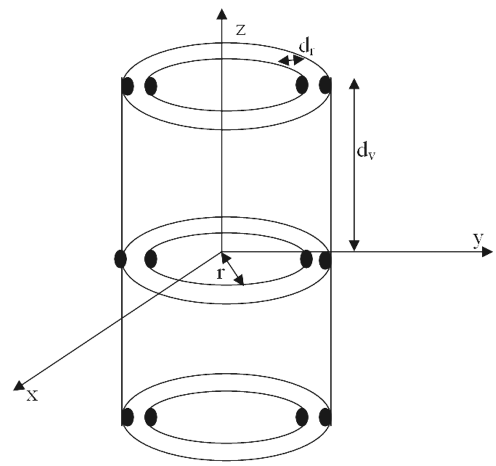

Fig. 3. Coaxial cylindrical array.

cylindrical and coaxial cylindrical arrays in the general case with a combination of various current distributions on linear and circular arrays. Furthermore, analytical approximation of simulation results for directivity and HPBW is presented, and comparisons are made between the results obtained with various current distributions.

Since this paper is a theoretical study of arrays of isotropic elements, mutual coupling between large numbers of radiating elements is not considered.

In section 2, the array factors of the three array types are derived. Section 3 studies their directivities, and section 4 is concerned with their half power beam widths. Each of the mentioned sections contains a discussion of the results obtained. Finally, conclusions are drawn in section 5.

II. Array Factors

A. Array Factor of the Concentric Circular Array

In this section, the structure of the circular grid array, i.e. an array whose elements lie on coplanar concentric circles as shown in Fig. 1, is considered. The elements along each circle form a circular array, and the elements along a radial direction constitute a linear array. Hence, the total array factor can be seen as the summation of circular or linear array factors.

Using M concentric circular arrays of N elements each, the total array factor is given by the sum of the M individual array factors

$$AF(\theta,\phi)=\sum_{m=1}^{M}\left[\sum_{n=1}^{N}c_{nm}e^{jkr_m\sin\theta\cos(\phi-\phi_n)}\right] \qquad (1)$$

In (1), the term between brackets is the array factor of one circular array in the x-y plane, c_{nm} are the excitation coefficients (magnitude and phase), N is the number of elements, and $\phi_n=2\pi(n-1)/N$ is the angle in the x-y plane between the x-axis and the nth element. Furthermore, r_m is the radius of the *m*-th circular array and it is given by

$$r_m=r+(m-1)d \qquad (2)$$

with "r" is the radius of the innermost circular array.

Substituting (2) into (1), yields

$$AF(\theta,\phi)=\sum_{m=1}^{M}\left[\sum_{n=1}^{N}c_{nm}e^{jk[r+(m-1)d]\sin\theta\cos(\phi-\phi_n)}\right] \qquad (3)$$

B. Array Factor of the Cylindrical Array

Instead of considering concentric circles, we consider the case when all the circles are of equal radius and they are placed one above the other, with an equal vertical separation between them. This geometry, shown in Fig. 2, will be a cylindrical array where the elements along a vertical line on the cylindrical surface form a linear array, and those lying in a transversal plane cutting the surface constitute a circular array. In this case, the total array factor would be the product of the array factor of a linear array aligned in z-direction by that of a circular array in the x-y plane.

Using a cylinder of M identical circular arrays, the total array factor is given by the sum of the M individual array factors

$$AF(\theta,\phi)=\sum_{m=1}^{M}\left[\sum_{n=1}^{N}c_{nm}e^{jkr\sin\theta\cos(\phi-\phi_n)}\right] \qquad (4)$$

In (4), we consider that in the far field region, the array factor of each circular array in Fig. 2 is the same as the array factor of the circular array in the x-y plane. However, the vertical distribution of the elements introduces a phase difference between the circular arrays identical to the phase difference between the elements of a linear array, since the elements along the vertical direction constitute a linear array with a distance d between the elements. Hence, in (4)

$$c_{nm}=I_n e^{j\alpha_n}\,b_m e^{j(m-1)(kd\cos\theta+\beta)} \qquad (5)$$

In (5), the term $b_m e^{j(m-1)(kd\cos\theta+\beta)}$ comes from the *m*-th vertical element and $I_n e^{j\alpha_n}$ is the excitation coefficient of the nth element of a circular array. Substituting (5) into (4), it becomes

$$\begin{aligned}AF(\theta,\phi)&=\sum_{m=1}^{M}b_m e^{j(m-1)(kd\cos\theta+\beta)}\sum_{n=1}^{N}I_n e^{j\alpha_n}e^{jkr\sin\theta\cos(\phi-\phi_n)}\\&=\sum_{m=1}^{M}b_m e^{j(m-1)(kd\cos\theta+\beta)}\sum_{n=1}^{N}I_n e^{j\{kr\sin\theta\cos(\phi-\phi_n)+\alpha_n\}} \qquad (6)\\&=AF_{linear}(\theta,\phi).AF_{circular}(\theta,\phi)\end{aligned}$$

