Intelligent Control of Inter-Area Oscillations in a Multimachine Network Employing LMI Based Wide Area TCSC Controller

D. Mondal, A. Sengupta, and A. Chakrabarti

Abstract—This paper proposes a Linear Matrix Inequality (LMI) based $H_\infty$ robust controller design method for enhancement of damping of inter-area oscillations in a multimachine power system network. A Four-input, Single-output $H_\infty$ controller is designed for a Thyristor Controlled Series Compensator (TCSC) employing Wide Area Measurement (WAM) based stabilizing signals as generator speed. The major concern of signal transmission delay in wide area measurement is overcome here using the novel concept of Synchronized Speed Phasor Measurement (SSPM) and Global Position Satellite System (GPS) technology. The controller design has been carried out based on $H_\infty$ mixed-sensitivity formulation in an LMI framework with pole-placement constraint. The performance of the controller has been examined employing eigenvalue as well as time domain analysis in the face of different operating scenarios of a 14 area real power system network comprising of 24 generators, 203 buses and 266 lines. The designed controller has been found to be robust against varying generation and load power demand as well as for transmission line outage.

Index Terms—$H_\infty$ robust controller, inter-area oscillations, linear matrix inequality, small signal stability, thyristor controlled series compensator, wide area measurement.

I. INTRODUCTION

The problem of inter-area oscillations (0.2-1.0 Hz) is a long-standing issue in electric power systems. These oscillations may sustain and grow up to cause severe system outage if adequate damping is not available [1]. Traditionally, potential benefits of using Power System Stabilizer (PSS) and Flexible AC Transmission System (FACTS) devices are well recognized [2], [3] to damp these oscillations for enhancing power system stability. Out of many FACTS devices, [4] Thyristor Controlled Series Compensator (TCSC) has been proven to be robust and effective means [5] for mitigation of small signal oscillations in long transmission lines of modern power systems.

It is well known that the design and synthesis of conventional damping controllers are simple but these controllers lack robustness even after optimal tuning. The ever expanding nature of the power systems and its rapid upgrade to smart grid technology aggravates the situation further. In order to ensure intelligent communication and control with varying operating conditions, there is a requirement of optimal and robust controller at critical transmission and distribution nodes.

The mixed-sensitivity based LMI approach using $H_\infty$ control techniques has been applied in [6] to design an inter-area damping controller employing Superconducting Magnetic Energy Storage (SMES). A multiple-input, single-output (MISO) robust controller has been designed in [7] for a TCSC to improve the damping of inter-area modes employing global stabilizing signals. A centralized multivariable-control algorithm is implemented in [8] for damping multiple swing modes employing remote feedback signals considering delay in signal transmission. The design of an adaptive Wide Area Control System (WACS) has been proposed in [9] to compensate wide range of communication delays and to provide robust damping in migrating system oscillations. A systematic and practical means to design a WAMS based HVDC damping control system has been reported in [10] in order to enhance damping of certain inter-area modes in a power system.

The developments of Wide Area Measurement (WAM) technologies using Synchronized Phasor Measurement (SPM) [11]-[12] have brought new opportunities of damping of inter-area oscillations in power systems. Inclusion of these SPM units as additional and remote inputs in feedback control loops, for PSS and FACTS devices, allows a coherent picture of the entire networks in real time [13]. A wide-area signals based hierarchical control has been successfully utilized in [14] to enhance power system stability and security. In [15] it has been reported that a combination of both SPM and FACTS technologies can result in very powerful potential for real time sensing, monitoring and counteracting the effect of large disturbances in power systems.

The earlier applications of SPM technology are basically based on the phasor measurement of bus voltage, current or frequency. But the concept of Synchronized Speed Phasor Measurement (SSPM) using GPS signal for designing wide area damping controllers has not been explored in the existing literatures. Moreover, design of a WACS considering signal transmission delay requires complicated mathematical model [8] and in several literatures [7] controller design was carried out assuming a constant overall transportation delay, which seems inaccurate in practical point of view. Therefore, there is a strong need to develop new controllers which are robust and can use synchronized input signals from remote nodes and have satisfactory contribution on control of inter-area oscillations. A Four-input, Single-output (MISO) robust controller has been designed in [6] to design an inter-area damping controller employing SMES. A multiple-input, single-output (MISO) robust controller has been designed in [7] for a TCSC to improve the damping of inter-area modes employing global stabilizing signals. A centralized multivariable-control algorithm is implemented in [8] for damping multiple swing modes employing remote feedback signals considering delay in signal transmission. The design of an adaptive Wide Area Control System (WACS) has been proposed in [9] to compensate wide range of communication delays and to provide robust damping in migrating system oscillations. A systematic and practical means to design a WAMS based HVDC damping control system has been reported in [10] in order to enhance damping of certain inter-area modes in a power system.

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Manuscript received November 11, 2011; revised June 11, 2012.

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Publisher Item Identifier S 1682-0053(12)1981
oscillations. This paper addresses this problem and a synchronized speed phasor measurement based multi-input \( H_n \) controller for a TCSC has been designed in an LMI framework which ensures satisfactory settling of inter-area oscillations following possible disturbances e.g., change in load, generation drop and transmission line outage. Once the designs and simulations have been performed, the next credible step would be to implement the closed-loop control and for this requirement a simple feedback control scheme has been proposed in this paper.

The paper is organized as follows; Section II describes the general small signal modeling of a multimachine system with a TCSC. A brief description of the study system has been given in Section III where part of the network is shown indicating location of the TCSC. Section IV explains the configuration of an SSPMU and speed measuring technology. The theory of mixed-sensitivity based robust controller design in LMI framework followed by design of robust damping controller for TCSC has been described in Section V and subsequently robust performance of the controller has been examined. Finally, Section VI presents a closed-loop feedback control scheme for intelligent control of inter-area oscillations.

II. MULTIMACHINE MODEL WITH TCSC

The small signal model of a multimachine system with IEEE-Type I exciter has been described in [16]. All equations relating to the performance of the machine with exciter, PSS and network power flow were linearized around a nominal operating condition to obtain the dynamic model of the system for eigenvalue analysis and are represented by the following state-space equations

\[
\Delta \dot{X} = A \Delta X + B \Delta \alpha + D \Delta V_g + E \Delta U
\]

(1)

\[
0 = C_1 \Delta X + D_1 \Delta \alpha + D_2 \Delta V_g
\]

(2)

\[
0 = C_2 \Delta X + D_3 \Delta \alpha + D_4 \Delta V_g + D_5 \Delta V_i
\]

(3)

\[
0 = D_6 \Delta V_g + D_7 \Delta V_i
\]

(4)

here (1) and (2) represent the linearized differential equations and linearized stator algebraic equations of the machine, while (3) and (4) correspond to the linearized network equations pertaining to the generator buses and the load buses. The variable \( \Delta X \) contains machine states and the states corresponding to the PSS.

The installation of a TCSC results in addition of state variables corresponding to the TCSC power flow equations in the general network (4). The basic model of a TCSC (Fig. 1) utilizes the concept of a variable series reactance \( X_{TCSC} \) which can be adjusted through appropriate variation of the firing angle \( \alpha \) in order to allow the specified amount of active power flow across the series compensated line. The linearized TCSC equivalent reactance can be obtained by the following relationship [17]

\[
\Delta X_{TCSC} = \{-2C_1(1+\cos(2\alpha)) + C_2 \sin(2\alpha)(\sigma \tan(\sigma(\pi-\alpha)) - \tan \alpha)
\]

\[
+ C_3(\sigma^2 \cos^2(\pi-\alpha) - \cos(\sigma(\pi-\alpha))))\Delta \alpha
\]

(5)

The TCSC linearized power flow equations at the node \( 's' \) can be obtained from the following expressions

\[
0 = \begin{bmatrix}
\frac{\partial P}{\partial \theta_s} & \frac{\partial P}{\partial V_s} & \frac{\partial P}{\partial \alpha} \\
\frac{\partial Q}{\partial \theta_s} & \frac{\partial Q}{\partial V_s} & \frac{\partial Q}{\partial \alpha}
\end{bmatrix} \begin{bmatrix}
\Delta \theta_s \\
\Delta V_s \\
\Delta \alpha
\end{bmatrix}
\]

(6)

where

\[
P_s = V_s^2 g_s - V_s^2 (g_s \cos \theta_s + b_s \sin \theta_s)
\]

(7)

and

\[
Q_s = -V_s^2 b_s - V_s^2 (g_s \sin \theta_s - b_s \cos \theta_s)
\]

(8)

similarly, the equations for node \( 'i' \) can be obtained by using \( 'i' \) in place of \( 's' \).