It is evident from (6) that the array factor of a cylindrical array is equivalent to the multiplication of the array factors of a linear array by that of a circular array, similarly to the fact that the array factor of a planar array is the product of the array factors of two linear arrays.

C. Array Factor of the Coaxial Cylindrical Array

If we combine the summation and the product, the case of coaxial cylindrical arrays is obtained. The geometry, shown in Fig. 3, forms an antenna array where the elements along a vertical line on the cylindrical surface form a linear array, and those lying in a transversal plane cutting the cylinder constitute a concentric circular array. The total array factor in this case should have the form of the convolution of the array factor of a linear array with that of a circular array, since convolution is a sum of products.

The array factor of a coaxial cylindrical array is the summation of the array factors of P cylindrical arrays, each with a different radius. Using (6), the array factor of P coaxial cylinders is given by

$$\begin{aligned} AF(\theta,\varphi) &= \sum_{p=1}^{P} AF_{cyl}(\theta,\phi) \\ &= \sum_{p=1}^{P}\sum_{m=1}^{M} b_m e^{j(m-1)(kd_v \cos\theta+\beta)} \sum_{n=1}^{N} I_{pn} e^{j\alpha_{pn}} e^{jkr_p \sin\theta\cos(\phi-\phi_n)} \\ &= \sum_{m=1}^{M} b_m e^{j(m-1)(kd_v \cos\theta+\beta)} \sum_{p=1}^{P}\sum_{n=1}^{N} I_{pn} e^{j\{kr_p \sin\theta\cos(\phi-\phi_n)+\alpha_{pn}\}} \\ &= AF_{linear}(\theta,\phi).\sum_{p=1}^{P} AF_{circular}(\theta,\phi) \end{aligned} \tag{7}$$

In (7), $r_p = r + (p-1)d_r$ is the radius of the p-th cylinder, d_r is the distance between two elements in the radial direction, and d_v is the distance between two elements in the vertical direction.

III. Directivity

In this section, the directivity of hybrid linear and circular antenna arrays is investigated. As finding a closed form expression for the directivity is a very tedious task, a closed form approximation for the directivity in the case of small radii of the circular arrays will be presented, and the directivity will be determined by simulation for more practical cases.

A. Directivity Approximation for Small Radii

The array factor of an N element circular array with uniform amplitude excitation I_0 and $N >> k_r$ can be approximated as in [1] by

$$AF(\theta,\phi) \approx NI_0 J_0(k\rho_0) \tag{8}$$

In (8)

$$\begin{aligned} \rho_0 = r[&(\sin\theta\cos\phi - \sin\theta_0\cos\phi_0)^2 \\ &+ (\sin\theta\sin\phi - \sin\theta_0\sin\phi_0)^2]^{1/2} \end{aligned} \tag{9}$$

J_0 is the Bessel function of the first kind of order zero, (θ_0,ϕ_0) is the direction of the peak of the main beam, and "r" is the radius of the circle. By simple substitution, the directivity can be written as

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} J_0^2(k\rho_0)\sin\theta d\theta d\phi} \tag{10}$$

Equation (10) shows that for a uniform circular array of radius r and for $N >> k_r$, the directivity is independent of the number of elements N. Hence, increasing N beyond a minimum value will increase the complexity of the system without any increase in directivity. The integral in the denominator cannot be obtained in closed form. However, a polynomial approximation of $J_0(x)$ for $-3 \le x \le 3$ is given in [8], and can lead to a closed form solution when we consider $\theta_0 = \phi_0 = 0$.