Here, admittance

\[
Y_a = \frac{1}{R_a + j(X_a - X_{TCSC})}
\]

(9)

Eliminating \( \Delta I_a \) from (1)-(3), the overall system matrix for an \( m \)-machine system with a TCSC can be obtained as

\[
\begin{bmatrix}
A_{TCSC} \end{bmatrix}_{(m+1)\times(m+1)} = \begin{bmatrix} A' & B' \end{bmatrix} \begin{bmatrix} D' \end{bmatrix}^T \begin{bmatrix} C' \end{bmatrix}
\]

(10)

where

\[
A' = A_t - B_t D_t^{-1} C_t , \quad B' = \begin{bmatrix} B_1 & B_2 D_2^{-1} D_2 & 0 \end{bmatrix}
\]

\[
C' = \begin{bmatrix} K_2 \\ 0 \end{bmatrix} , \quad D' = \begin{bmatrix} K_1 & D_1 \\ D_0 & D_1 \end{bmatrix}
\]

with

\[
K_1 = [D_4 - D_3 D_3^{-1} D_2] , \quad K_2 = [C_2 - D_3 D_3^{-1} C_1]
\]

In the following section this system matrix (10) has been used for computation of eigenvalues and small signal stability analysis of the proposed test system.

III. DESCRIPTION OF THE TEST SYSTEM

The power system under consideration (Fig. 2) is one of the largest power network of Eastern India. It is a 14 area, 24 machine system consisting of 203 buses with 266 branches. It has 108 numbers 132 kV lines, 30 numbers 220 kV lines, 15 numbers 400 kV lines, and 6 numbers 66 kV lines. The whole network includes 35 numbers 3-windings line transformers and 37 numbers 2-windings load transformers. There are 6 generators (#1, #2, #3, #5, #17, and #20) having higher capacity (540 MW-600 MW) while 8 generators (#4, #6, #7, #10, #11, #12, #13, and
The proposed system has a total 168 eigenvalues without PSS and TCSC dynamics for the base case. Here 23 eigenvalues are identified as swing modes among which 11 have the frequency range 0.2-1.0 Hz and are represented in Table I. It is evident from this table that the damping ratio of the swing mode #4 is smallest compared to other swing modes and thus referred to as the critical swing mode ($\lambda_D$). The behavior of this mode is of prime concern for the study of small signal oscillation problem of the system. The mode frequency and right eigenvector analysis suggest that this mode is an inter-area mode involved with almost all machines and in particular it has strong association with machines #4, #13, #24, and #20 which are belong to four different areas (Fig. 2). The TCSC has been installed in a line between bus #152 and #154. It is to be noted that TCSC has been placed with the line associated with the highest load bus #154 and is nearer to the machine #20 which has highest participation in the critical mode of interest. The initial value of firing angle ($\alpha$) of the TCSC is kept within the capacitive zone with compensation of the TCSC being 56%. The size of the TCSC has been specified in the Appendix.

### Table I

<table>
<thead>
<tr>
<th>#</th>
<th>Swing modes</th>
<th>Frequency ($f$)</th>
<th>Damping ratio ($\zeta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.0431 ± j6.0689</td>
<td>0.9659</td>
<td>0.4482</td>
</tr>
<tr>
<td>2</td>
<td>-3.3906 ± j5.8905</td>
<td>0.9375</td>
<td>0.4988</td>
</tr>
<tr>
<td>3</td>
<td>-3.3215 ± j5.7429</td>
<td>0.9140</td>
<td>0.5006</td>
</tr>
<tr>
<td>4</td>
<td>-1.0363 ± j4.3800</td>
<td>0.6971</td>
<td>0.2302</td>
</tr>
<tr>
<td>5</td>
<td>-3.4131 ± j5.0370</td>
<td>0.8016</td>
<td>0.5609</td>
</tr>
<tr>
<td>6</td>
<td>-3.2044 ± j4.9464</td>
<td>0.7872</td>
<td>0.5437</td>
</tr>
<tr>
<td>7</td>
<td>-3.8535 ± j4.0865</td>
<td>0.6503</td>
<td>0.6860</td>
</tr>
<tr>
<td>8</td>
<td>-3.2680 ± j4.5548</td>
<td>0.7249</td>
<td>0.5829</td>
</tr>
<tr>
<td>9</td>
<td>-3.3041 ± j4.3813</td>
<td>0.6973</td>
<td>0.6019</td>
</tr>
<tr>
<td>10</td>
<td>-3.3031 ± j4.4201</td>
<td>0.7034</td>
<td>0.5986</td>
</tr>
<tr>
<td>11</td>
<td>-2.9858 ± j2.5168</td>
<td>0.4005</td>
<td>0.7646</td>
</tr>
</tbody>
</table>

#19) are having medium capacity (150 MW-380 MW) and rest 10 are of low capacity (20 MW-90 MW). All machines are assumed to be equipped with IEEE Type –I excitation system and speed input single-stage power system stabilizer to ensure adequate damping of local modes. All loads are assumed to be of constant power type. The nodal voltage magnitudes and angles are obtained in the simulation by the conventional N-R load flow while a separate subprogram has been incorporated at the end of each iteration to update the state variables of the TCSC.

## IV. CONCEPT OF SYNCHRONIZED SPEED PHASOR MEASUREMENT UNIT (SSPMU)

In synchronized phasor measurement, by synchronizing the sampling processes for different signals which may be acquired from hundreds of miles apart, it is possible to put their phasors on the same phasor diagram. Fig. 3 shows a functional block diagram of a typical Synchronized Speed Phasor Measuring Unit (SSPMU). The digital speed signals derived from the Proximity Sensor (PS) (Fig. 4) are applied to the SSPMU which uses synchronization signals from GPS satellite system. The received GPS signals are then converted to a suitable Intermediate frequency (IF) in the front end of the receiver, and then processed by microprocessor. The GPS receiver provides the 1 pulse-per-second (pps) signal and a time tag, which consists of the year, day, hour, minute, and second. The 1-pps signal is...
usually divided by phase-locked oscillator into the required number of pulses per second for sampling of the analog signals. The microprocessor determines the positive sequence phasors according to the recursive algorithm [18], and the timing message from GPS system. The computed string of phasors, one for each of the positive sequence measurements, is assembled in a Phasor Data Concentrator (PDC) and this data stream from PDC is then transmitted over a dedicated communication line through the modems [19]. A 4800 or 9600 baud communication line can support this transmission of the phasor stream at the rate of about every 2-5 cycles (40-100 msec) of the fundamental frequency (50 Hz), depending upon the number of positive sequence phasors being transmitted. Considering that the usual power system dynamics fall in the range of 0.2 to 2.5 Hz, it is possible to observe in real-time the power system dynamic phenomena with high fidelity at the control centre.

The SSPMU may be suitable to install in both substations and in power plants to measure not only the voltage and current phasors but also it can be utilized to measure the rotor speed (ω). The digital data generated by the proximity sensor is applied to the SSPMU. Once accurate timing data and the navigation message is available to the SSPMU through GPS receiver, the microprocessor can complete its required tasks.

The most common technique for determining the phasor representation is to use sampled data taken from the input signals, and apply the Discrete Fourier Transform (DFT) to compute the phasor, if \( m_k \) \((k=1, 2, ..., N)\) are \( N \) samples of the input speed signal taken over one period, then the phasor representation is given by [12]

\[
M = \frac{\sqrt{2}}{N} \sum_{n=1}^{N} m_n e^{-\frac{2\pi n t}{T}} \tag{11}
\]

where \( M \) is the phasor. Consider the case at the instant \( t = 0 \) (Fig. 4) is the time tag of the measurement started. The SSPMU must then provide the phasor given by (11) using the sampled data of the input signal.

For continuous measurement each new sample of speed signal is acquired, a new phasor is obtained with a data window including the latest sample.