The directivity of a cylindrical array with M element linear arrays and N element circular arrays is expressed as

$$D = \frac{4\pi[(AF_{linear}(\theta,\phi))^2(AF_{circular}(\theta,\phi))^2]_{\max}}{\int_0^{2\pi}\int_0^{\pi}[(AF_{linear}(\theta,\phi))^2(AF_{circular}(\theta,\phi))^2]\sin\theta d\theta d\phi} \tag{11}$$

The approximation in (8) is considered to be applicable to all the circular arrays that constitute the cylindrical array. Hence, substituting (8) into (11), using the expression of the array factor of a uniform linear array given in [1], and letting $\theta_0 = \phi_0 = 0$, (11) becomes

$$D = \frac{2M^2}{\int_0^{\pi}\left\{J_0^2(kr\sin\theta)\left[\frac{\sin(\frac{M}{2}kd(\cos\theta-1))}{\sin(\frac{1}{2}kd(\cos\theta-1))}\right]^2\sin\theta\right\}d\theta} \tag{12}$$

For a uniform circular array with a radius $r = 0.4683\lambda$, and $N = 50$, the directivity is 8.4977. It is the same for $N = 100$. For a uniform cylindrical array with $N = 5$, $M = 10$ and $d = 0.5\lambda$, the directivity is 12.2834. For $M = N = 10$, it is 17.0246. Hence, the problem of directivity saturation of uniform circular arrays is solved with uniform cylindrical arrays.

The integral in the denominator of (12) can be obtained in closed form for small inter element spacing when $\sin(x) \approx x$ and the approximation of $J_0(x)$ for $-3 \le x \le 3$ is used. As this approximation does not correspond to practical cases, we proceed via simulation.

B. Directivity Simulations

After fixing some parameters and varying others, the directivity is computed numerically, and a closed form expression for the directivity by curve fitting is obtained. The fixed parameters and their values are shown in Table I. Considering a uniform cylindrical array, dv, Nc, Nv, and (θ_0,φ_0) are fixed as in Table I and the radius r is varied from 0.1λ to 10λ in increments of 0.1λ. For each value of r, the array factor is derived and the directivity is computed numerically. The plot of the directivity vs. kr is shown in Fig. 4. It has the form of the response of an under damped second order system whose parameters are numerically obtained

$$D(kr) = 100[1 - \frac{\exp(-0.08156kr)}{0.89}\sin(0.1592kr + 1.097)] \tag{13}$$

The maximum approximation error between the numerical results and the formula in (13) is 14 % for $r > \lambda$. More accurate results can be obtained if polynomial curve fitting is performed. With a polynomial of degree 20, the approximation error is 3.5 % for $r > \lambda$, and the results are notably more accurate for lower values of r.

The numerical curve in Fig. 4 was approximated by the following polynomial

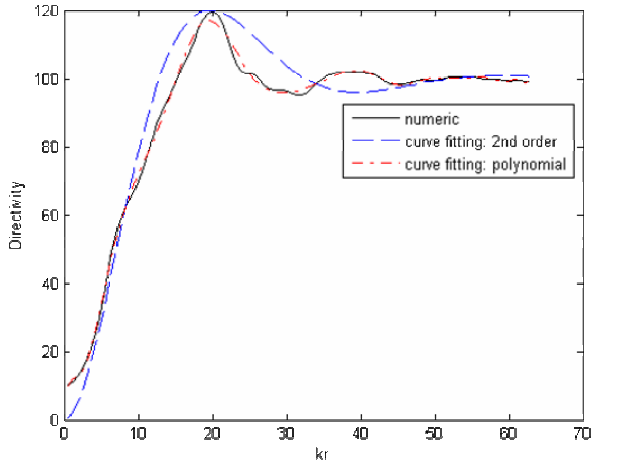

Fig. 4. Comparison of the estimated and computed directivity of a uniform cylindrical array with $Nc = Nv = 10$ and $dv = 0.5\lambda$ vs. the radius.

TABLE I
PARAMETERS OF THE HYBRID LINEAR AND CIRCULAR ANTENNA ARRAYS USED IN THE SIMULATIONS.

Parameter	Definition	Value
dv	inter element spacing in the vertical direction	$dv = 0.5\lambda$
dr	inter element spacing in the radial direction	$dr = 0.5\lambda$
Nc	number of elements on each circular array	$N_c = 10$
Nr	number of elements on each linear array in the radical direction	$N_r = 10$
Nv	number of elements on each linear array in the vertical direction	$N_v = 10$
(θ_0, φ_0)	direction of maximum radiation (in degrees)	$(\theta_0 = 0, \varphi_0 = 0)$

$$\begin{aligned} p(x) &= x^{20} + 5.5x^{19} - 15.3x^{18} - 78.1x^{17} + 99.7x^{16} + 472.1x^{15} \\ &-371.1x^{14} - 1567.6x^{13} + 878.5x^{12} + 3085.5x^{11} - 1375.4x^{10} \\ &-3594.3x^{9} + 1395.1x^{8} + 2282.7x^{7} - 816.9x^{6} - 584.6x^{5} \\ &+175.4x^{4} - 26.6x^{3} + 27.4x^{2} + 10.7x + 96.6 \end{aligned} \tag{14}$$

where $x = [(r/\lambda) - 5.05)/2.9011$.

Hence, it is concluded that to design such antenna arrays, closed form approximations can be used to obtain the optimal radius leading to the desired directivity without the need to perform extensive simulations.

The same approach is applied to concentric circular arrays. The result is shown in Fig. 5, where the directivity is maximal for $kr = \pi$ with an overshoot of 7.4%. It is found that increasing the radius doesn't affect much the directivity, which oscillates sinusoidally around a value equal to the number of elements. Hence, a convenient radius can be chosen, not too small (impractical) and not too large (too big antenna), corresponding to a maximum on the curve. Note that the maximum approximation error for $r > 0.3\lambda$ is 4%. The directivity is approximated by

$$\begin{aligned} D(kr) &= 100[1 - \frac{\exp(-0.0828kr)}{0.77}\sin(kr + 0.879)] \\ &+ 4.2\sin(\frac{2\pi}{10.5}kr + 0.278) \end{aligned} \tag{15}$$

The directivity was approximated accurately enough by (15). Therefore, polynomial curve fitting was not performed in this case.

The same method was used to find the directivity of cylindrical arrays where the distribution along each circular

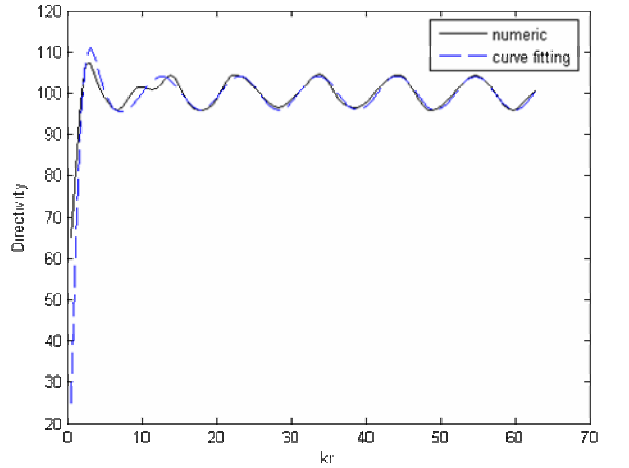

Fig. 5. Comparison of the estimated and computed directivity of a uniform concentric circular array with $Nc = Nr = 10$ and $dr = 0.5\lambda$ vs. the radius.

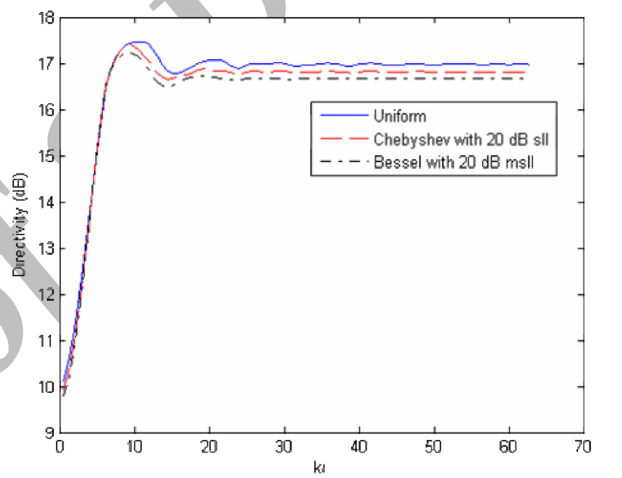

Fig. 6. Directivity of cylindrical arrays with $Nc = 5$, $Nv = 10$ and $dv = 0.5\lambda$ vs. the radius for various current distributions.

array is uniform, and the distribution along each linear array is Chebyshev (Bessel) with 20 dB side lobe level (maximal side lobe level). An example is shown in Fig. 6 for $Nc = 5$ elements, $Nv = 10$ elements, and $d = 0.5\lambda$.