The most efficient method of dealing with continuous monitoring of the sampled data input is to use a recursive form of the phasor equation [18]. It should be noted that since sampled data are used to represent the input signals, it is essential to include antialiasing filters to the analog input signals before data samples are taken in order to meet Nyquist criterion.

V. MIXED-SENSITIVITY BASED ROBUST CONTROLLER DESIGN: AN LMI APPROACH

The design objective in this paper is to find an internally stabilizing damping controller that satisfies an infinity norm constraint, while ensuring that the closed-loop poles lie in a certain location in the complex plane. The closed-loop system together with the \( H_c \) controller based on the standard mixed-sensitivity problem is proposed in Fig. 5. In this figure \( G(s) \) is the open-loop plant, \( K(s) \) is the controller to be designed, and \( W_1(s) \) and \( W_2(s) \) are weights for shaping the characteristics of the closed-loop plant. The input signal to the controller is the normalized speed deviation \( (\Delta \theta) \), and output signal is the deviation in thyristor conduction angle \( (\Delta \sigma) \).

The state space description of the augmented plant is represented by [20]

\[
\dot{x} = A_k x + B_k u \tag{12}
\]

\[
z = C_k x + D_k u \tag{13}
\]

\[
y = C_k x + D_k u \tag{14}
\]

where \( x \) is the state vector of the plant, \( u \) is the plant input, \( y \) is the measured signal modulated by the disturbance input \( d \), \( z \) is the controlled output. The controller \( K(s) \) can be realized by the following state space equations

\[
\dot{\chi} = A_k \chi + B_k d \tag{15}
\]

\[
u = C_k \chi + D_k d \tag{16}
\]

The state space representation of the closed-loop plant is then given by

\[
\dot{x} = A_{cl} \chi + B_{cl} d \tag{17}
\]

\[
z = C_{cl} \chi + D_{cl} d \tag{18}
\]

where

\[
A_{cl} = \begin{bmatrix} A_k & 0 \\ B_k & 0 \end{bmatrix}, \quad B_{cl} = \begin{bmatrix} B_k & B_k C_k \\ B_k & 0 \end{bmatrix}, \quad C_{cl} = \begin{bmatrix} 0 & B_k \\ 0 & 0 \end{bmatrix}
\]

Without loss of generality, \( D_{cl} \) can be set to zero to make the derivation simpler.

The transfer function between \( d \) to \( z \) can be described as

\[
T_{ud} = \begin{bmatrix} W_1(s)S(s) \\ W_2(s)K(s)S(s) \end{bmatrix} = C_{cl} (s I - A_{cl})^{-1} B_{cl} + D_{cl} \tag{19}
\]

The objective of the mixed-sensitivity problem is to find an internally stabilizing controller \( K(s) \) that minimizes the transfer function between \( d \) to \( z \) and meets the following requirement [21]

\[
\| F_{ud}\| < \gamma \tag{20}
\]

where \( \gamma \) is the bound on \( H_\infty \) norm and \( S(s) \) is the sensitivity transfer function. In an LMI formulation, the
equivalent objective (20) can be achieved in the sub-optimal sense if there exists a solution \( X_{cl} = X_{cl}^T > 0 \) such that the bounded real lemma [22] given by

\[
\begin{bmatrix}
A_{cl}^T X_{cl} + X_{cl}^T A_{cl} & B_{cl}^T & X_{cl}^T \hat{C} \\
B_{cl} & -I & D_{cl}^T \\
C_{cl}^T X_{cl} & D_{cl} & -\gamma^T I
\end{bmatrix} < 0 \tag{21}
\]

is satisfied and the resulting controller design problem reduces to an LMI problem. Pole clustering in LMI regions can be formulated as an LMI optimization problem. It has been reported in [23] that the state matrix, \( A_{cl} \) of the closed loop system, has all its poles inside the conical sector (Fig. 6) if and only if there exists \( X_{cl}^T > 0 \) such that

\[
\begin{bmatrix}
\sin \left( \frac{\theta}{2} (A_{cl}^T X_{cl} + X_{cl}^T A_{cl}) \right) & \cos \left( \frac{\theta}{2} (A_{cl}^T X_{cl} - X_{cl}^T A_{cl}) \right) \\
\cos \left( \frac{\theta}{2} (X_{cl}^T A_{cl} - A_{cl}^T X_{cl}) \right) & \sin \left( \frac{\theta}{2} (A_{cl}^T X_{cl} + X_{cl}^T A_{cl}) \right)
\end{bmatrix} < 0 \tag{22}
\]

or equivalently this can be expressed in Kronecker product form given by

\[
\eta \otimes A_{cl}^T X_{cl} + \eta^T \otimes X_{cl}^T A_{cl}^T < 0 \tag{23}
\]

where

\[
\eta = \begin{bmatrix}
\sin \frac{\theta}{2} & \cos \frac{\theta}{2} \\
-\cos \frac{\theta}{2} & \sin \frac{\theta}{2}
\end{bmatrix}
\]

The inequalities in (21) and (23) are not jointly convex as the solutions \( X_{cl} \neq X_{cl}^T \). The convexity can be accomplished by seeking a common solution, \( X_{cl} = X_{cl}^T = X_{cl}' \). It is to be noted that the inequalities in (21) and (23) contain non-linear terms \( A_{cl}^T X_{cl} \) and \( C_{cl}^T X_{cl} \) (\( A_{cl} \) and \( C_{cl} \) contain unknown matrices of the controller in (15)-(16)) and the resulting problem therefore cannot be handled by LMI optimization directly. To convert the problem into a linear one, a change of controller variables is necessary and the transformation [23], [24] gives the following simplified LMI's in terms of new controller variables

\[
\begin{bmatrix}
Q & I \\
I & S
\end{bmatrix} > 0 , \begin{bmatrix}
\Pi_{11} & \Pi_{12} \\
\Pi_{12}^T & \Pi_{22}
\end{bmatrix} < 0
\]

\[
[\eta \otimes \Psi + \eta^T \otimes \Psi^T] < 0
\]

where

\[
\Pi_{11} = \begin{bmatrix}
A_{cl} Q + DA_{cl} & B_{cl} \hat{C} + \hat{C}^T B_{cl} & B_{cl} + B_{cl} \hat{D} D_{cl} \\
(B_{cl} + B_{cl} \hat{D} D_{cl})^T & -\gamma I
\end{bmatrix}
\]

\[
\Pi_{12} = \hat{A} + (A_{cl} + B_{cl} \hat{D} C_{cl}) \hat{C} \quad \hat{B}_{cl} + \hat{B}_{cl} \hat{D} \hat{D}_{cl}
\]

\[
\Pi_{22} = A_{cl} S + SA_{cl} + \hat{B}_{cl} C_{cl} + C_{cl} \hat{B}_{cl}^T (C_{cl} + D_{cl} \hat{D} C_{cl})^T \\
C_{cl}^T + D_{cl} \hat{D} C_{cl}
\]

\[
\Psi = \begin{bmatrix}
A_{cl} Q + B_{cl} \hat{C} & A_{cl} + B_{cl} \hat{D} C_{cl} \\
\hat{A} & SA_{cl} + \hat{B}_{cl} C_{cl}
\end{bmatrix}
\]

The new controller variables are then defined as

\[
\hat{A} = N A_{cl} M^T + N B_{cl} C_{cl} Q + S B_{cl} C_{cl} M^T \tag{24}
\]

\[
\hat{B} = N B_{cl} + S B_{cl} D_{k} \tag{25}
\]

\[
\hat{C} = C_{cl} M^T + D_{k} C_{cl} Q \tag{26}
\]

\[
\hat{D} = D_{k} \tag{27}
\]

where \( Q, S, N \), and \( M \) are submatrices of the solution \( X_{cl} \). The matrices \( A, \hat{B}, \hat{C}, \) and \( D \) are solved form the LMIs employing interior-point optimization methods [25]. Once \( \hat{A}, \hat{B}, \hat{C}, \) and \( \hat{D} \) are obtained the controller variables \( A_k, B_k, C_k, \) and \( D_k \) are recovered by solving (24)-(27).