Fig. 6 shows that for a given radius, the uniform cylindrical array has the largest directivity, followed by the Chebyshev then the Bessel cylindrical arrays. This is due to the fact that, for linear arrays, as seen in Fig. 7, decreasing the side lobe level broadens the main beam and hence lowers the directivity in the direction of the peak of the main beam. The side lobes for the uniform case are higher than 20 dB. Chebyshev linear arrays are characterized by a constant side lobe level. Bessel linear arrays have a controllable maximal side lobe level, but not constant; i.e. the other minor lobes are lower than the first side lobe. Figure 8 shows the array factor of the uniform circular array. The normalized array factor magnitudes of the three types of cylindrical arrays are shown in Fig. 9. These are due to the multiplication of the pattern in Fig. 8 with the corresponding pattern of Fig. 7. The directivities of the uniform, Chebyshev, and Bessel cylindrical arrays of Fig. 6 for $kr = 10$ are 55.6258 (17.45 dB), 54.6214 (17.37 dB), and 52.5876 (17.21 dB), respectively. For the same number of elements, we need a uniform linear array of 50 elements and inter element spacing of 0.5λ to reach a directivity of

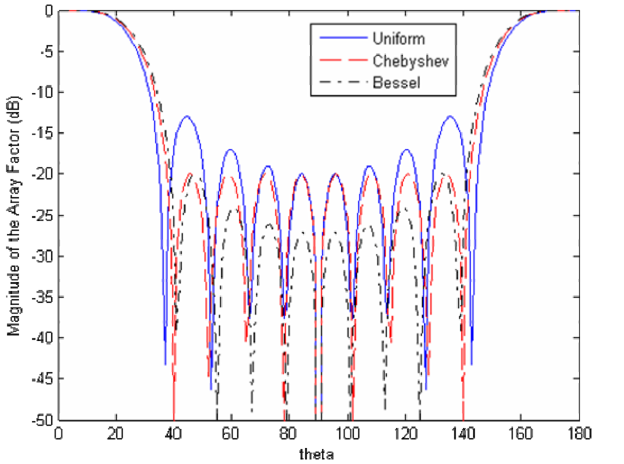

Fig. 7. Normalized Array Factors of the 3 types of Linear Arrays with $N = 10$ elements and $d = 0.5\lambda$.

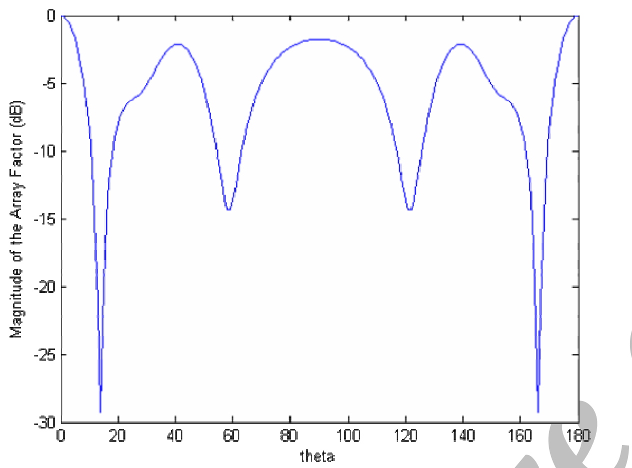

Fig. 8. Normalized Array Factor of the Circular Array with $N = 5$ elements and $kr = 10$ (plane $\varphi = 0$).

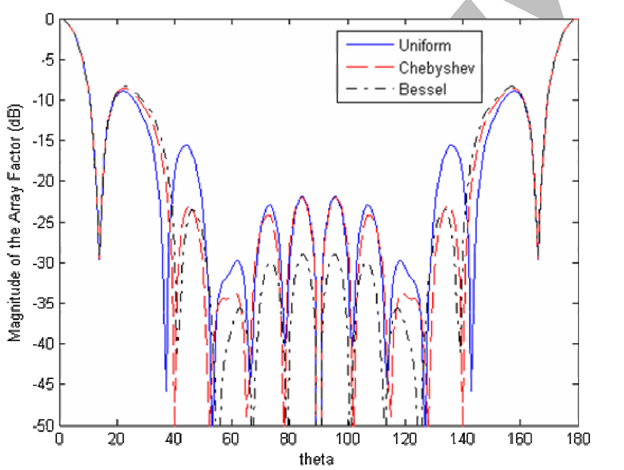

Fig. 9. Normalized Array Factors of the 3 types of Cylindrical Arrays with $Nc = 5$, $Nv = 10$, $dv = 0.5\lambda$ and $kr = 10$.

50. The length of the array will be 25λ instead of 5λ for the cylindrical arrays. A uniform circular array with 50 elements needs a radius r such that $kr = 24.14$ to reach a directivity of 50. Hence, we see that combining linear and circular antenna arrays leads to arrays with higher directivities and smaller dimensions.

Fig. 10 presents the plots of the radii leading to maximum directivity vs. the number of elements for the three types of cylindrical arrays. The radii increase linearly with the number of elements, and for a given number of

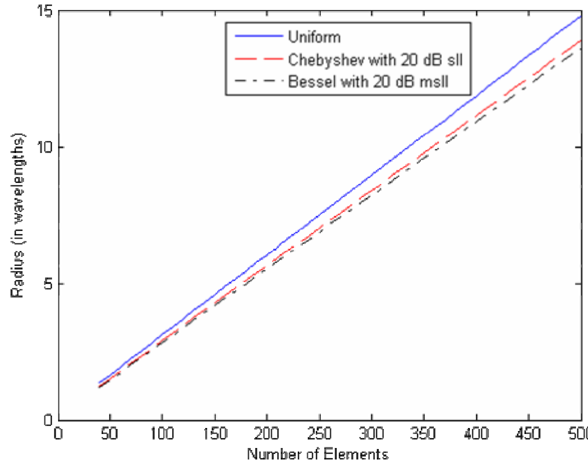

Fig. 10. Radii leading to maximum directivity vs. the number of elements for the three types of cylindrical arrays.

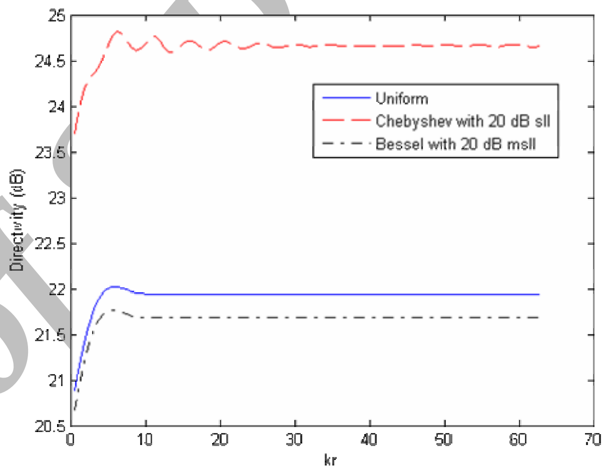

Fig. 11. Directivity in dB of coaxial cylindrical arrays with $Nc = 4$, $Nv = 10$ and $dv = dr = 0.5\lambda$ and vs. the radius for various current distributions.

elements, the Bessel cylindrical array reaches the peak directivity with a slightly smaller radius than the Chebyshev cylindrical array, which needs a smaller radius than the uniform cylindrical array.

The plots of the directivity of coaxial cylindrical arrays where the distribution along each concentric circular array is uniform, and the distribution along each linear array is uniform, Chebyshev, or Bessel are shown in Fig. 11 for $Nc = 4$ elements, $Nr = Nv = 10$ elements and $dr = dv = 0.5\lambda$. There's a 3 dB difference in directivity between the Chebyshev and Bessel coaxial cylindrical arrays, and 0.25 dB between the uniform and Bessel coaxial cylindrical arrays.

IV. Half Power Beam Width (HPBW)

Similarly to the directivity, the half power beam width of hybrid linear and circular antenna arrays can be approximated in closed form in very specific cases that don't correspond to practical situations. Therefore, the HPBW will be determined by simulation using the fixed values of Table I and a radius variation from 0.1λ to 10λ in increments of 0.1λ. For each value of r, the array factor is derived and the HPBW is computed numerically. The plots of the HPBW vs. r for uniform, Chebyshev, and Bessel current distributions are shown in Fig. 12.