**A. Design of Robust Damping Controller**

The LMI formulations described in Section V are now applied to the 14 area, 24 machine study system. Here an \( H_{\infty} \) FISO controller was designed for the TCSC using four input signals, machine speed \( (\Delta \omega) \) from four different remote locations of the study system (Fig. 2). The resulting system has a total of 193 states including one state for the
### TABLE II

<table>
<thead>
<tr>
<th>Power system disturbances</th>
<th>Critical inter-area mode</th>
<th>Damping ratio</th>
<th>Critical inter-area mode</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load increase 40% more than nominal</td>
<td>-0.75328 ± j4.9973</td>
<td>0.14905</td>
<td>-2.7942 ± j6.6364</td>
<td>0.38805</td>
</tr>
<tr>
<td>Gen. drop total 40% (Gen #2, 3, 5, &amp; 20)</td>
<td>-0.1615 ± j4.9112</td>
<td>0.0328</td>
<td>-2.9344 ± j8.2818</td>
<td>0.3339</td>
</tr>
<tr>
<td>Line outage #42-53, #118-122, &amp; #145-149</td>
<td>0.06065 ± j3.9016</td>
<td>-0.0015</td>
<td>-3.0273 ± j7.1389</td>
<td>0.3904</td>
</tr>
</tbody>
</table>

The weights $W_i(s)$ and $W_j(s)$ are worked out to be

$$W_i(s) = \frac{2}{s + 1.5}, \quad W_j(s) = \frac{0.5s + 1}{0.25s + 1}$$

The multi-objective $H_\infty$ synthesis program for disturbance rejection and control effort optimization features of LMI was accessed by suitably chosen arguments of the function hinfinx of the LMI Toolbox in MATLAB. The pole placement objective in LMI has been achieved by defining the conical sector with $\theta/2 = 67.5^\circ$, which provides a desired minimum damping $\zeta = 0.39$ for all the closed-loop poles. The order of the controller obtained from the LMI solution was equal to the reduced plant order plus the order of the weights, which was quite high (18-th order) posing difficulty in practical implementation. Therefore, the controller was reduced to a seventh-order one by the balanced truncation without significantly affecting the frequency response. The state variable representation of the four-input, one-output controller for the TCSC is given in the Appendix. This reduced-order controller has been tested on the full-order system against varying generation, load power change and transmission line outage.

### B. Performance Evaluation of the Controller

The performance of the controller is evaluated here in the face of large variations of system disturbances, that include real and reactive load increase (40% more than nominal) in selected buses, drop in real power generations (Total 40%) in some designated generators buses and simultaneous tripping of three transmission lines (#42-53, #118-122, and #145-149). It has been observed that the WAM TCSC controller exhibits good damping characteristics in all these cases of contingencies. The damping ratio of the critical inter-area mode without and with controller has been represented in Table II.

It is evident from this table that simultaneous occurrence of three tie-line outage pushes the critical inter-area mode to the right half of the $s$-plane resulting instability of the system. In this situation installation of the WAM controller shows significant improvement of damping and brings back the system under stable operating condition establishing the need of robust TCSC controller.

The dynamic behavior of the system has also been investigated for a simulation time 10 sec. The angular speed response of Generator #20 with different power system disturbances has been plotted in Fig. 7. It is visible that the application of the LMI based WAM TCSC controller introduces a remarkable improvement on system oscillations and provide a reasonable settling time (2-3 sec) for all three cases of disturbances. A view of the Fig. 7(c) shows that the designed controller achieves a higher level of damping for the tie-line outage oscillations compared to the case of load increase and generation drop.
VI. CLOSED-LOOP CONTROL SCHEME OF INTER-AREA OSCILLATIONS

The objective in this section is to model a closed-loop monitoring and control system using WAM based four-input, single-output TCSC controller for mitigation of inter-area oscillations in the proposed study system. It has been assumed that the four control inputs of the controller (generators speed) are measured from four generating stations. The proposed possible schematic diagram of WAM based control system has been depicted in a block diagram (Fig. 8). The measurement mechanism of remote substations variable, machine speed \( \Delta \omega \) is assumed to be based on SPM with Global Position Satellite System (GPS) technology. To accomplish this, the transmission system needs to install four numbers of SSPMU for collection of auxiliary control inputs, rotor speed data corresponding to the machines (#4, #13, #24, and #20) have high participation in inter-area oscillations and are located in four different areas or substations. Generator speed signals measured by the Proximity Sensor (PS) from four areas are fed to the Synchronized Speed Phasor Measuring Units (SSPMU). The synchronized speed signals are then modulated for transmission through the wide area communication network. The Ethernet or high speed fibre optic link may suitable for this wide area communication. In the receiving end of the centralize control station transmitted signals are demodulated and filtered out to input to the TCSC controller. The controller produces output signal, thyristor conduction angle \( \sigma \) which introduces additional damping to the inter-area mode executing TCSC reactance in phase with the speed difference of the generators.