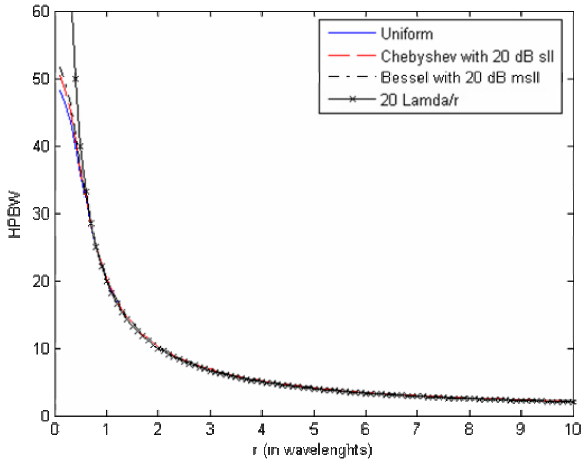

Fig. 12. HPBW of a cylindrical array with $Nc = 5$, $Nv = 10$ and $dv = 0.5\lambda$ vs. the radius various current distributions.

Fig. 12 shows that for $r > \lambda$, the HPBW has the form C / r, where C is a constant. Finding C that minimizes the mean squared error with the simulated values, yields $C = 20$. Hence, the HPBW can be closely approximated by $20\lambda/r$. The simulations showed that this approximation is valid for cylindrical arrays with $Nv = 10$ and dv=0.5λ irrespective of the value of Nc. Hence, we conclude that we can use the current distribution that leads to the desired directivity or side lobe level, without any concern about the HPBW, since the HPBW doesn't depend on the current distribution of the linear arrays nor on the number of elements of the circular arrays. For small radii ($r < \lambda$), the uniform cylindrical array has the smallest HPBW, followed by the Chebyshev then the Bessel cylindrical arrays. This is due to the fact that decreasing the side lobe level broadens the main beam and hence increases the HPBW.

The same method was used to find the HPBW of uniform concentric circular arrays and of coaxial cylindrical arrays where the distribution along each concentric circular array is uniform, and the distribution along each linear array is uniform, Chebyshev with 20 dB side lobe level (sll), or Bessel with 20 dB maximum side lobe level (msll). Simulation results for coaxial cylindrical arrays are shown in Fig. 13. The HPBW of the coaxial cylindrical arrays with the various current distributions overlap exactly with each other. In fact, Fig. 13 shows that this overlap is also with uniform concentric circular arrays with the same Nc. Hence, the HPBW of coaxial cylindrical arrays is completely determined by that of the concentric circular arrays that constitute the coaxial cylindrical array.

V. Conclusion

Hybrid linear and circular antenna arrays were presented. They include concentric circular, cylindrical and coaxial cylindrical antenna arrays. Their array factors were computed. The array factor of a cylindrical array was found to be equivalent to the product of the array factor of a linear array by that of a circular array. This is a very important property that allows cylindrical arrays to combine the properties of linear and circular arrays. The directivity and HPBW were computed. Closed form solutions, except for very special cases, were hard to find. Instead, simulations were conducted and closed form expressions were found by curve fitting. The proposed

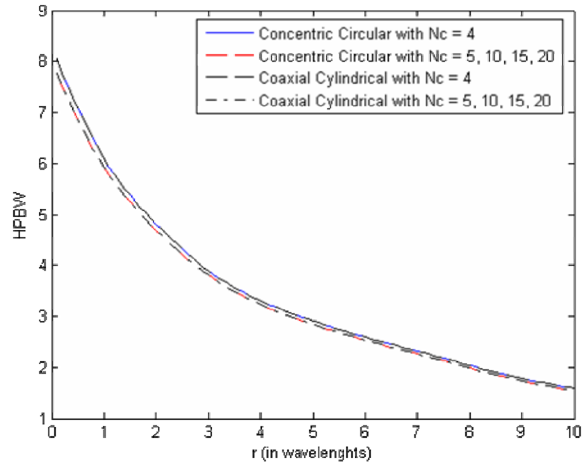

Fig. 13. Comparison of the HPBW of concentric circular arrays with $Nr = 10$, $Nr = 10$ and $dr = 0.5\lambda$ and the HPBW of coaxial cylindrical arrays with $Nv = Nr = 10$ and $dr = dv = 0.5\lambda$ vs. the radius for different values of Nc.

arrays show good directivity properties. For a given number of elements on the linear arrays, the HPBW of cylindrical arrays depends only on the radius of the circular arrays that constitute the cylinder. The elements in the vertical direction that constitute linear arrays have the role of enhancing the directivity. The obtained results imply that the proposed arrays are promising and deserve further investigation. Cylindrical arrays could be used, for example, to increase user capacity and coverage in 3G cellular systems where beam steering is used, since circular arrays would provide 360 degrees coverage in the azimuth plane, and stacking them in a cylindrical structure would increase the directivity.

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