A novel concept of Synchronized Speed Phasor Measuring (SSPM) technology has been introduced in order to access remote input signals and centralize control of inter-area oscillations which may overcome the problem of transportation delay in wide area measurement.

A feedback control scheme has been proposed for application of the wide area controller in order to realize the existing power system control into intelligent control system.

The proposed technique can be implemented for the design of other FACTS controllers for any multimachine power system.

APPENDIX

A. Parameters of PSS and TCSC Module

\[ K_{PSS} \text{ (Gain)} = 10 \]
\[ T_{PSS} \text{ (Lead time)} = 0.4 \text{ sec} \]
\[ T_{LSS} \text{ (Lag time)} = 0.15 \text{ sec} \]
\[ X_1 \text{ (TCSC)} = 0.000526 \text{ pu} \]
\[ X_2 \text{ (TCSC)} = 0.00526 \text{ pu} \]
\[ X_{TCSC} = 0.0130 \text{ pu} \]
Firing angle \( \alpha \) = 155 deg.
\[ T_{TCSC} \text{ (TCSC delay)} = 17 \text{ msec} \]

B. LMI Based WAM TCSC Controller

The state-space representation \( (A_{K_{TCSC}}, B_{K_{TCSC}}, C_{K_{TCSC}}, D_{K_{TCSC}}) \) of the four-input, single-output WAM controller for the TCSC

\[
\begin{bmatrix}
11.886 & 9.1837 & 7.5342 & 1.7663 \\
249.64 & -201.2 & -734.31 & -1463.8 \\
0.4711 & 0.2125 & 0.5264 & 0.1518 \\
& & & \\
0.1889 & 0.3467 & 0.4495 & 0.2202 \\
1.160 & -9.6148 & 3.2768 & 0.3184 \\
2.0539 & 1.5970 & -1.0910 & -2.5106 \\
1.6676 & 0.1484 & 0.2874 & 0.2088 \\
11.886 & 9.1837 & 7.5342 & 1.7663 \\
249.64 & -201.2 & -734.31 & -1463.8 \\
0.4711 & 0.2125 & 0.5264 & 0.1518 \\
& & & \\
0.1889 & 0.3467 & 0.4495 & 0.2202 \\
1.160 & -9.6148 & 3.2768 & 0.3184 \\
2.0539 & 1.5970 & -1.0910 & -2.5106 \\
1.6676 & 0.1484 & 0.2874 & 0.2088 \\
\end{bmatrix}
\]

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A \_K = \frac{\text{TCSC}}{\text{FACTS}} = \begin{bmatrix}
-0.24504 & -1854.6 & -0.02038 & 0.01947 & -0.29701 & 0.10962 & 0.04946 \\
1836.20 & -2429.8 & -2.2824 & 1.79380 & -29.793 & 11.0130 & 4.98410 \\
-0.01863 & 1.1139 & -0.00620 & 0.03802 & -0.16872 & 0.06247 & 0.02840 \\
-0.01692 & 1.0728 & -0.03760 & -0.00604 & 0.19162 & -0.07132 & -0.03225 \\
0.08566 & 1.24420 & 0.00195 & 0.03433 & -24.094 & 18.1920 & 8.44730 \\
-0.04555 & -8.1658 & -0.00913 & 0.00526 & -0.8256 & -6.7779 & -5.9537 \\
-0.04074 & 0.22424 & -0.02213 & -0.01573 & -2.1844 & -0.05223 & -3.8335 \\
\end{bmatrix}

C \_K = \begin{bmatrix}
16.897 & -1668.7 & -0.7532 & 0.6376 & -10.229 & 3.7765 & 1.7115 \\
\end{bmatrix}

REFERENCES


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